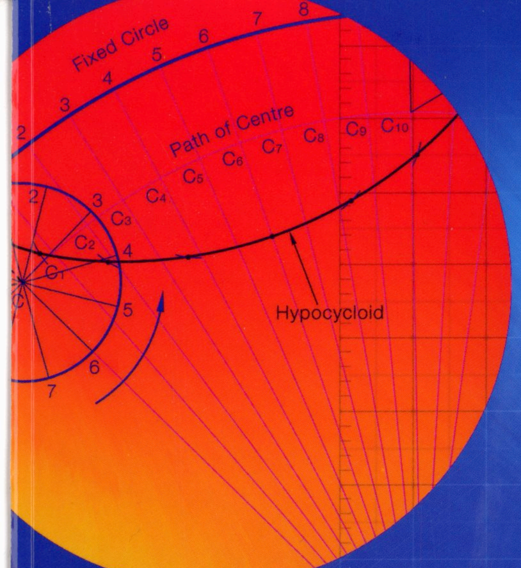
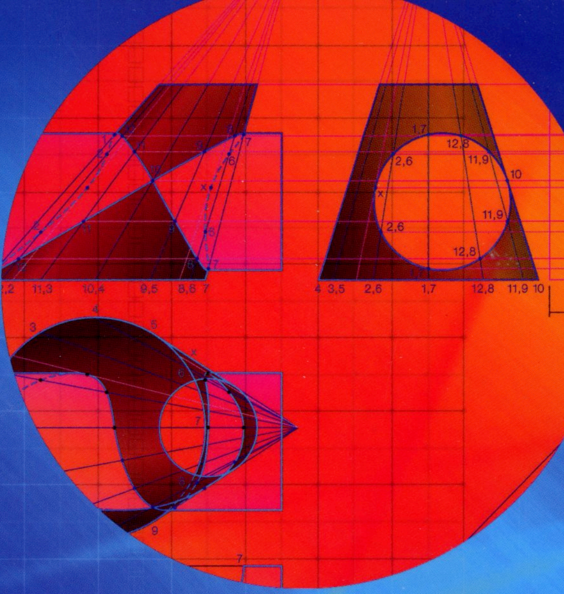
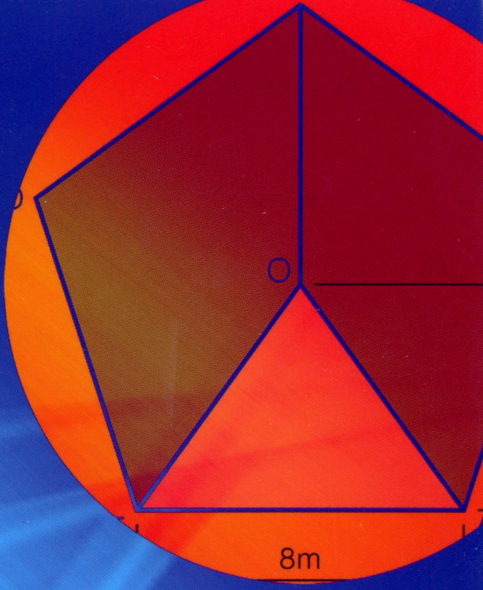
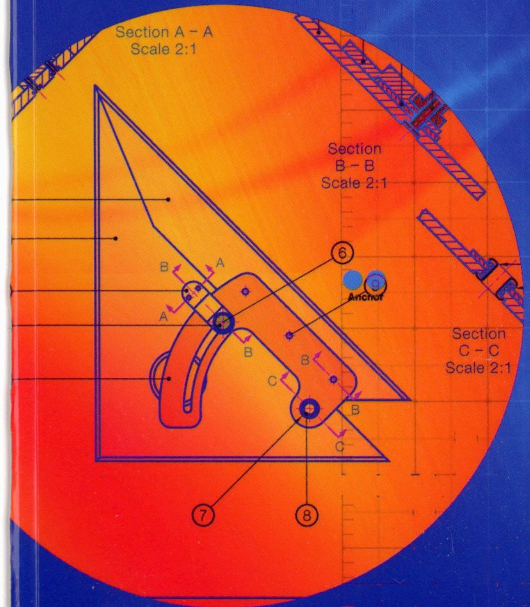


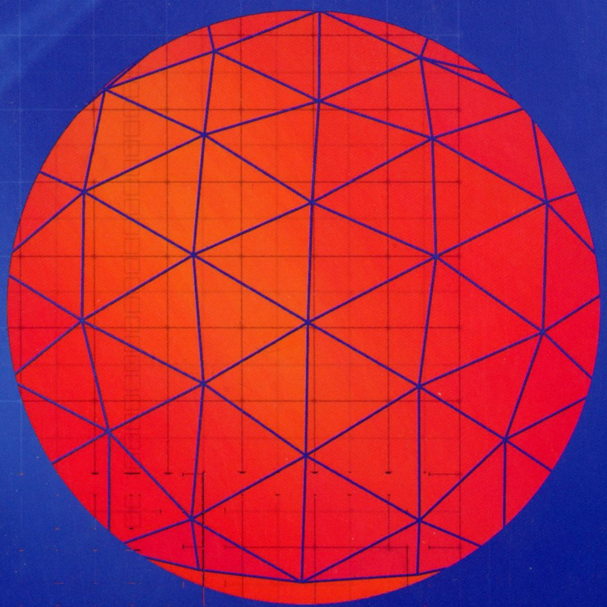


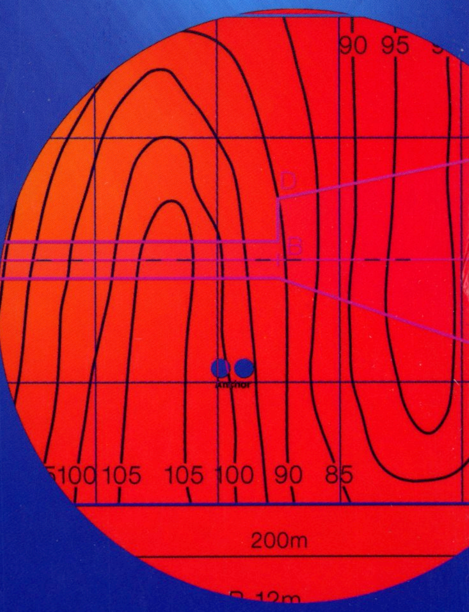


GRAPHICS IN DESIGN & COMMUNICATION 2

CAD AND APPLIED GRAPHICS







DAVID ANDERSON

18 Structural Forms

SYLLABUS OUTLINE

Areas to be studied (in an applied context):

• Structural forms, natural and manufactured. • Singly and doubly ruled surfaces. • The hyperbolic paraboloid as a ruled surface. • *The hyperbolic paraboloid as a surface of translation.* • *Plane directors.* • The hyperboloid of revolution, projections and sections. • Sections through singly and doubly ruled surfaces. • *The geodesic dome of not more than four points of frequency.*

Learning outcomes

Students should be able to:

Higher and Ordinary levels

- Investigate the development of structural forms in a historical context.
- Identify the key structural forms including arches, domes, vaults, frames and surface structures.
- Produce line drawings of the basic structural forms.
- Produce two-dimensional drawings of arches, domes, vaults and surface structures.
- Construct a hyperbolic paraboloid as a ruled surface.
- Determine the true shape of sections through curved surfaces.
- Project views and sections of a hyperboloid of revolution.

Higher level only

- *Relate the key properties of structural forms to their design and construction.*
- *Produce three-dimensional drawings of arches, domes, vaults and surface structures.*
- *Determine plane directors for ruled surfaces, and construct ruled surfaces given plane directors and directrices.*
- *Project views of a hyperbolic paraboloid defined as a surface of translation.*
- *Construct geodesic domes of not more than four points of frequency.*
- *Investigate and represent structural forms as they occur in the environment.*

In this chapter we will be looking at the historical development of some common structural forms including the arch, the dome and the vault. We will then move on to look at some structural forms of special interest, the hyperbolic parabaloid and the hyperboloid of revolution.

Column and Beam

Using columns and beams is the simplest way to make an opening in a wall. The column or post is the vertical member and the beam is the horizontal member. The beam supports the weight (load) above it and its own weight. This weight is then transferred to the columns and from these to the lower structure. This type of construction was used in prehistoric times and is still used in modern day structures.

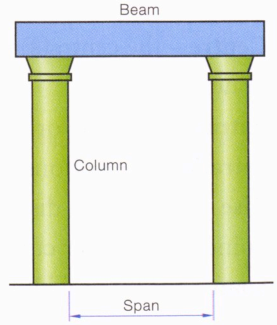


Fig. 18.1

Hyperbolic Paraboloid as a Surface of Translation

As has been explained earlier at the introduction to this topic, the hyperbolic paraboloid can be seen as a structure made up of straight line elements obeying certain rules or as a parabolic curve sliding on its vertex along an inverted parabolic curve. We will now look at some problems based on this type of model.

Fig. 18.80 shows a pictorial view of a shell structure. The surface of the structure is generated by translating the parabola ABC in a vertical position along the parabola BE whose vertex is at E.
Draw the plan and elevation of the structure.

Scale 1:200

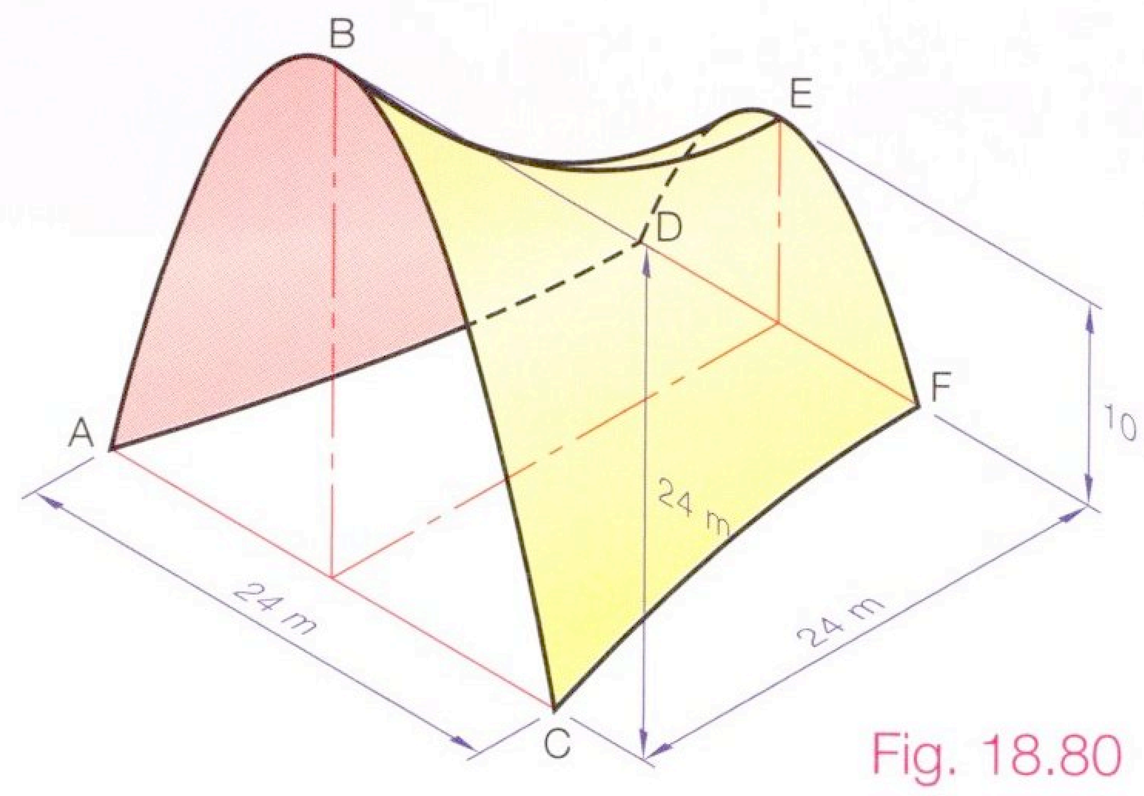


Fig. 18.80

(1) Draw the parabola ABC.
(2) In front elevation draw point B and E.
(3) Construct the parabola BE ensuring that the vertex is at E.
(4) All vertical sections will produce parabolas which are part of the ABC parabola. The end curve DEF is part of the ABC parabola. The width w is found in plan by stepping height h down from the top of parabola ABC giving width w.
(5) Curves AD and CF are hyperbolas and are constructed as explained earlier.

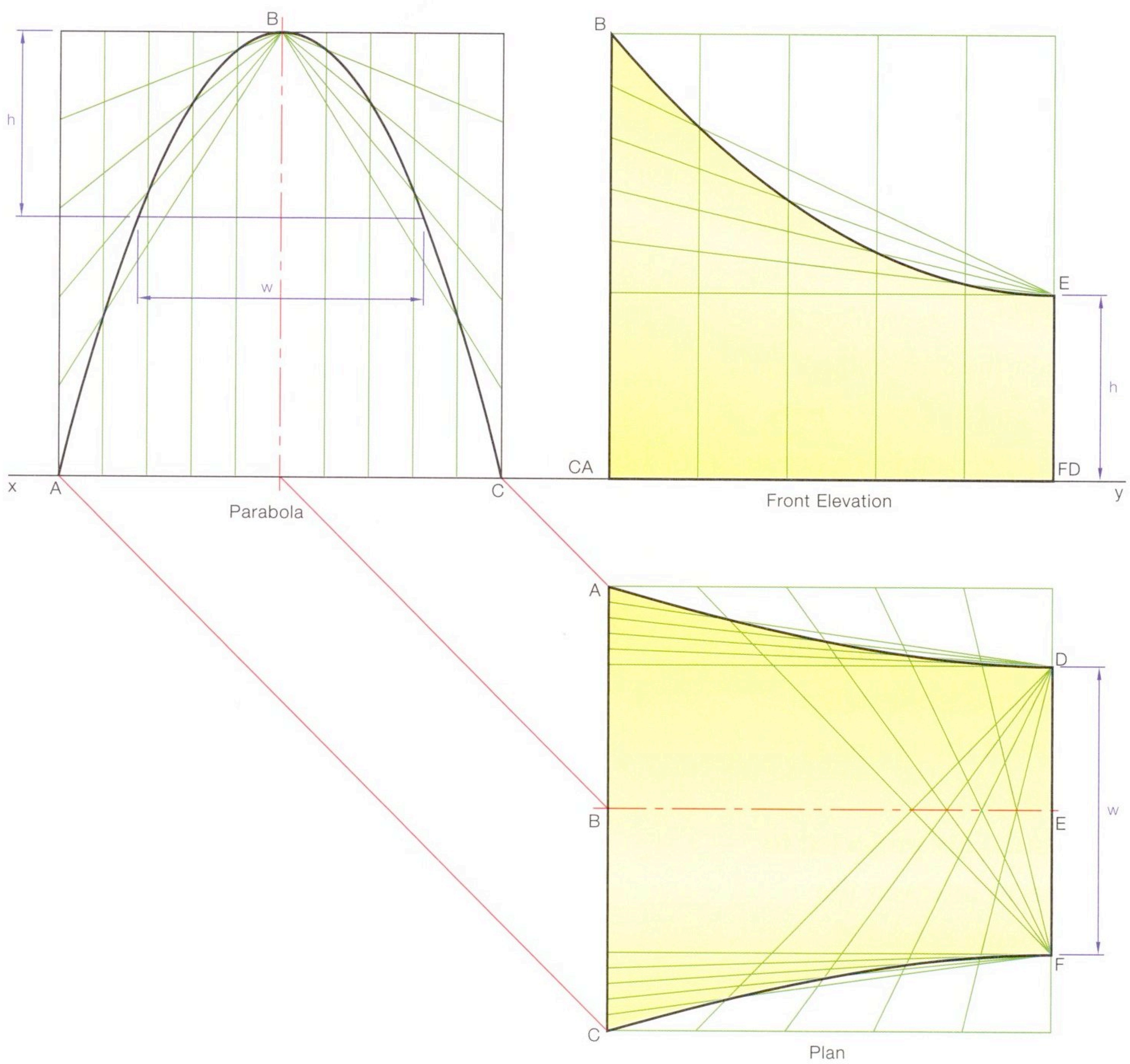


Fig. 18.81

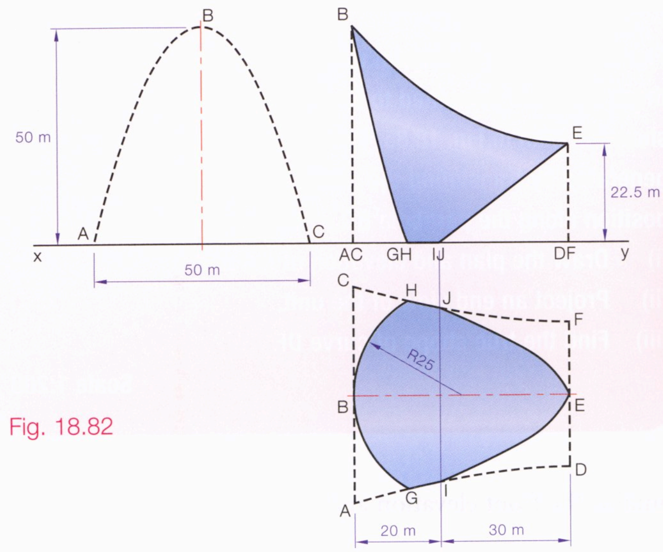


Fig. 18.82

Fig. 18.82 shows the plan and elevation of a shell structure which is in the form of a hyperbolic paraboloid. It is formed by sliding parabola ABC in a vertical position along the parabola BE whose vertex is at E. The shell has been cut as shown.

(i) Draw the plan and elevation of the unit.

(ii) Project an end view of the unit.

Scale 1:500

(1) Draw the parabola ABC.

(2) Construct the parabola BE in elevation having its vertex at E.

(3) In the plan, the width of the end DEF is not given and must be found. Take the height of E in elevation and step it **down** from the top of parabola ABC. This gives the **width** of DF in plan.

(4) The left side of the plan can be completed and the right side of the elevation.

(5) w_1 and w_2 are taken from plan, stepped out from the axis of parabola ABC to find h_1 and h_2 which are stepped down from BE.

(6) h_3 and h_4 are taken from elevation, stepped down from the vertex of parabola ABC to find w_3 and w_4 which find points in the plan.

(7) The end view is projected from front elevation and plan.

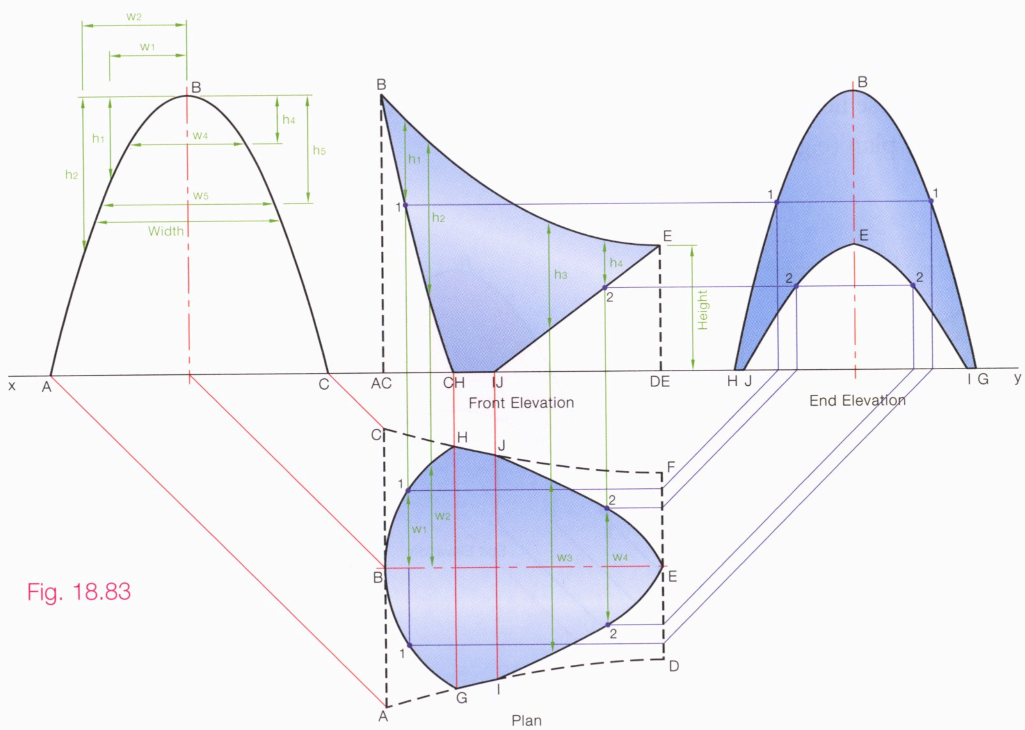


Fig. 18.83

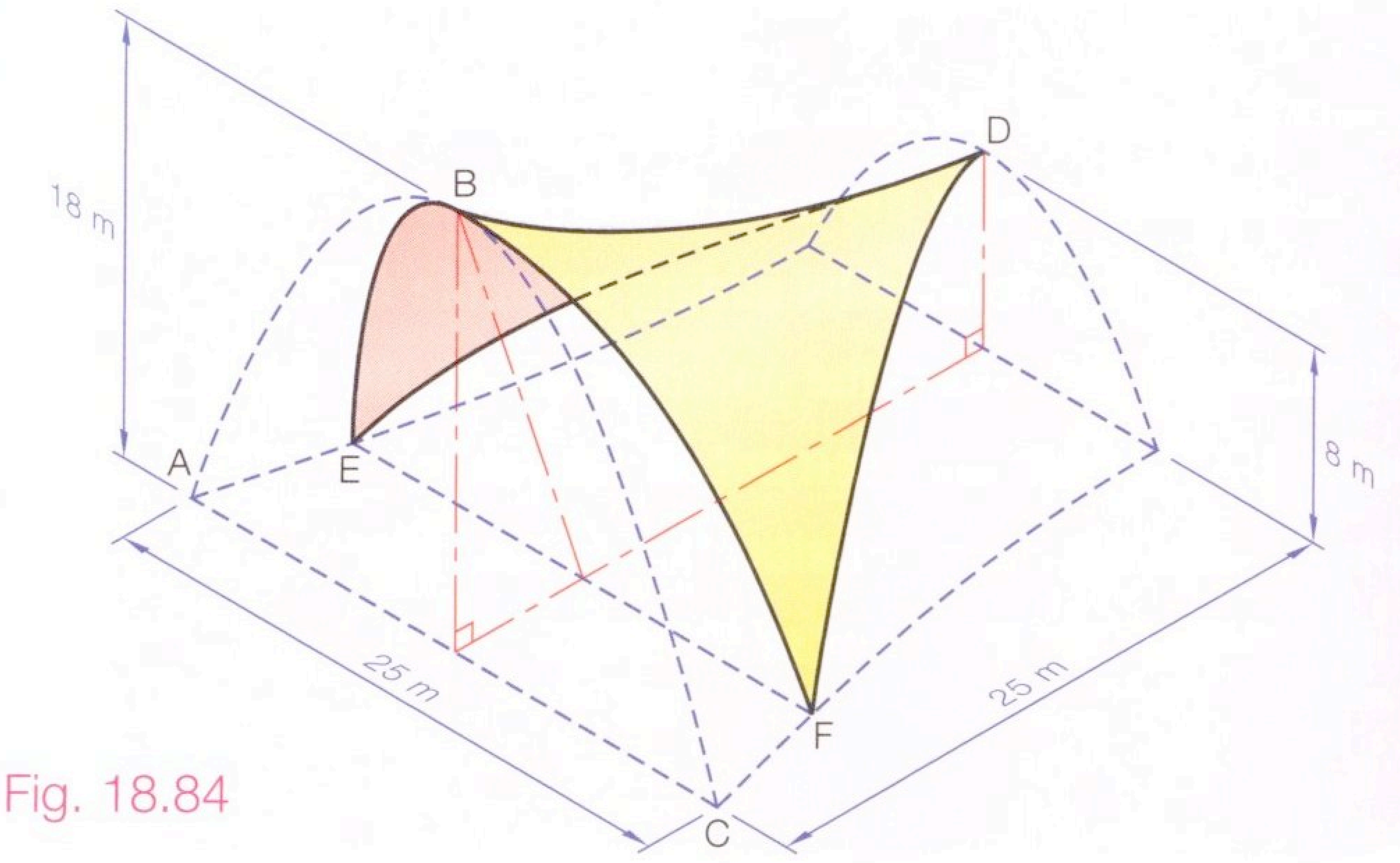


Fig. 18.84

Fig. 18.84 shows a pictorial view of a shell structure. Six of these units are combined to form a total roof surface as shown in plan in Fig. 18.85. The surface of the unit is generated by translating the parabola ABC in a vertical position along the parabola BC whose vertex is at D.

(i) Draw the plan and elevation of the unit.

(ii) Project an end view of the unit.

(iii) Find the true shape of curve DF.

Scale 1:200

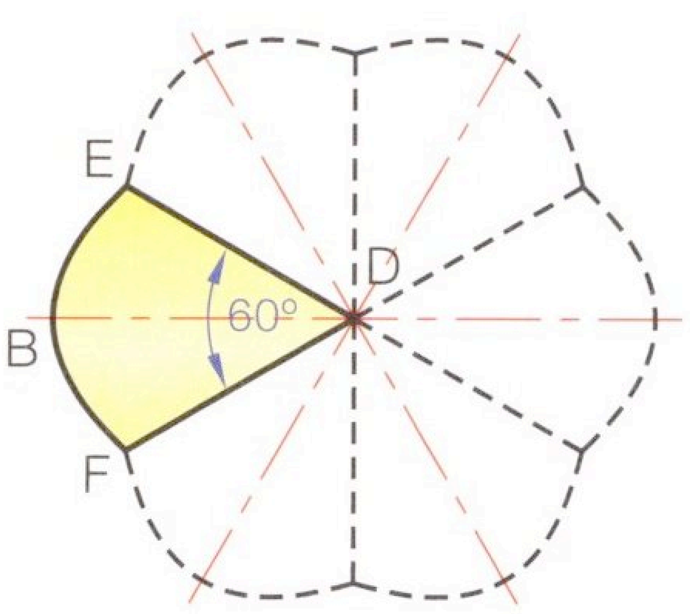


Fig. 18.85

(1) Draw parabola ABC.

(2) Draw the rectangle that contains the front elevation and construct the half parabola BD.

(3) Draw the plan of the uncut shell structure by taking heights from parabola BD to the xy line (e.g. H_1) and step these heights **down** from the top of parabola ABC to give widths (e.g. W_1) which are used in the plan.

(4) Cut the shell structure in plan to form an inclusive angle of 60° thus finding E and F.

(5) Project E and F to the xy line and join to B. The left of the front elevation is completed and the right of the plan is completed.

(6) Take heights in elevation from parabola BD to straight line BFE (e.g. H_2).

(7) Step these heights **down** from the top of parabola ABC to find widths which are used in the plan (e.g. W_2).

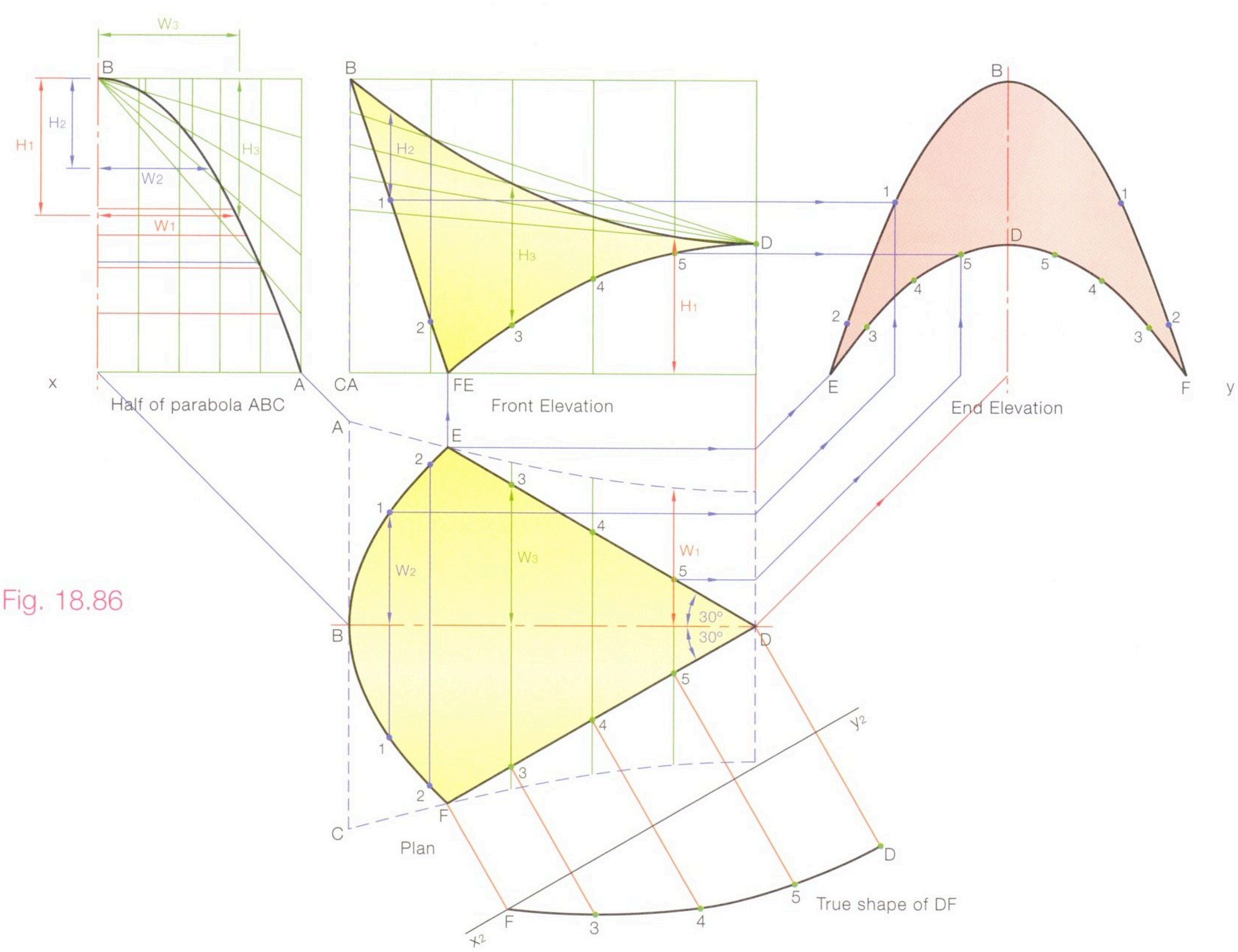


Fig. 18.86

(8) Widths are now taken in plan from the axis to line DE (e.g. W_3).
(9) Use these widths on the parabola ABC to find heights **from the top of the parabola down** to the curve (e.g. H_3).
(10) The end view is projected in the normal way.
(11) The true shape of curve DF is found by projecting an auxiliary elevation with $x_1 y_1$ parallel to line DF in plan.

Geodesic Domes

A geodesic dome is a type of structure shaped like a piece of a sphere. This structure is made up of a complex network of triangles that form a roughly spherical surface. The more triangles, the more closely the dome approximates the shape of a true sphere. The geodesic dome was invented by Buckminster Fuller in the late 1940s. He hoped to use such domes to improve the housing of humanity and envisaged that giant domes would cover whole cities.

Geodesic domes are light structures yet very rigid and inherently strong. They have no need for internal supports, as they are self-supporting and therefore leave the internal floor area completely open and unobstructed. They are attractive aesthetically and philosophically because you are getting more internal area using less building materials. There are, unfortunately, some severe practical difficulties in constructing these 'perfect buildings', which has limited their use to public spaces, exhibition halls and enthusiast projects. With so many edges and joints it is difficult and expensive to waterproof these domes. Rain seems to find a way in, no matter what precautions are taken. Furthermore, the fact that the internal surfaces are curved can lead to its own problems in a man-made world that favours rectangular, more modular furniture.

Spherical Geometry

The term 'geodesic' comes from the Greek *geo*, earth, and *daiesthai*, to divide. We have earth-dividing domes. For a sphere, the shortest distance between two points A and B on the surface, travelling along the sphere's surface, is called a **geodesic**. A geodesic is always part of the circumference of a circle which has its centre at the centre of the sphere. Such a circle is called a **great circle**. An unlimited number of great circles can be drawn on the sphere's surface. Other smaller circles can be drawn on the surface but their centres will not coincide with the sphere's centre. These lesser circles play little or no part in dome theory.

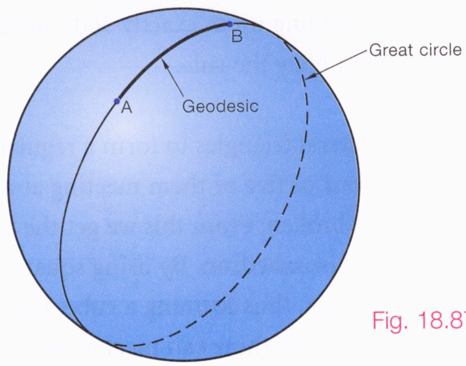
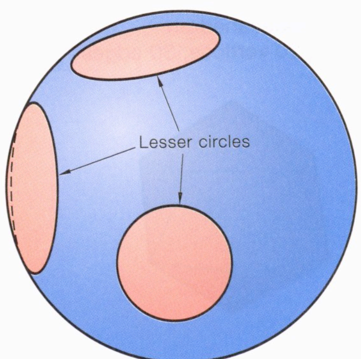


Fig. 18.87



Lesser circles

Fig. 18.88

The two points on a sphere that the geodesic line connects can also be connected by a chord that cuts through the sphere. Geodesic domes use these chords as struts. The struts form triangles and it is from these triangles that the dome forms its strength.

Before looking at the make-up of the triangles in a geodesic dome we must first look at much more simple shapes, on which the more complex geodesic shapes are based, **the platonic solids**.

The Five Regular Polyhedra

There are only five regular polyhedra: the tetrahedron, cube, octahedron, dodecahedron and icosahedron. For each of these solids all the faces are similar regular polygons, all the edges are equal in length and the same number of faces meet at every vertex.

Why only five regular polyhedra?

In any convex polyhedron the sum of the face angles at a vertex is always less than 360°.

In order to form a three-dimensional solid there must be a minimum of three faces meeting at any one vertex. By looking at the interior angles of regular polygons which can act as faces, it is evident that only three polygons may be used.

(1) Equilateral triangle with interior angles of 60°.
(2) Square with interior angles of 90°.
(3) Regular pentagon with interior angles of 108°.

A regular hexagon which is the next polygon in line has an interior angle of 120°. Three hexagonal faces meeting at a vertex will have interior angles adding up to exactly 360°, not less than 360° and therefore do not obey the rule.

By using equilateral triangles to form a regular polyhedron, there can be three, four or five of them meeting at a vertex, any more and the rule will be broken. From this we get the tetrahedron, octahedron and isosahedron. By using squares, there can only be three at each vertex, thus forming a cube. Finally, by using pentagons, again only three can meet at each vertex, thus forming a dodecahedron.

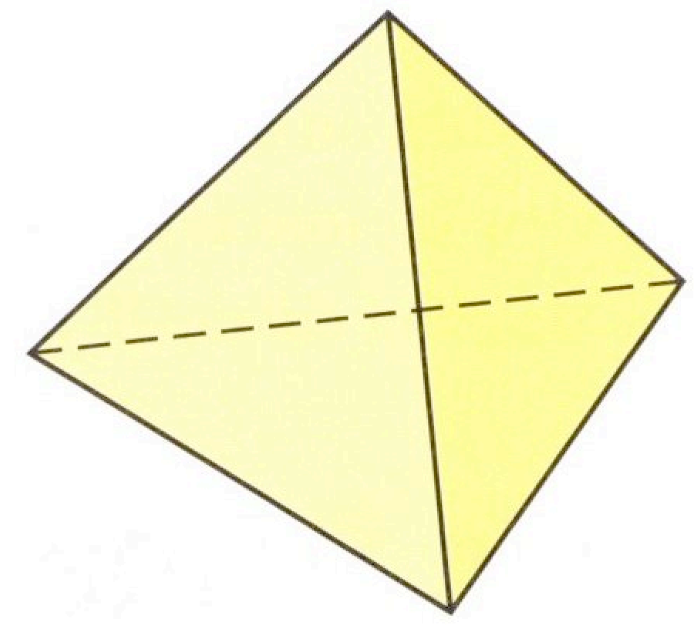


Fig. 18.89 Tetrahedron
4 faces, 4 vertices, 6 edges

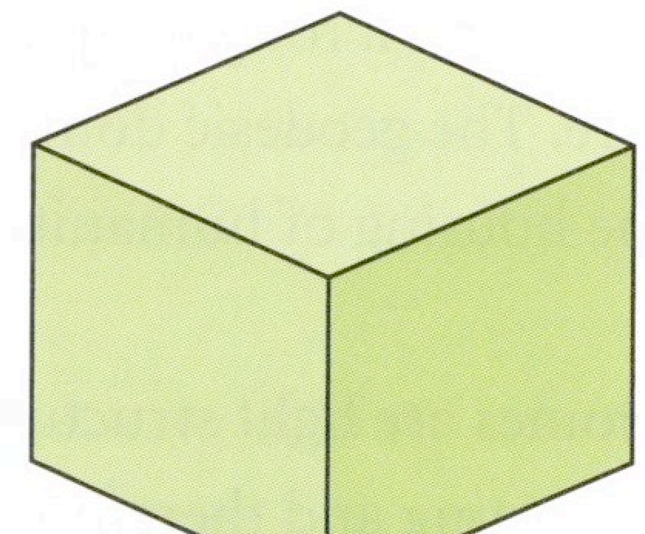


Fig. 18.90 Cube
6 faces, 8 vertices, 12 edges

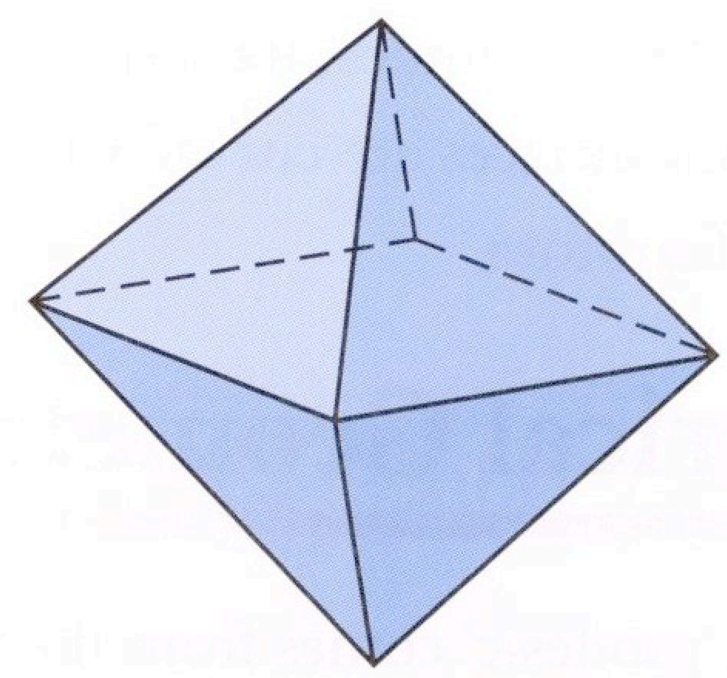


Fig. 18.91 Octahedron
8 faces, 6 vertices, 12 edges

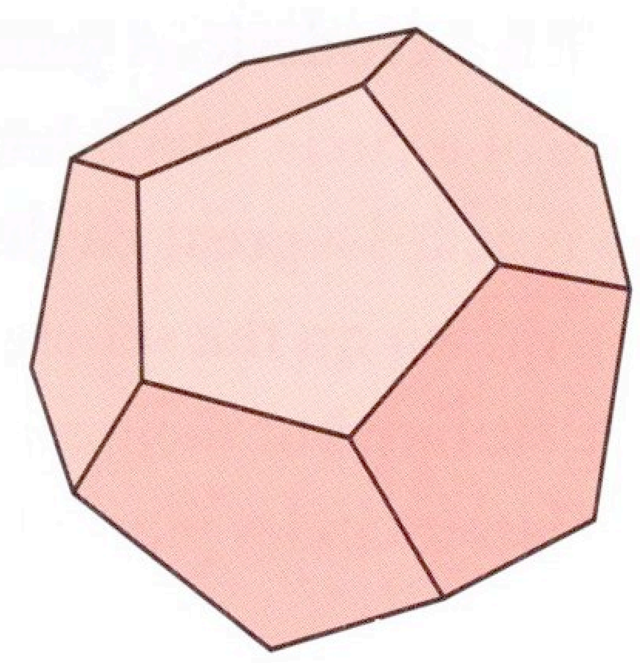


Fig. 18.92 Dodecahedron
12 faces, 20 vertices, 30 edges

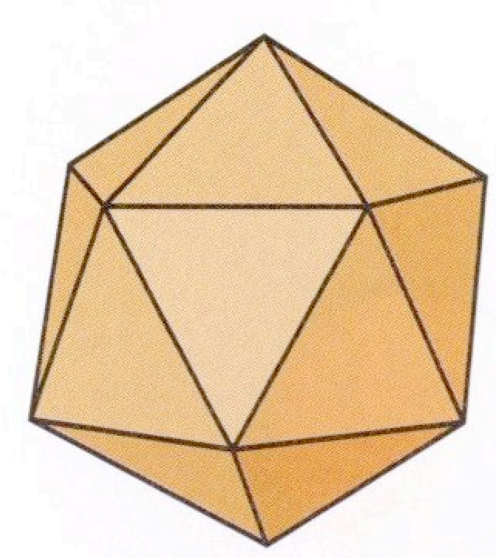


Fig. 18.93 Icosahedron
20 faces, 12 vertices, 30 edges

Faces, Vertices and Edges

There is obviously a relationship between the number of faces, vertices and edges for each of these regular polyhedra. This relationship was discovered by Leonhard Euler (1707–83).

Faces + Vertices – Edges = 2

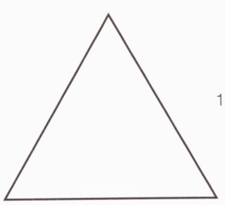
F + V – E = 2

This relationship stands true for the five regular polyhedra mentioned earlier, but also for any polyhedra, regular or not, which can be enclosed in a sphere having each vertex touching the sphere surface.
This formula is useful when working on domes because it can help to count vertices.

Platonic Solids and the Sphere

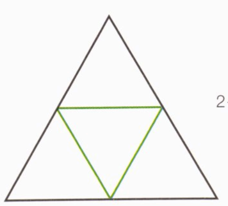
Each of the platonic solids can be circumscribed by a sphere, such that each vertex of the solid rests on the sphere surface and each edge of the surface forms a chord joining two such points. As such these five solids form the basis of geodesic dome construction. They are, however, poor approximations to a sphere. Of the five the tetrahedron, octahedron and icosahedron offer better stability because of their triangular make-up. If we look at the icosahedron which has 20 triangular faces, the vertices are all equidistant from the centre so they determine a sphere. If we subdivide each of the triangular faces into smaller triangles, then some of the vertices of the smaller triangles lie inside the sphere rather than on it. By pushing these points radially outwards from the icosahedron until they meet the sphere, we arrive at the vertices of our geodesic sphere. Obviously by pushing the corners of the smaller triangles out to the sphere surface we are both changing the lengths of the triangle sides and their angles.

Frequency of a Geodesic Dome



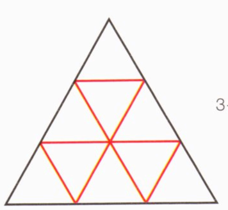
1-Frequency

Fig. 18.94



2-Frequency

Fig. 18.95



3-Frequency

Fig. 18.96

The frequency of a geodesic dome is the measure of the number of triangles into which each face is subdivided. A 1-frequency dome or sphere is just an icosahedron (for example) or part of an icosahedron whose faces have not been subdivided. For a 2-frequency dome, the sides of each icosahedral face are divided into two so that each face is divided into four smaller triangles. For a 3-frequency dome the sides are divided in three, so that the faces become nine smaller triangles and so on.

Fig. 18.97

The roundness of the dome is improved by dividing the edges into shorter lengths and raising more points to the surface of the sphere. Raising the midpoint of an edge to the sphere creates two shorter edges, both equal, which give a better approximation to the curvature of the sphere. Increasing the subdivisions to 3-frequency or 4-frequency produces better approximations and struts of varying lengths. The lengths of these struts may be calculated or can be found from tables.

Fig. 18.98

HIGHER LEVEL

Strut Factor and Tables

The lengths of the sides of the various triangles making up a dome may be calculated from tables. The tables are generally given based on a dome/sphere of one-metre radius. To make a dome of another radius it is simply a matter of multiplying the chord factor by the required radius.

Strut length = Dome/Sphere radius × Strut factor

1-FREQUENCY (ICOSAHEDRAL)

Strut	Strut factor	Req. for dome	Req. for sphere
A	1.05146	25	30

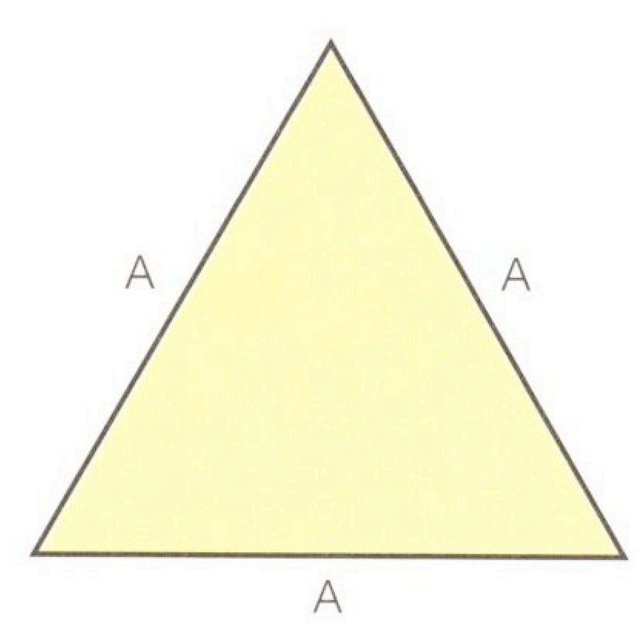


Fig. 18.99

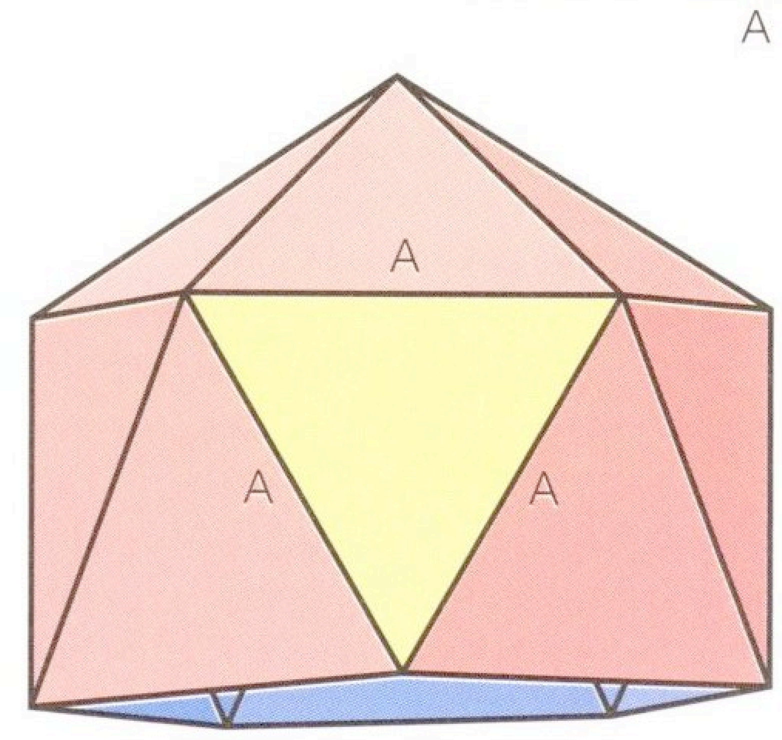


Fig. 18.100

Icosahedral dome
1-frequency

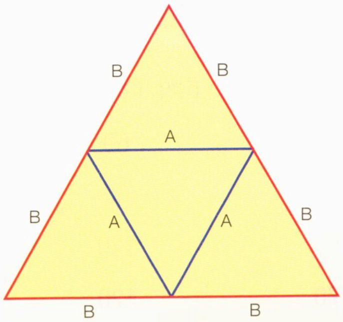
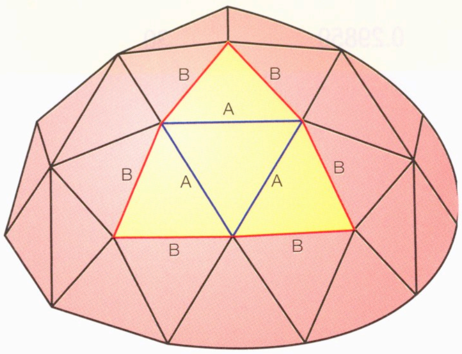


Fig. 18.101

2-FREQUENCY (ICOSAHEDRAL)

Strut	Strut factor	Req. for dome	Req. for sphere
A	0.61803	35	60
B	0.54653	30	60



Icosahedral dome
2-frequency

Fig. 18.102

3-FREQUENCY (ICOSAHEDRAL)

Strut	Strut factor	Req. for dome 3/8	Req. for dome 5/8	Req. for sphere
A	0.34862	30	30	60
B	0.40355	40	55	90
C	0.41241	50	80	120

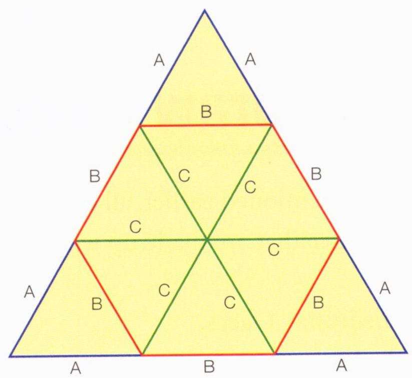


Fig. 18.103

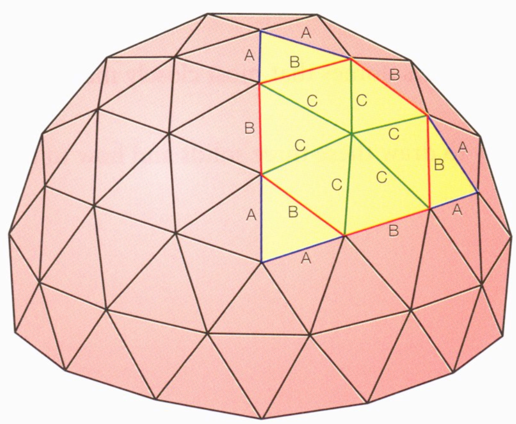


Fig. 18.104

5/8 Icosahedral dome
3-frequency

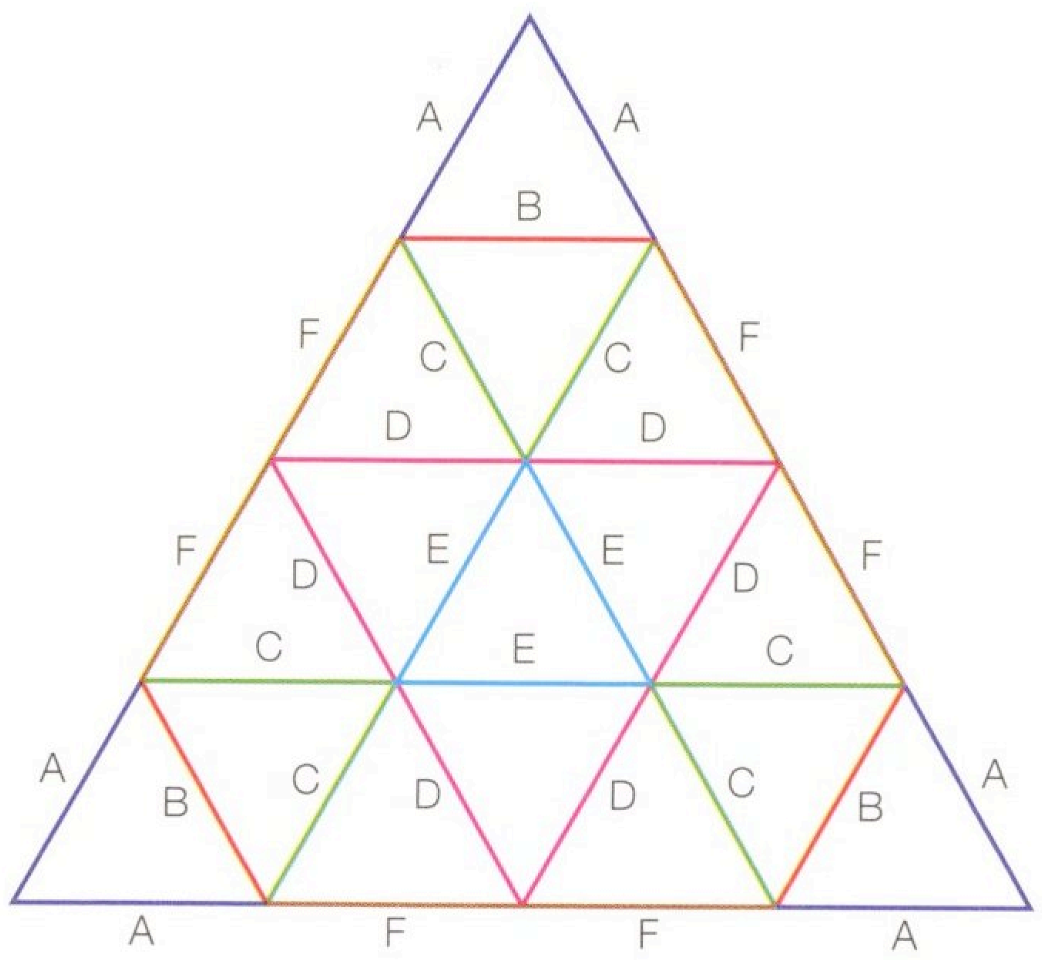


Fig. 18.105

4-FREQUENCY (ICOSAHEDRAL)

Strut	Strut factor	Req. for dome	Req. for sphere
A	0.25318	30	60
B	0.29524	30	60
C	0.29453	60	120
D	0.31285	70	120
E	0.32492	30	60
F	0.29859	30	60

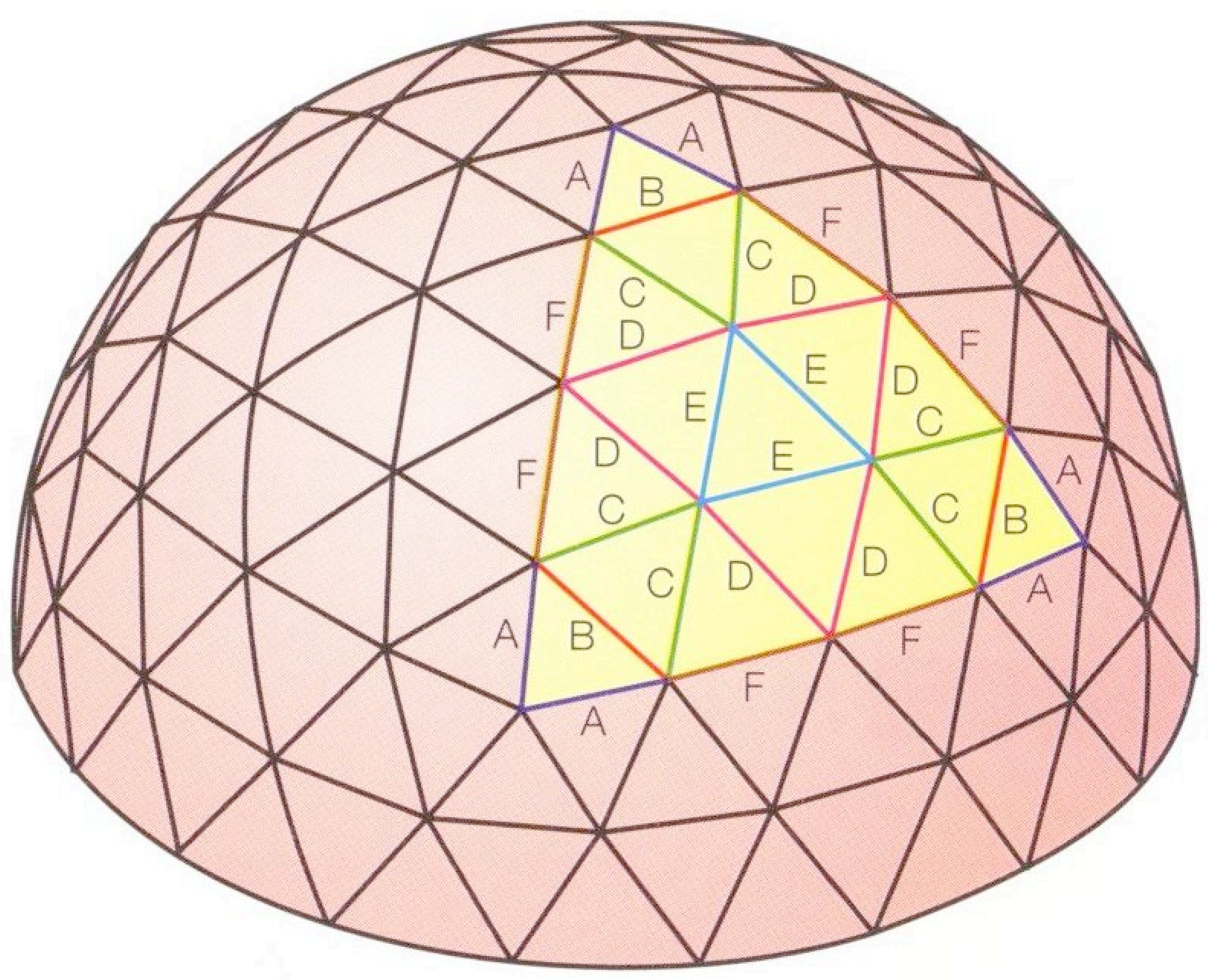


Fig. 18.106

Icosahedral dome
4-frequency

HIGHER LEVEL

How to Draw a Geodesic Dome

As has been mentioned earlier, all geodesic domes are derived from one or other of the platonic solids. Of these five solids, the three that are made up with triangles are most favoured:

the tetrahedron – 4 faces,
the octahedron – 8 faces,
the icosahedron – 20 faces.

The icosahedron produces domes that most closely match a sphere.

We first look at how to draw these three solids and how to find their circumscribing spheres.

Tetrahedron

(1) The plan of a tetrahedron resting on one of its faces is an equilateral triangle and its apex 0 is found by bisecting the angles.

(2) The end view can be used to find the tetrahedron's height as it shows edge 0,3 as a true length.

(3) Complete the front elevation.

(4) The centre of the circumscribing sphere lies on the axis of the solid and touches all vertices. In the end view, bisect true length 0,3 to intersect the axis through 0 to locate c. Draw the sphere in all views.

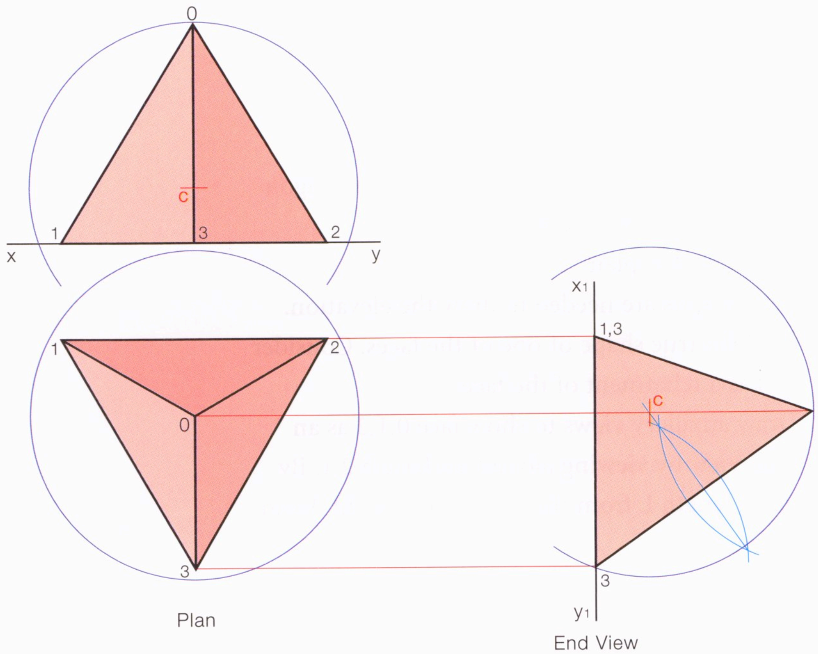


Fig. 18.107

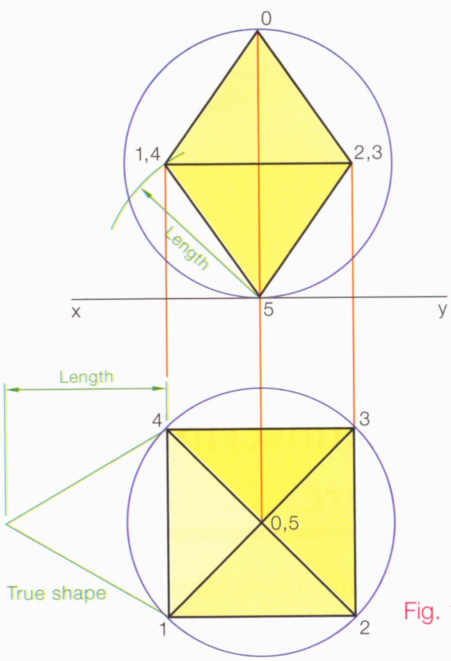


Fig. 18.108

Octahedron

(1) The plan of the octahedron appears as a square with the diagonals joined.

(2) The height for the elevation is found by drawing the true shape of one of the faces and using its length as shown to locate points 1 and 4.

(3) The circumscribing sphere is drawn in elevation and then in plan.

Icosahedron

(1) The plan of the top five triangles of an icosahedron having its apex facing straight up, will form a perfect pentagon. The sides of the pentagon equal the true length of each edge of the icosahedron and the edges leading up to the apex appear as spokes of this pentagon leading into the centre.

(2) Construct the pentagon in plan.

(3) Draw lines from each of the vertices of the pentagon to its centre to form the spokes.
(4) Find a starting point for the lower pentagon by extending one of these spokes, e.g. 3,0 to intersect a circumscribing circle about the pentagon. The vertices for the second pentagon will lie on the same circle.
(5) Complete the plan.
(6) Two heights are needed to draw the elevation. Find the true shape of one of the faces. Consider this as a rebatment of the face.
(7) Draw auxiliary views to show face 0,1,2 as an edge view by viewing along true length 2,1. By using length L from the true shape height A can be found.
(8) Similarly, an edge view of surface 1,2,3 will find height B.
(9) Construct elevation.

HIGHER LEVEL

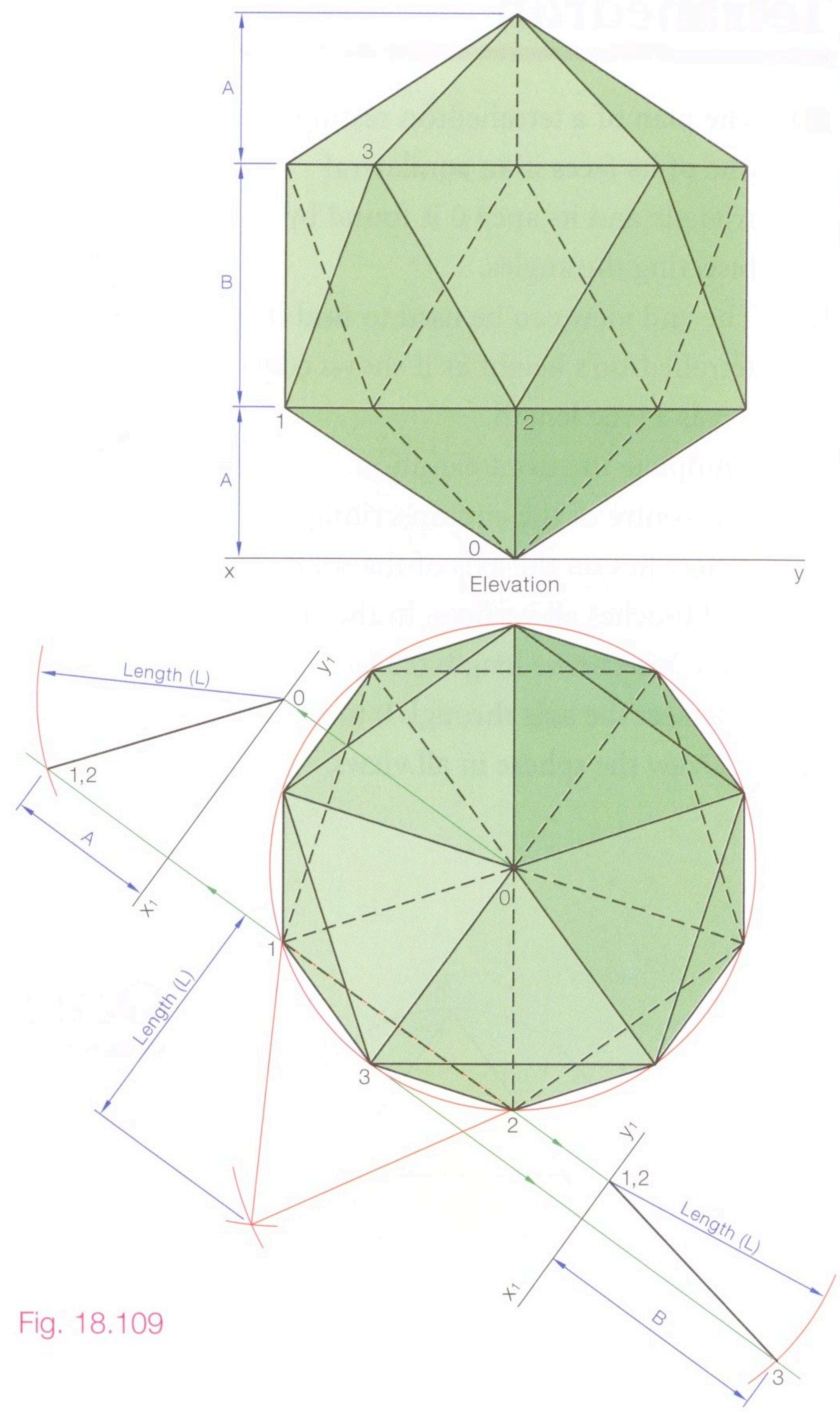


Fig. 18.109

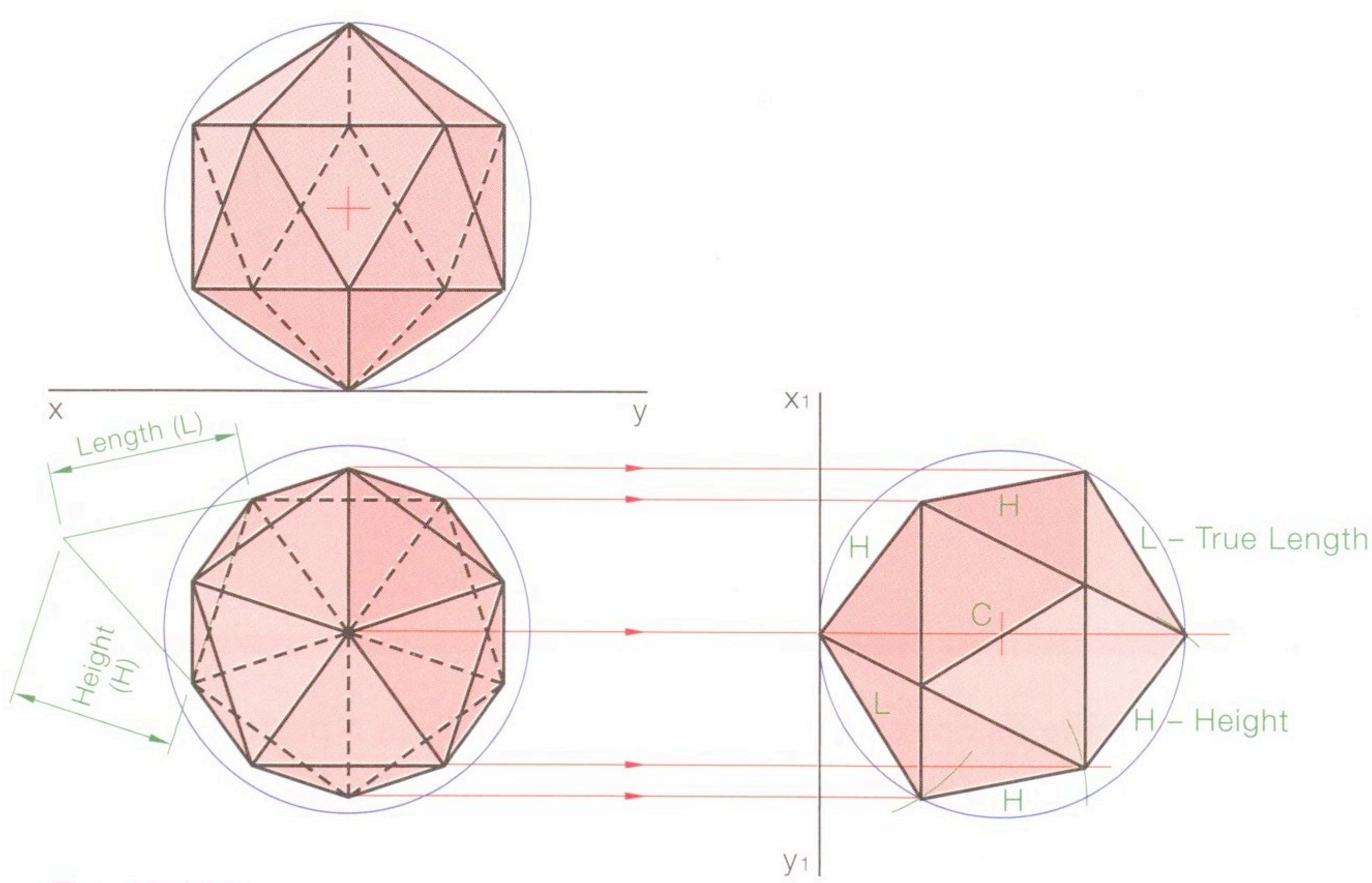


Fig. 18.110

Circumscribing Sphere

The circumscribing sphere can easily be found in elevation. If only a partial elevation is to be drawn, centre C can be found by projecting a view showing one face edge on and one edge as a true length. By bisecting the true length and extending to cross the axis centre, C is located.