4 Orthographic Projection of Solids in Contact

SYLLABUS OUTLINE

Areas to be studied:

- Right solids in contact.

Learning outcomes

Students should be able to:

Higher and Ordinary levels

- Construct views of up to three solids having curved surfaces and/or plane surfaces in mutual contact.
- Determine point of contact for surfaces in mutual contact.
- Construct views of solids given the point of contact.
- Depict the solutions of two-dimensional problems in three-dimensional format.

Higher Level only

- *Model various problems involving solids in contact, planes of reference and auxiliary planes.*

Solids in Contact

In this chapter you will draw the orthographic views of spheres, cones, cylinders, pyramids and prisms in contact with each other. Before this, we must examine how to find points on the surface of cylinders, cones and spheres.

The elevation of a cylinder is shown in Fig. 4.1 and the position of a point P on its surface. Draw the plan of the cylinder and point P.

By using an end view the location of point P in the plan can be easily found, see Fig. 4.2.

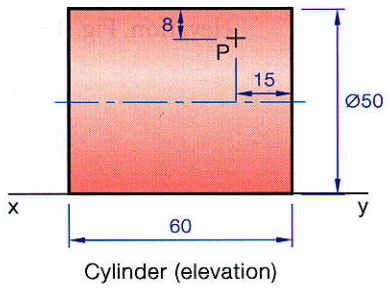


Fig. 4.1 Cylinder (elevation)

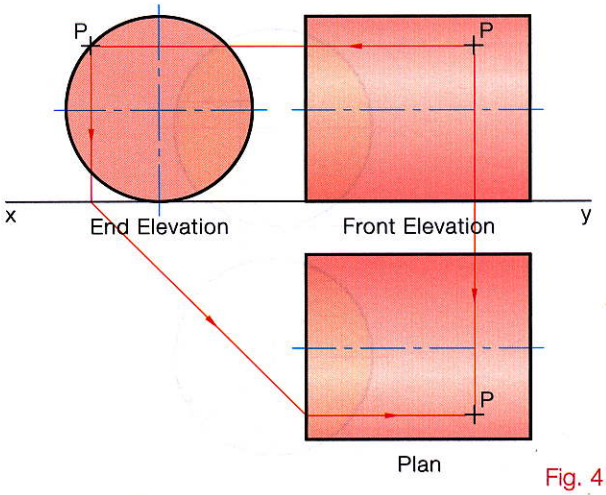


Fig. 4.2

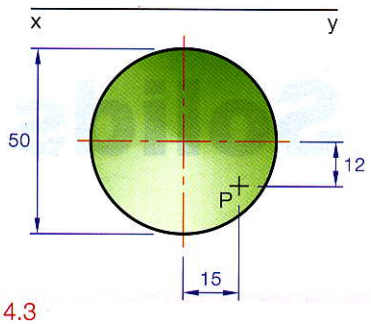


Fig. 4.3

The plan of a cone of altitude 60 mm is shown in Fig. 4.3. Also shown is a point P on its surface. Draw the elevation and locate point P.

There are two methods shown to solve this problem.

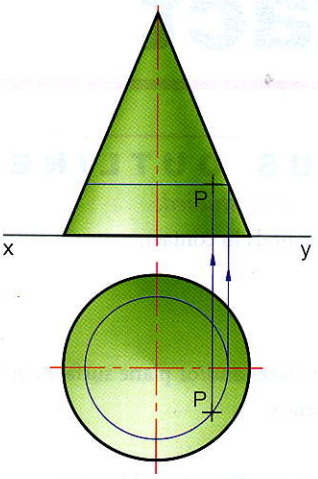
Method 1 (Fig. 4.4a)

Rotate point P in plan onto the horizontal axis. Project to the side of the cone in elevation giving the height of point P. Project the height horizontally and bring point P up from plan onto it.

Method 2 (Fig. 4.4b)

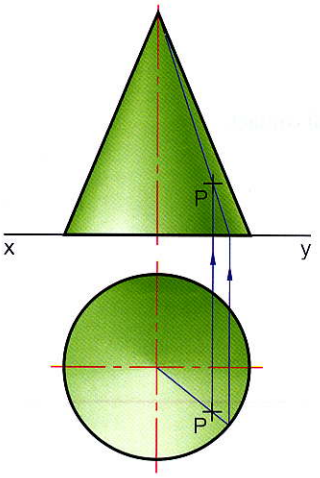
Draw the generator through point P in plan. Project this generator to elevation. Point P is projected onto this generator in elevation.

The horizontal section method is the preferred method.



Horizontal Section

Fig. 4.4a



Generator

Fig. 4.4b

The elevation of a sphere with a point P on its surface is shown in Fig. 4.5. Draw the plan and locate point P on it.

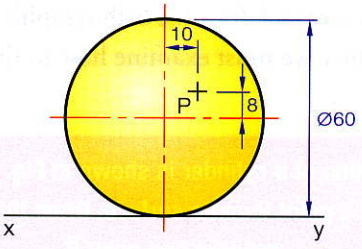


Fig. 4.5

As in the previous example, a horizontal section is used. Project P horizontally to touch the side of the sphere. Project down to the horizontal axis of the plan and rotate round. Drop point P from elevation, Fig. 4.6.

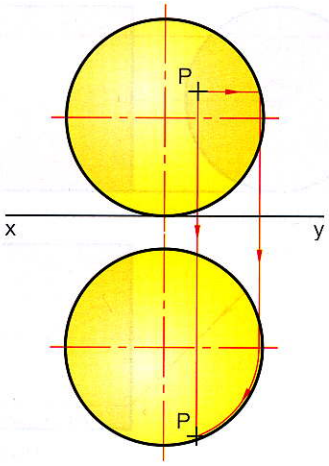


Fig. 4.6

The plan of a cone of altitude 70 mm is shown in Fig. 4.7. Also shown is a point P on the cone's surface. A sphere which rests on the horizontal plane touches the cone at point P. Draw the plan and elevation of the two solids in contact.

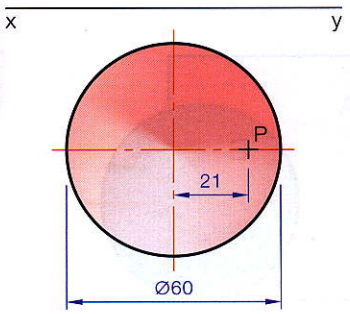


Fig. 4.7

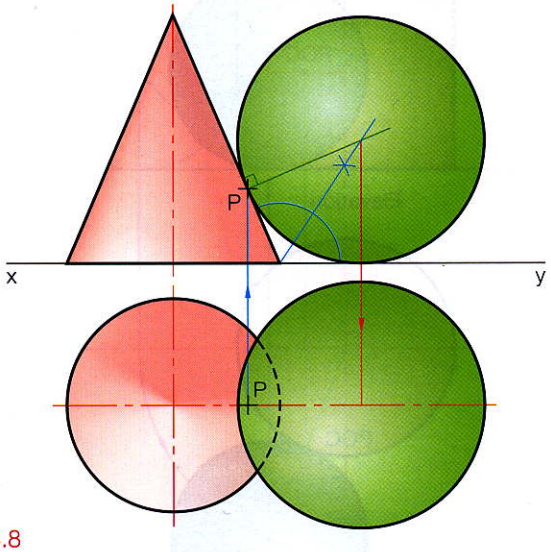


Fig. 4.8

(1) Draw the plan and project the elevation. Locate point P in elevation.

(2) Since the cone is tangential to the sphere, its edge will form a tangent to the sphere in elevation. A perpendicular to the side of the cone from point P will therefore pass through the sphere's centre.

(3) Since the sphere touches the horizontal plane and the cone edge, its centre will be on the bisector of the angle between the two, in elevation, Fig. 4.8.

(4) Draw the sphere in elevation and plan.

The plan of a cone of 70 mm altitude is shown in Fig. 4.9. Also shown is a point P on the cone's surface.

(i) Draw the plan and elevation of the cone and the point P.

(ii) Draw the plan and elevation of a sphere that rests on the horizontal plane and touches the cone at point P.

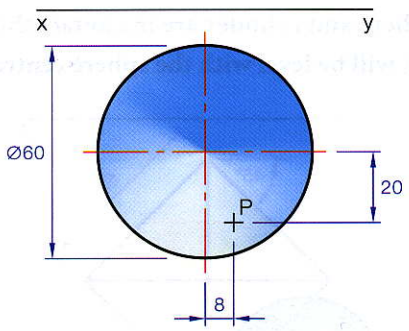


Fig. 4.9

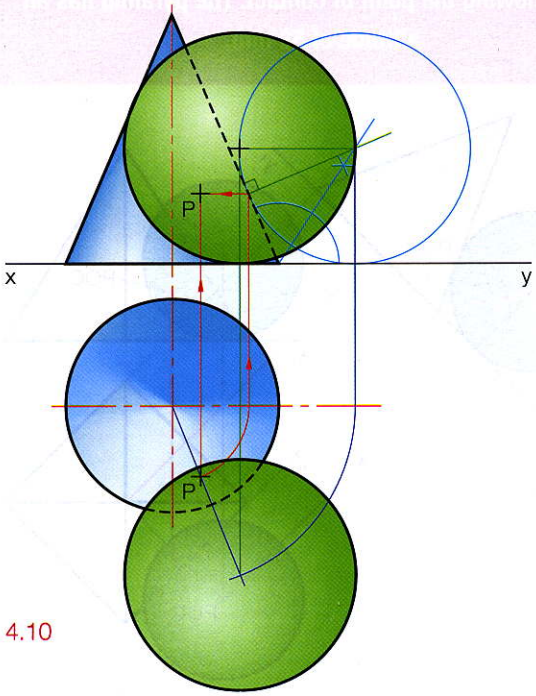


Fig. 4.10

(1) Draw the plan and elevation of the cone.

(2) Rotate point P in plan about the cone centre onto the axis. Project up to the side of the cone in elevation and then project across horizontally. Locate point P on this horizontal.

(3) The sphere required to touch the cone at point P and touch the horizontal plane is constructed at the side of the cone. The construction is the same as in the previous example.

(4) Once the sphere centre is located, it is dropped to plan and rotated onto a line drawn from the cone centre through point P.

The centre of the cone, the point of contact and the sphere centre will form a straight line.

(5) Draw the sphere in position in plan. Project the sphere centre to elevation and draw the sphere, Fig. 4.10.

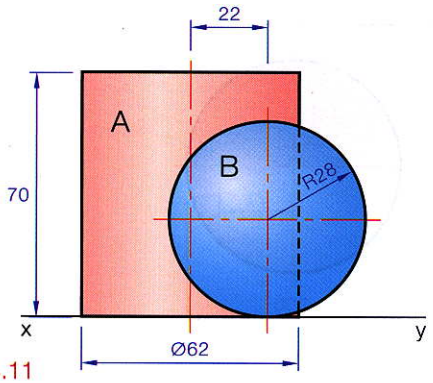


Fig. 4.11

Shown in Fig. 4.11 is the elevation of a cylinder A and a sphere B. Both solids are in contact with each other and rest on the horizontal plane. Draw the elevation and plan of the solids and the point of contact.

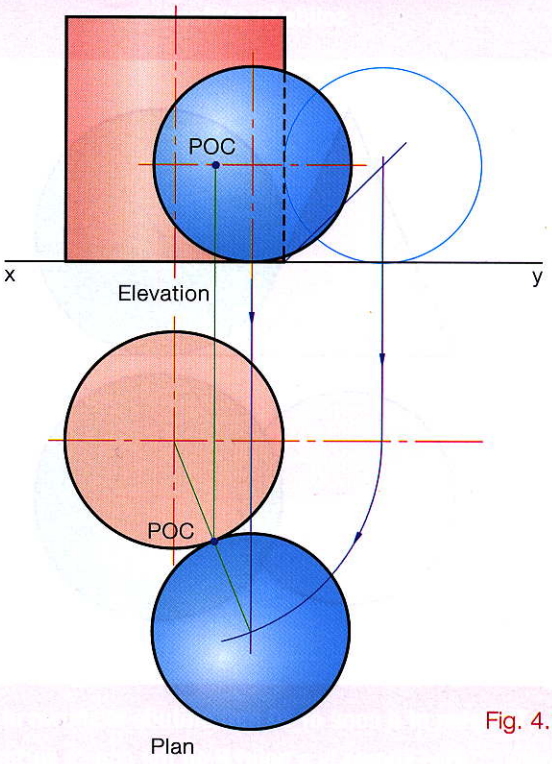


Fig. 4.12

(1) Draw the given elevation of both solids and project the plan of the cylinder.
(2) The sphere is drawn at the side of the cylinder. The point of contact (POC) can be clearly seen.
(3) Drop the centre of the sphere to the plan's horizontal axis and rotate it into the correct position. Draw the sphere in plan.
(4) Join the centres of the solid in plan thus locating the point of contact.
(5) Project the point of contact up from the plan and across from the constructional sphere, Fig. 4.12.

When a sphere and cylinder are in contact the point of contact will be level with the sphere centre.

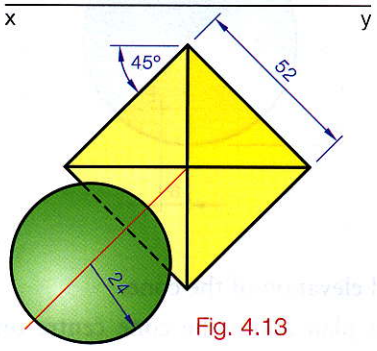


Fig. 4.13

The plan of a square-based pyramid and a sphere, resting on the horizontal plane, are shown in Fig. 4.13. They are in contact with each other. Draw the plan and elevation of the solids showing the point of contact. The pyramid has an altitude of 58 mm.

(1) Draw the plan and elevation of the pyramid.
(2) Project an auxiliary elevation to show the face that is in contact with the sphere as an edge view.
(3) Construct the sphere in the auxiliary. The radius is 24 mm so the centre is 24 mm above the x_1y_1 line. Also bisect the angle between the x_1y_1 and the side of the pyramid.
(4) Once the sphere is located in auxiliary it is projected back to plan and elevation.
(5) The point of contact is first found in the auxiliary by drawing a line from the sphere centre, perpendicular to the face of the pyramid. It is projected to plan and then elevation. The height of the POC in elevation equals the height of the POC in the auxiliary elevation, Fig. 4.14.

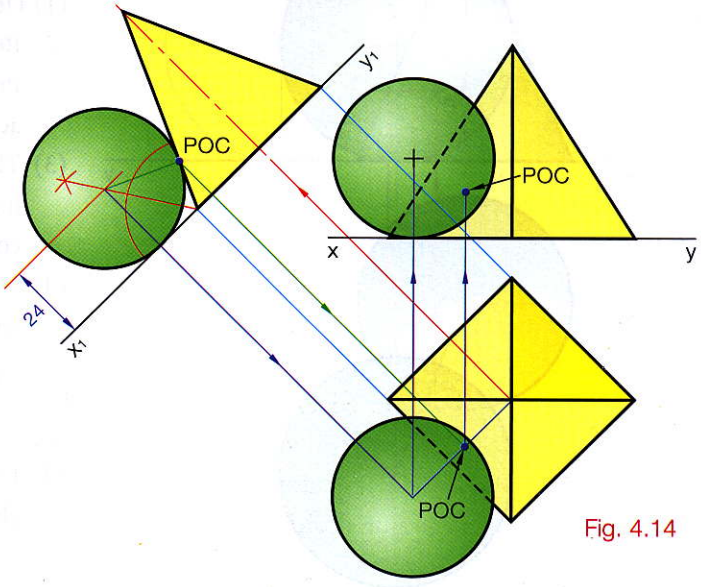


Fig. 4.14

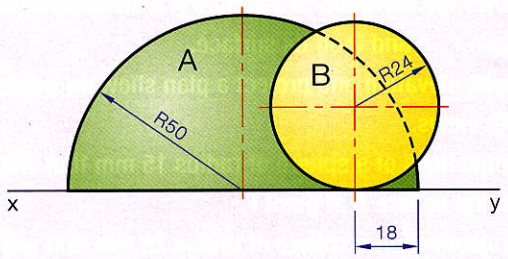


Fig. 4.15

The elevation of a hemisphere A and a sphere B resting on the horizontal plane is shown in Fig. 4.15. The solids are in contact. Draw the plan and elevation of the solids and show the point of contact in both views.

(1) Draw the given elevation and draw the plan of the hemisphere A.

(2) Sphere B is drawn to the side of the hemisphere in elevation. The centre is located by projecting the centre of sphere B horizontally. This horizontal line is then cut by an arc drawn from the centre of the hemisphere and equal to the two radii added together, i.e. 50 mm + 24 mm = 74 mm.

(3) The point of contact is located between the hemisphere and the constructional sphere by joining their centres.

(4) Both point of contact and sphere centre are dropped to plan and rotated into position.

(5) The point of contact is located in elevation, Fig. 4.16.

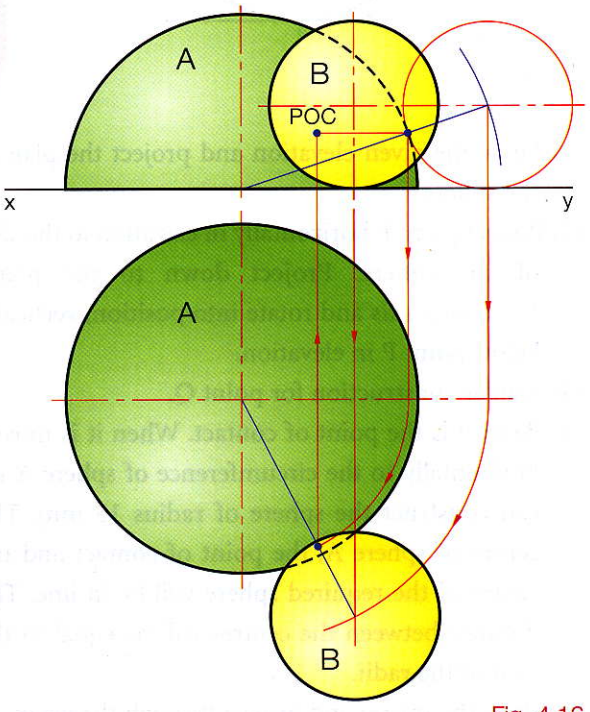


Fig. 4.16

Fig. 4.17 shows the plan of a cone, resting on the horizontal plane, having an altitude of 60 mm. Also shown is a point P on the cone's surface. Draw the plan and elevation of the cone. Draw the plan and elevation of a sphere that will touch point P and also rest on the horizontal plane.

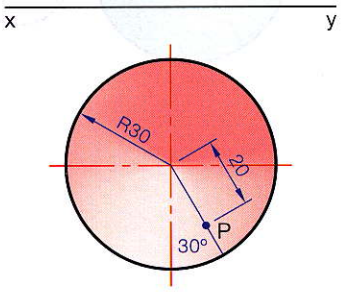


Fig. 4.17

(1) Draw the given plan and project the elevation. Point P is rotated in plan onto the horizontal axis, projected to the side of the cone and then horizontally.

(2) The sphere is constructed to the side of the cone in elevation. Point P is brought horizontally to the side of the cone and a perpendicular constructed to the cone edge. The angle between the xy and the cone side is bisected. The intersection between the perpendicular and the bisector gives the sphere centre.

(3) The sphere is rolled into position and drawn in both views, Fig. 4.18.

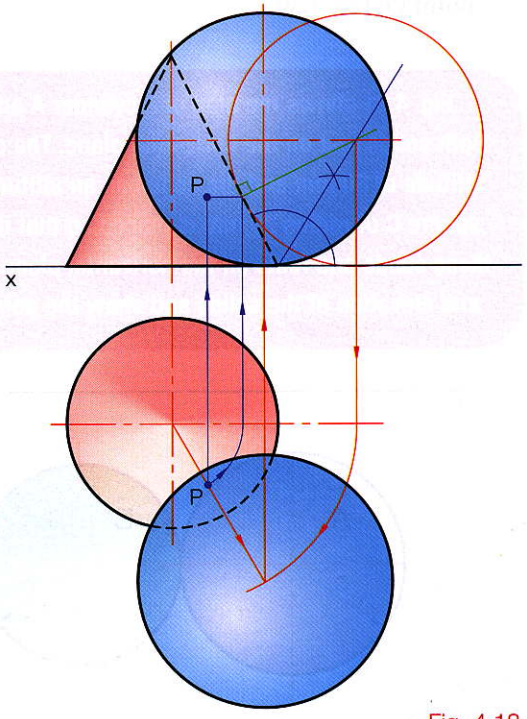


Fig. 4.18

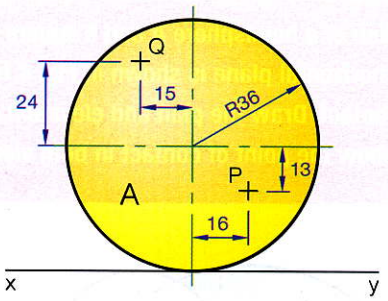


Fig. 4.19

Fig. 4.19 shows the elevation of a sphere A with two points, P and Q on its surface.

(i) Draw the given elevation and project a plan showing points P and Q in both views.

(ii) Show the projections of a sphere of radius 15 mm that will be in contact with the sphere A at point P.

(iii) Show the projections of a sphere of radius 28 mm that will be in contact with sphere A at point Q.

(1) Draw the given elevation and project the plan of the sphere.

(2) Project point P horizontally in elevation to the side of the sphere. Project down to the plan's horizontal axis and rotate into position, vertically below point P in elevation.

(3) Similar construction for point Q.

(4) Point P is the point of contact. When it is moved horizontally to the circumference of sphere A we can construct the sphere of radius 15 mm. The centre of sphere A, the point of contact and the centre of the required sphere will be in line. The distance between the centres will be equal to the sum of the radii.

(5) Draw the sphere and project through the views.

(6) Similar construction for the sphere touching at point Q, Fig. 4.20.

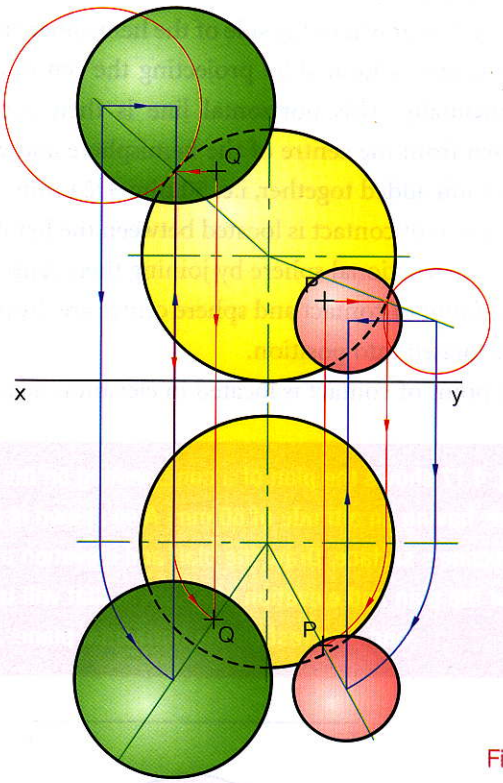


Fig. 4.20

Fig. 4.21 shows the plan of a cylinder A and a cone B. Both solids rest on the horizontal plane. The cylinder has an altitude of 80 mm and the cone has an altitude of 50 mm. A sphere C of radius 20 mm is placed so that it touches both these solids and rests on the horizontal plane. Draw the plan and elevation of the solids and show the points of contact.

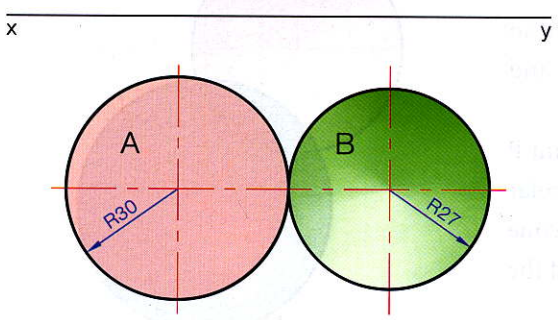


Fig. 4.21

(1) Draw the elevation and plan of the cone and cylinder.

(2) Draw the sphere C touching the side of the cylinder. Project the centre to the plan and rotate it about the cylinder. Similarly for the cone, draw the sphere C touching its side in elevation.

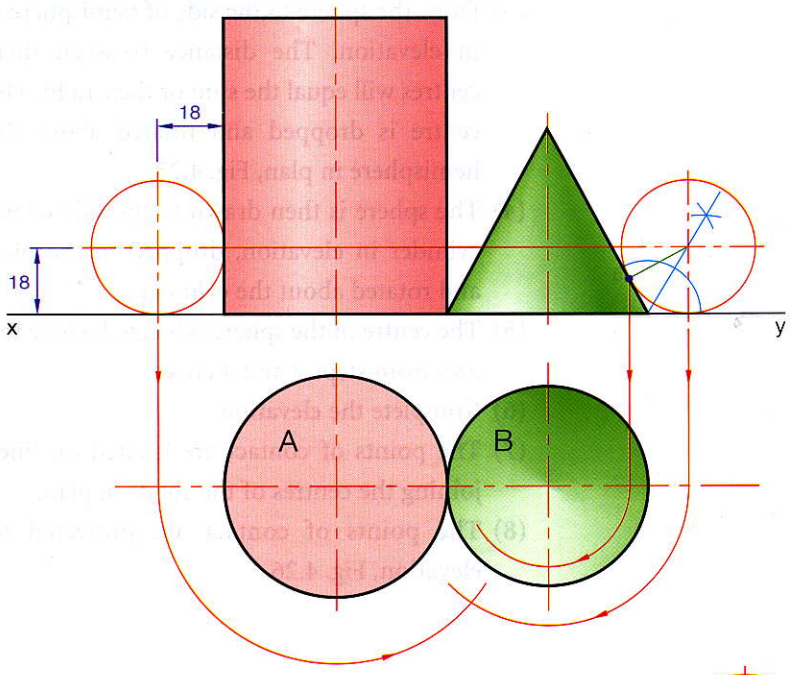


Fig. 4.22

(3) The required sphere has its centre where these two arcs cross in plan, Fig. 4.22.

(4) The point of contact between the sphere and cone is found at the side of the cone in elevation, dropped to the plan and rotated into position. the construction is evident from Fig. 4.23.

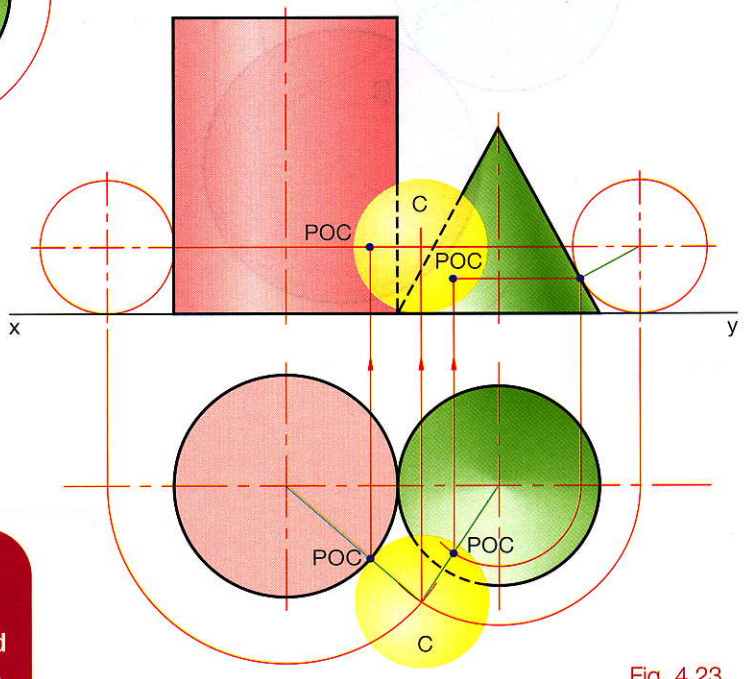


Fig. 4.23

Shown in Fig. 4.24 is the elevation of a cylinder A and a hemisphere B in contact with each other and resting on the horizontal plane. A sphere of 50 mm diameter is placed in position C. The sphere is to be in contact with the other two solids and the horizontal plane. Draw the plan and elevation of the solids showing all points of contact.

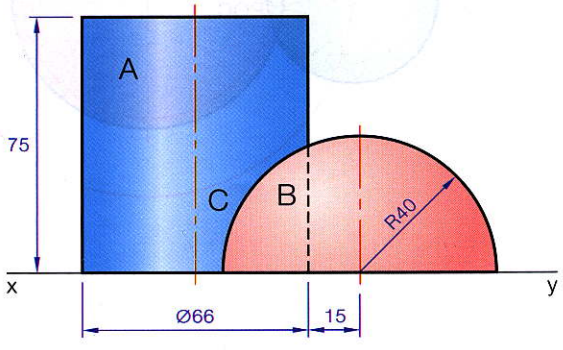


Fig. 4.24

(1) Draw the given elevation and project the plan of the cylinder.

(2) The centre of hemisphere B is projected to plan. The distance between the centres of A and B in plan will equal the sum of their radii.

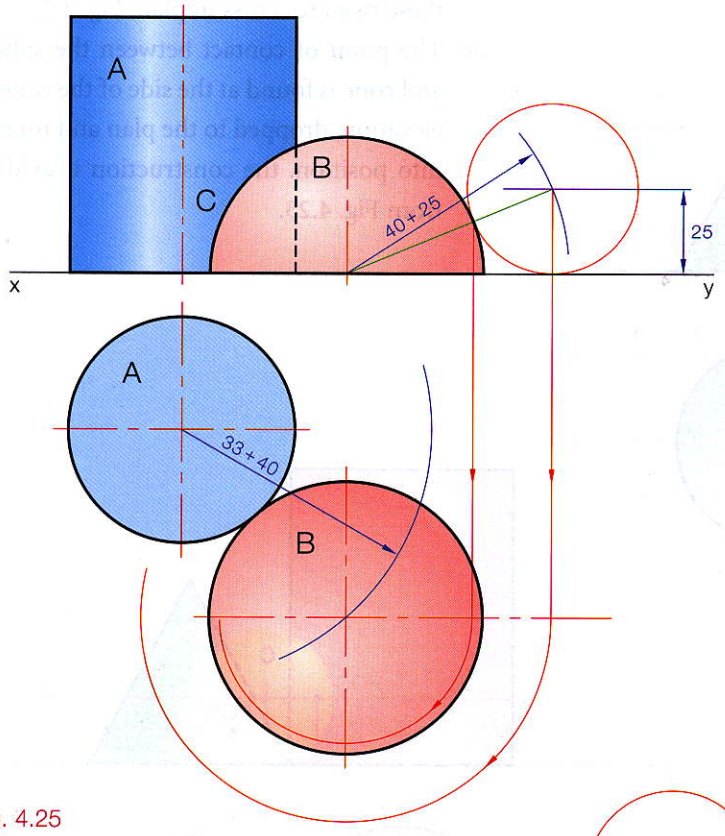


Fig. 4.25

(3) Draw the sphere to the side of hemisphere B in elevation. The distance between their centres will equal the sum of their radii. The centre is dropped and rotated about the hemisphere in plan, Fig. 4.25.
(4) The sphere is then drawn to the side of the cylinder in elevation, dropped to the plan and rotated about the cylinder.
(5) The centre of the sphere is located where the arcs from steps 3 and 4 cross.
(6) Complete the elevation.
(7) The points of contact are located on lines joining the centres of the solids in plan.
(8) The points of contact are projected to elevation, Fig. 4.26.

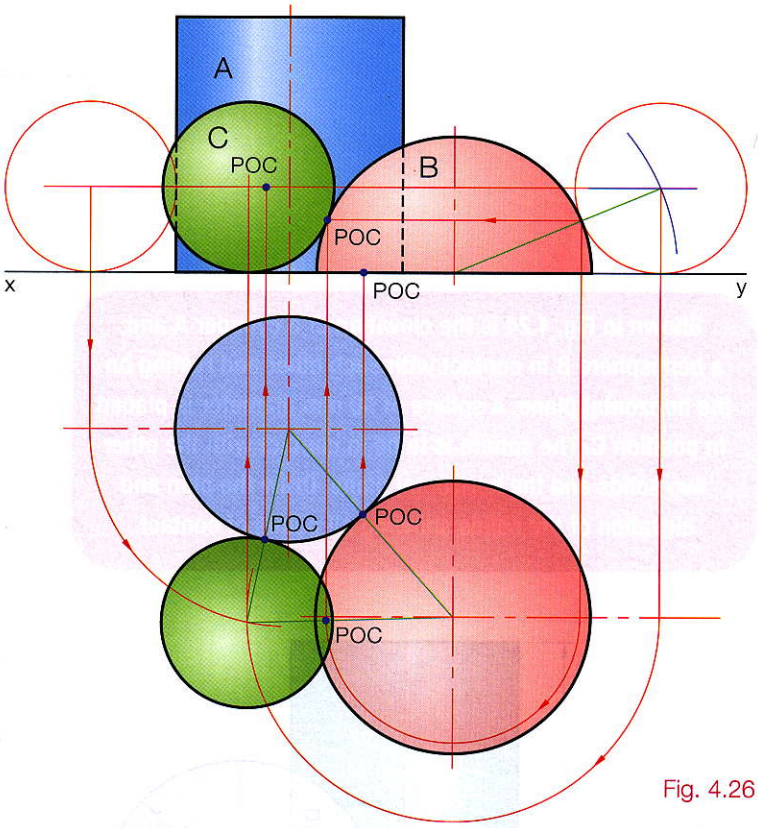


Fig. 4.26

Fig. 4.27 shows the elevation of a cone A and a sphere B resting on the horizontal plane and in contact with each other.

(i) Draw the elevation and plan of the two solids.

(ii) Draw the projections of another sphere C, of 40 mm diameter, which is in contact with the sphere and cone in position S. The centre of the sphere is to be 50 mm above the horizontal plane.

(iii) Show all points of contact.

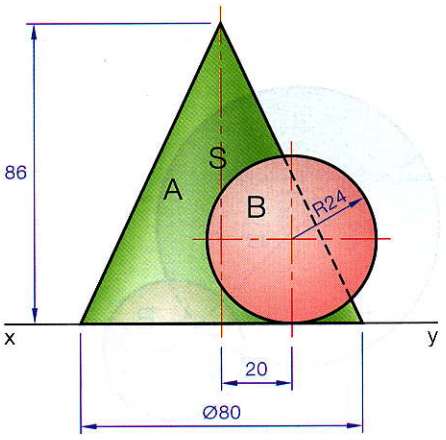


Fig. 4.27

(1) Draw the given elevation and project the plan of the cone.

(2) Sphere B must be drawn to the side of the cone, touching it, projected to plan and rolled into position, Fig. 4.28.

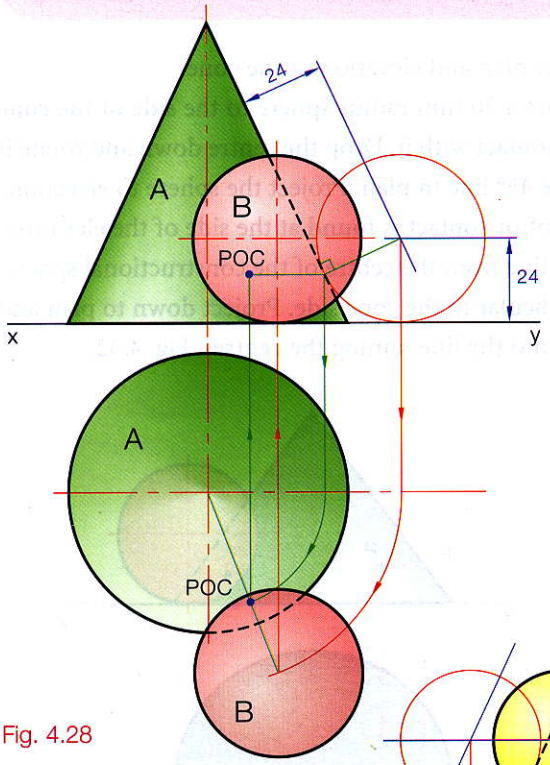


Fig. 4.28

(3) The point of contact is found at the side of the elevation, projected to plan and rotated until it hits the line joining the centres. Project to elevation.

(4) Sphere C has its centre 50 mm above the horizontal plane, Fig. 4.29. Draw the sphere to the side of, and touching, the cone A. Project to plan and rotate about the cone.

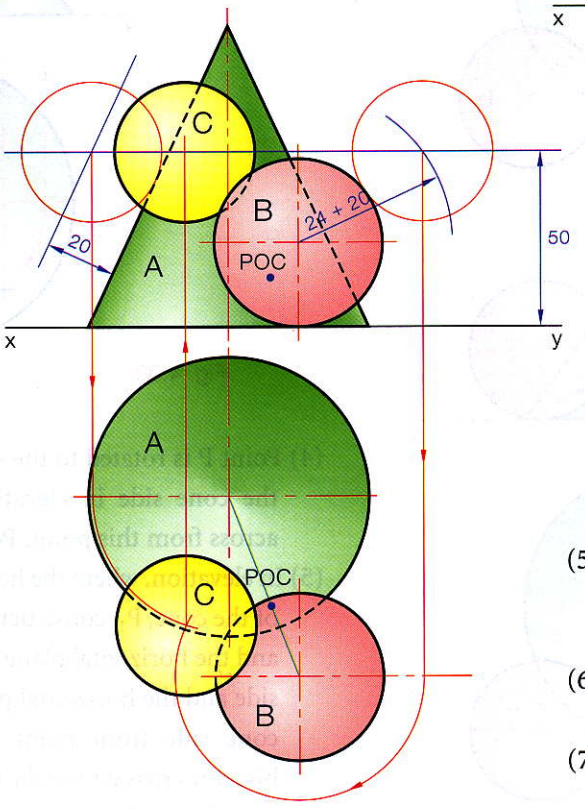


Fig. 4.29

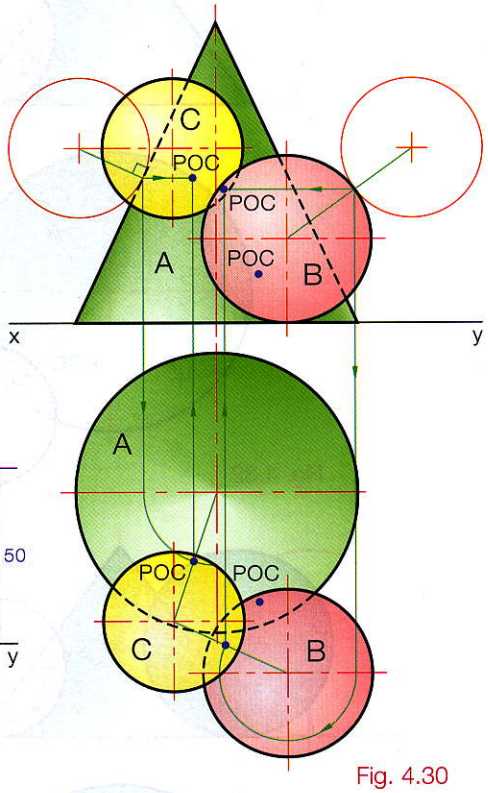


Fig. 4.30

(5) Draw the sphere C to the side of, and touching, sphere B. Project to plan and rotate about sphere B.

(6) Draw the sphere C on its correct position in both views.

(7) The points of contact between the solids are found in the usual way as can be seen in Fig. 4.30.

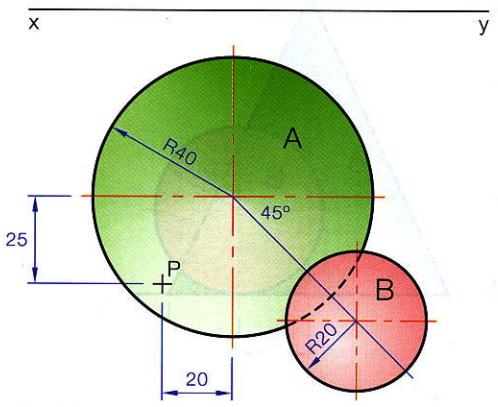


Fig. 4.31

Fig. 4.31 shows the plan of a right cone A in contact with a sphere B. The cone's altitude is 55 mm. Both solids rest on the horizontal plane. The position of a point P on the cone's surface is also given.

(i) Draw the cone A and sphere B in plan and elevation showing the point of contact.

(ii) Locate point P in elevation.

(iii) Another sphere C is placed on the horizontal plane and is in contact with cone A at the point P. Draw this sphere in plan and elevation.

(1) Draw the plan and elevation of the cone.

(2) Construct a 20 mm radius sphere to the side of the cone and in contact with it. Drop the centre down and rotate it onto the 45° line in plan. Project the sphere to elevation.

(3) The point of contact is found at the side of the elevation. Draw a line from the centre of the constructional sphere, perpendicular to the cone side. Project down to plan and rotate onto the line joining the centres, Fig. 4.32.

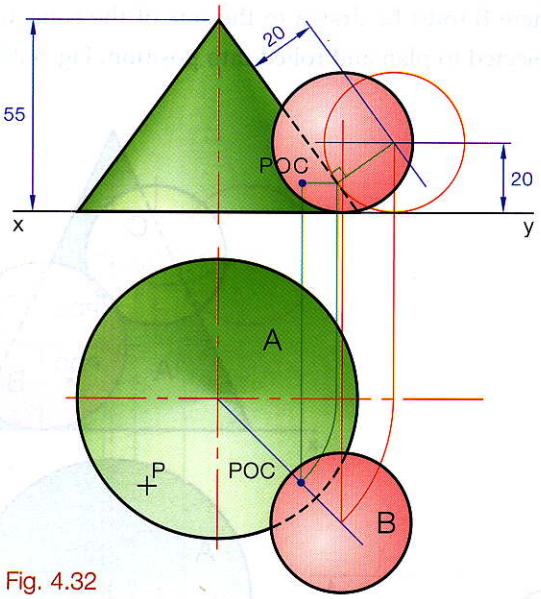


Fig. 4.32

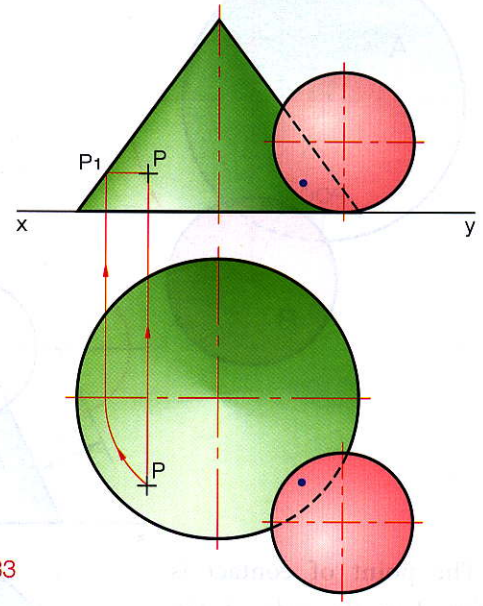


Fig. 4.33

(4) Point P is rotated to the side of the cone, then projected to the cone side in elevation, P_1. A horizontal is brought across from this point. Point P is on this line, Fig. 4.33.

(5) In elevation, where the horizontal from point P hits the side of the cone, P_1, construct a sphere that will touch this point and the horizontal plane. Bisect the angle between the cone side and the horizontal plane. Draw a perpendicular to the cone side from point P_1. The perpendicular and the bisector cross giving the sphere centre.

(6) Find the final position of this sphere in the usual way, Fig. 4.34.

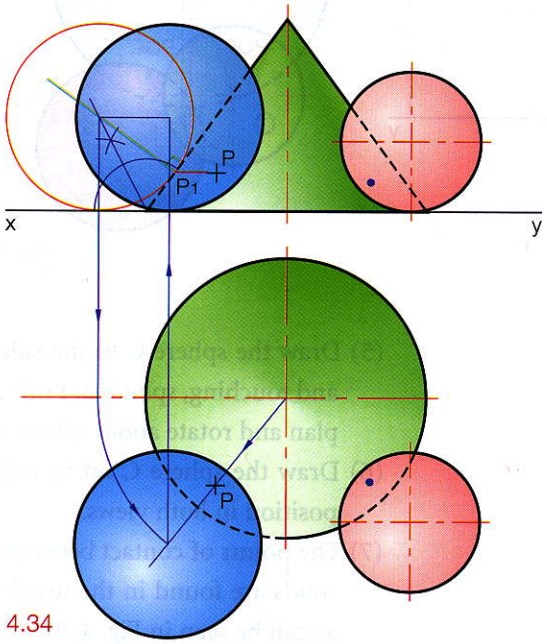


Fig. 4.34

Fig. 4.35 shows the plan of a rectangular-based pyramid of 55 mm height. A sphere of 20 mm radius is in contact with the horizontal plane and the pyramid. Also shown is a point P on the surface of the pyramid.

(i) Draw the elevation and plan of the two solids showing the point of contact.

(ii) Show the plan and elevation of a sphere that rests on the horizontal plan and is in contact with the pyramid at point P.

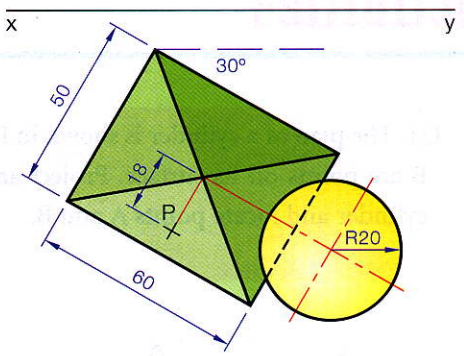


Fig. 4.35

(1) Draw the plan and elevation of the pyramid.

(2) Project an auxiliary elevation that will show the surface in contact with the sphere as an edge view.

(3) Construct the sphere in the auxiliary. Bisect the angle between the pyramid face and the xy line. Draw a horizontal line 20 mm above the xy line. These two lines cross giving the required sphere's centre point. Draw the sphere.

(4) Find the point of contact and project both centre and POC back to plan and elevation, Fig. 4.36.

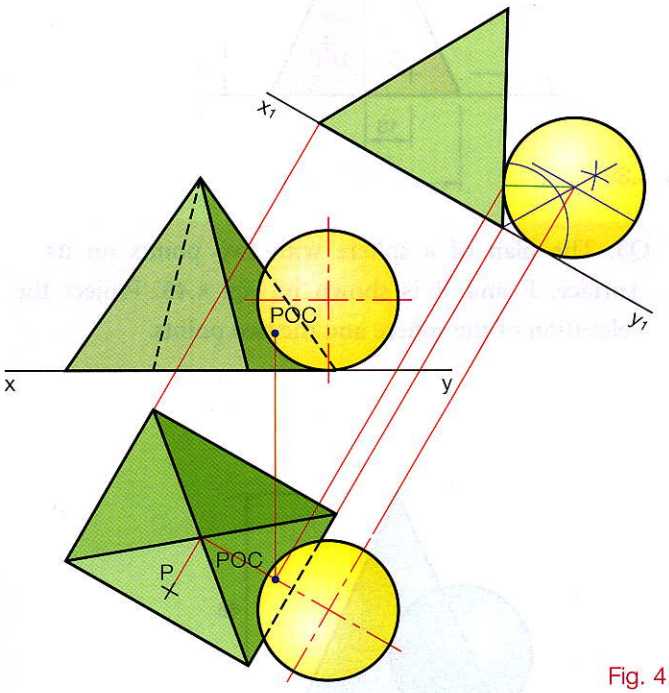


Fig. 4.36

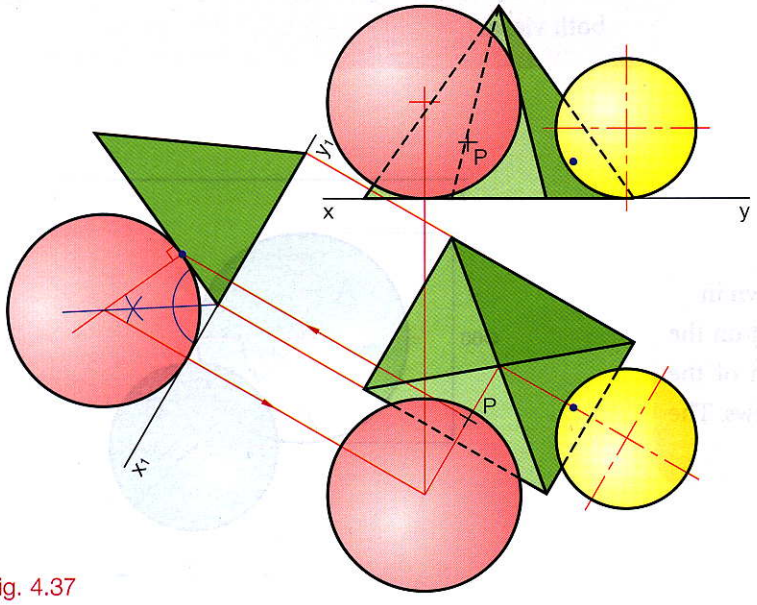


Fig. 4.37

(5) To find the sphere that touches point P. Project an auxiliary that will show the surface containing point P as an edge view. Find the sphere centre in the auxiliary by drawing a perpendicular to the pyramid side from point P and bisecting the angle the pyramid surface makes with the xy line, Fig. 4.37.

Activities

Q1. The plan of a cylinder is shown in Fig. 4.38. A and B are points on its surface. Project an elevation of the cylinder and locate points A and B.

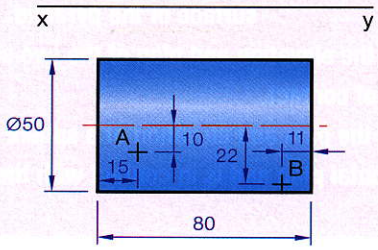


Fig. 4.38

Q2. The elevation of a cone is shown in Fig. 4.39. Two points C and D are shown on the surface. Project a plan showing clearly how the two points are located.

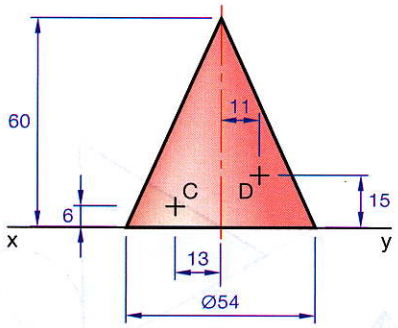


Fig. 4.39

Q3. The plan of a sphere with two points on its surface, E and F, is shown in Fig. 4.40. Project the elevation of the sphere and the two points.

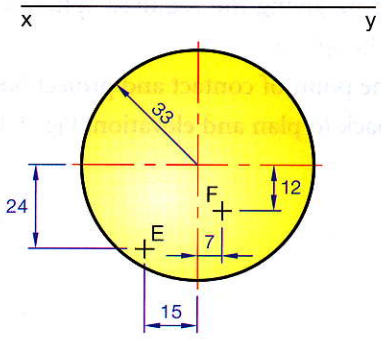


Fig. 4.40

Q4. The elevation of a sphere and cone are shown in Fig. 4.41. The two solids are in contact. Draw the given view and project the plan. Show the point of contact in both views.

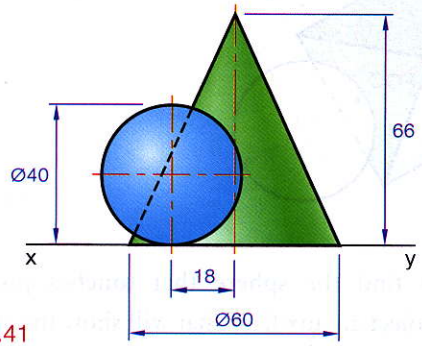


Fig. 4.41

Q5. The plan of a cone A and a sphere B are shown in Fig. 4.42. The two solids are in contact and rest on the horizontal plane. Draw the plan and elevation of the solids and show the point of contact in both views. The cone has an altitude of 60 mm.

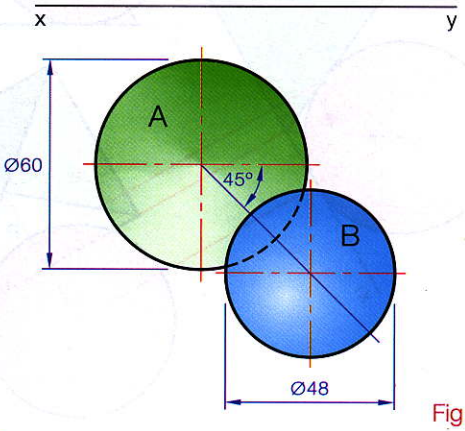


Fig. 4.42

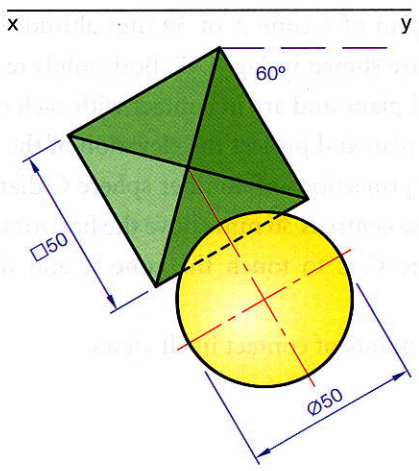


Fig. 4.43

Q6. Fig. 4.43 shows the plan of a square-based pyramid of altitude 70 mm. Also shown is a sphere which is in contact with the pyramid. Both solids rest on the horizontal plane. Draw the given plan and project the elevation. The point of contact should be clearly shown in both views.

Q7. The elevation of a hemisphere is shown in Fig. 4.44. A sphere S is in contact with the hemisphere. Draw the plan and elevation of the two solids showing the point of contact clearly in both views.
Also shown is a point P on the surface of the hemisphere. Draw the plan and elevation of point P.

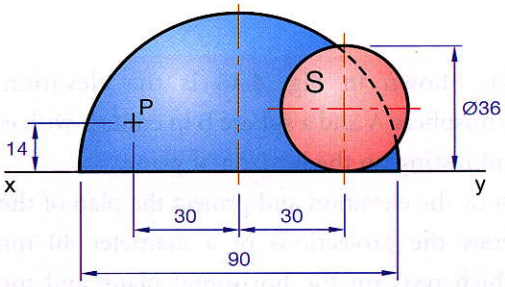


Fig. 4.44

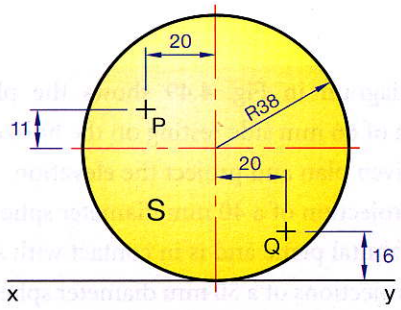


Fig. 4.45

Q8. The elevation of a sphere S is shown in Fig. 4.45 with two points on its surface, points P and Q.
(i) Draw the plan and elevation of the sphere and points.
(ii) Draw the plan and elevation of a sphere of radius 25 mm that has point P as its point of contact with sphere S.
(iii) Draw the plan and elevation of a sphere of radius 8 mm that has point Q as its point of contact with sphere S.

Q9. The plan of a cylinder A and a cone B are shown in Fig. 4.46. Both solids rest on the horizontal plane and are in mutual contact.
(i) Draw the plan and elevation of the two solids and show the point of contact.
(ii) A sphere C rests on the horizontal plane and is in contact with the cone B and the cylinder A. Sphere C has a radius of 12 mm. Project the views of this solid and show all points of contact.

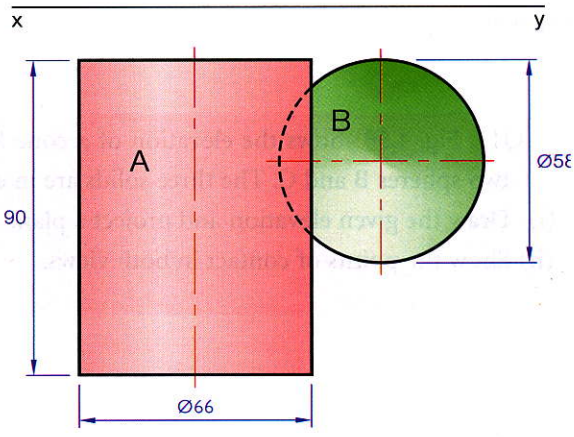


Fig. 4.46

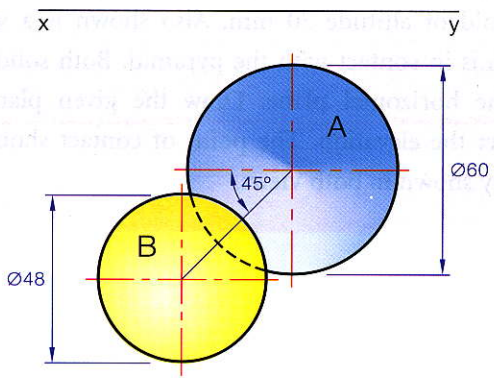


Fig. 4.47

Q10. The plan of a cone A of 50 mm altitude and a sphere B are shown in Fig. 4.47. Both solids rest on the horizontal plane and are in contact with each other.

(i) Draw the plan and project the elevation of the solids.

(ii) Draw the projections of another sphere C diameter 30 mm, whose centre is 36 mm above the horizontal plane. The sphere C is to touch the cone A and the other sphere B.

(iii) Show the points of contact in all views.

Q11. Shown in Fig. 4.48 is the elevation of a hemisphere A and a sphere B in contact with each other and resting on the horizontal plane.

(i) Draw the elevation and project the plan of the solids.

(ii) Draw the projections of a diameter 40 mm sphere which rests on the horizontal plane and touches the hemisphere A and sphere B.

(iii) Show all points of contact.

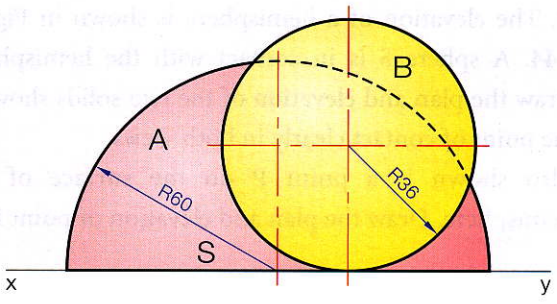


Fig. 4.48

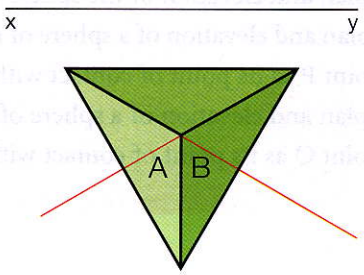


Fig. 4.49

Q12. The diagram in Fig. 4.49 shows the plan of a tetrahedron of 66 mm side resting on the horizontal plane.

(i) Draw the given plan and project the elevation.

(ii) Draw the projection of a 40 mm diameter sphere that rests on the horizontal plane and is in contact with surface A.

(iii) Draw the projections of a 50 mm diameter sphere that rests on the horizontal plane and is in contact with surface B. Show all points of contact.

Q13. Fig. 4.50 shows the elevation of a cone A and two spheres B and C. The three solids are in contact.

(i) Draw the given elevation and project a plan.

(ii) Show the points of contact in both views.

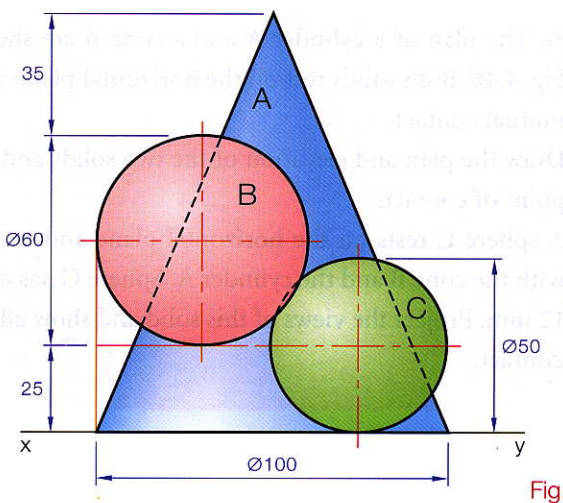


Fig. 4.50

Problems

PROBLEM 1

Fig. 4.51 shows the plan of a sphere with a point P on its underside.

(i) Draw the plan and elevation of the sphere and find the projections of point P.

(ii) Find the projections of the sphere which rests on the horizontal plane and has point P as its point of contact.

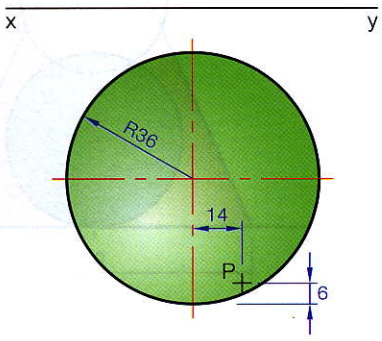


Fig. 4.51

(1) Draw the sphere in plan and elevation.

(2) Find point P in elevation in the usual way. Rotate point P onto the horizontal axis in plan and project to the sphere's circumference in elevation giving P_1.

(3) Draw a line from C through P_1 and extend.

(4) Draw a locus of points equidistant from the circumference and the xy line.

(5) The locus and CP_1 extended cross giving the sphere centre, Fig. 4.52.

(6) Draw the sphere and roll it into position, Fig 4.53.

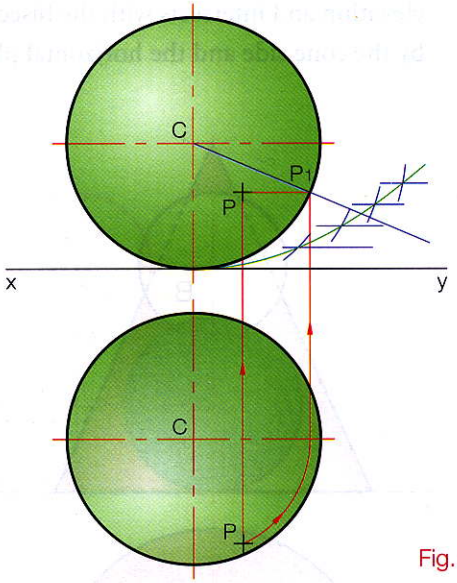


Fig. 4.52

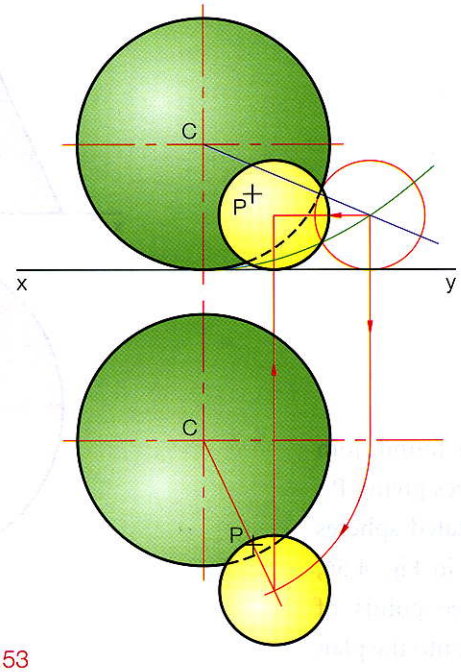


Fig. 4.53

HIGHER LEVEL

PROBLEM 2

The diagram in Fig. 4.54 shows the elevation of two spheres and a cone in contact with one another. Draw the elevation and plan of the solids showing all points of contact.

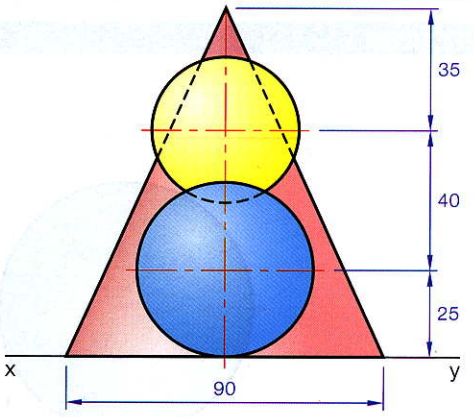


Fig. 4.54

(1) Draw the plan and elevation of the cone. The elevation of sphere A can also be drawn.

(2) The location of sphere A in plan is found by rolling the sphere to the side of the cone, dropping it to the side of the plan and rotating it into position. The centre point is brought across in elevation and intersects with the bisector of the angle formed by the cone side and the horizontal plane, Fig. 4.55.

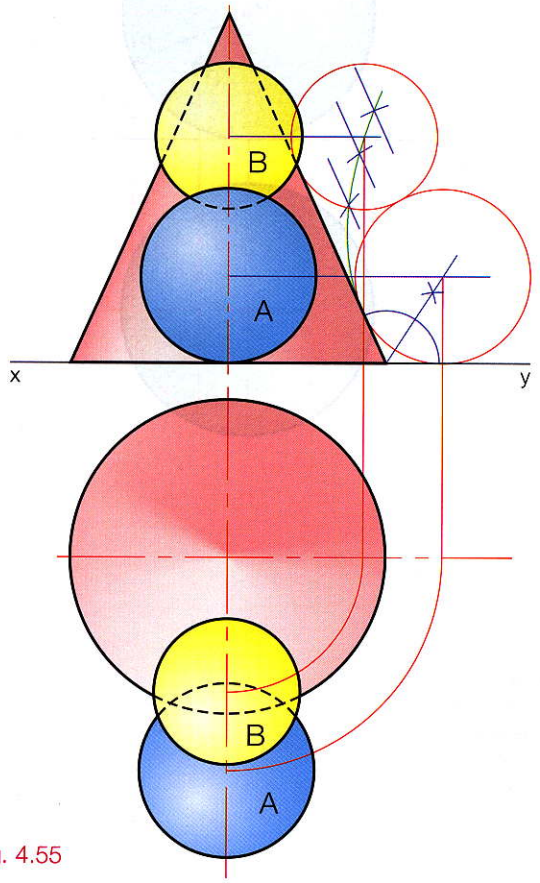


Fig. 4.55

(3) The radius of sphere B is not given. It too must be brought to the side of the cone. Project the given centre point across to the side. This line intersects with the locus of points which are equidistant from the cone side and the circumference of sphere A rotated.

(4) Project the sphere centres back to the elevation and plan, Fig. 4.55.

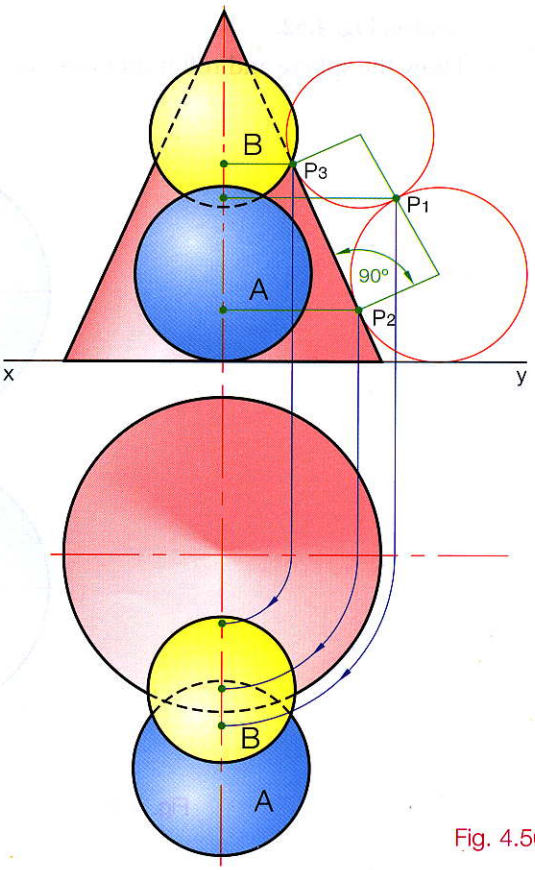


Fig. 4.56

(5) The points of contact must now be found. Join the centres of the two rotated spheres giving P_1. Draw a perpendicular from the rotated spheres centres to the cone side, as shown in Fig. 4.56, thus locating P_2 and P_3. The three points of contact may then be projected back onto the plan and elevation.

PROBLEM 3

Fig. 4.57 shows the plan and elevation of a sphere A and a cylinder B.

(i) Draw the elevation and plan of the solids.

(ii) Draw the elevation and plan of a cone of 60 mm base diameter and 60 mm height, which rests on the horizontal plane in position S and is in contact with the given solids. Show all points of contact.

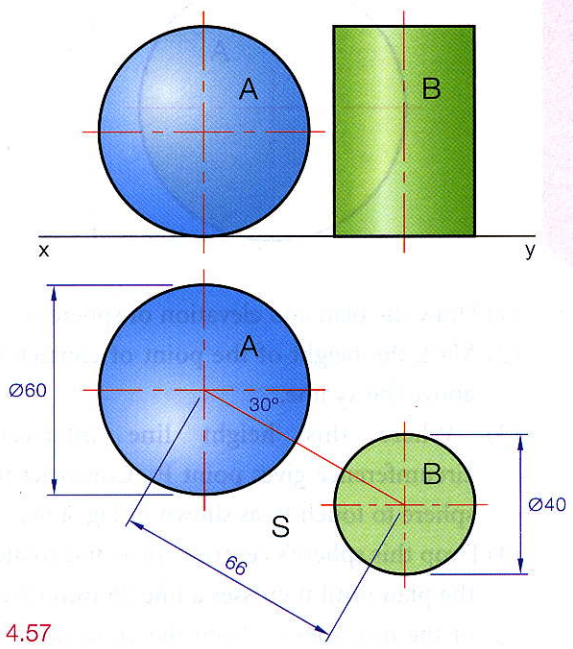


Fig. 4.57

(1) Draw the required cone to the side of the elevation and slide it across to come into contact with sphere A.

(2) The point of contact is located.

(3) Drop the cone centre and point of contact down to plan and rotate about the plan of sphere A, Fig. 4.58.

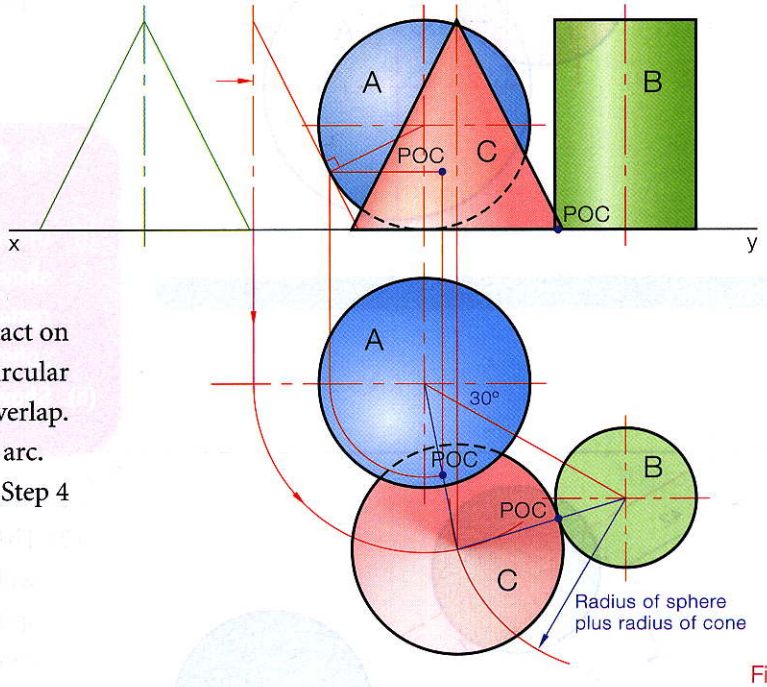


Fig. 4.58

(4) The cone and cylinder will make contact on the horizontal plane and their circular plans will make contact but not overlap. Add their radii in plan and scribe an arc.

(5) Where the arcs cross from Step 3 and Step 4 gives the centre of the required cone.

(6) Complete the views in the usual way.

PROBLEM 4

The diagram shows the projection of a sphere A which rests on the horizontal plane, Fig. 4.59.

(i) Draw the sphere in plan and elevation.

(ii) Draw the projections of a sphere of 50 mm diameter which touches the sphere A at a point 60 mm above the horizontal plane and also touches the vertical plane. Show the point of contact in both views.

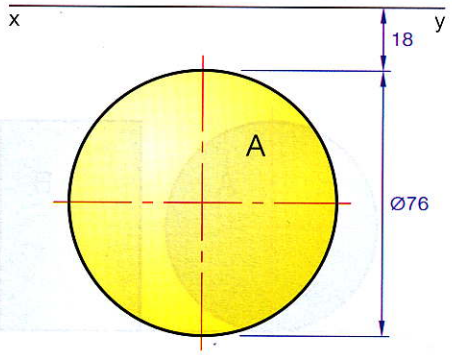


Fig. 4.59

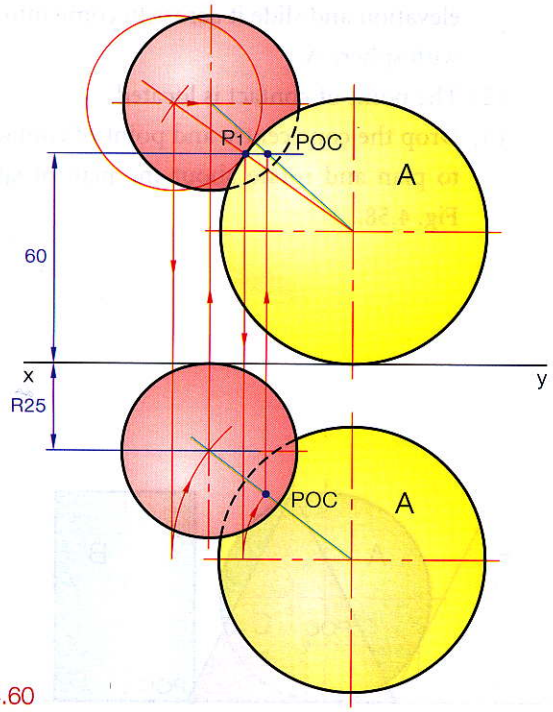


Fig. 4.60

(1) Draw the plan and elevation of sphere A.

(2) Mark the height of the point of contact, 60 mm above the xy line.

(3) Where this height line intersects the circumference gives point P_1. Construct the new sphere to touch P_1 as shown in Fig. 4.60.

(4) Drop this sphere's centre to plan and rotate about the plan until it crosses a line 25 mm (the radius of the new sphere) from the xy line, the vertical plane.

(5) Draw the elevation of this sphere and find the point of contact in the usual way, Fig. 4.60.

PROBLEM 5

Fig. 4.61 shows the plan of two spheres A and B resting on the horizontal plane.

(i) Draw the plan and elevation of the two spheres and show the projections of the smallest sphere which rests on the horizontal plane and touches both sphere A and sphere B.

(ii) Show the points of contact in all views.

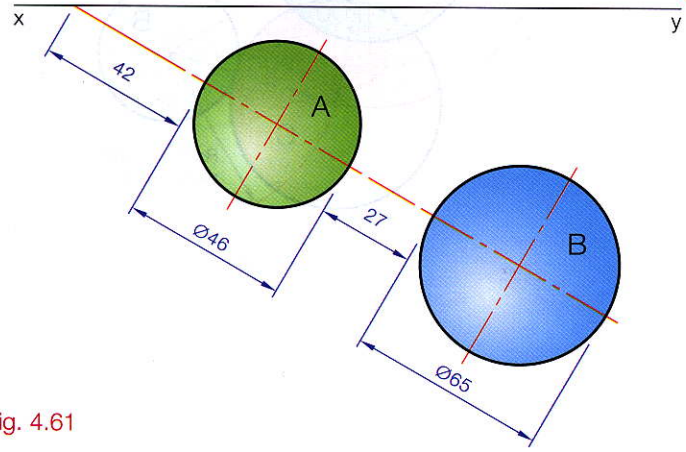


Fig. 4.61

(1) Draw the plan and elevation of the solids.

(2) The smallest sphere to touch both existing solids will have its centre in line with the centres of sphere A and sphere B in plan. Project an auxiliary elevation with the x_1y_1 line parallel to the line joining the centres of sphere A and B. This auxiliary will show the space between the spheres.

(3) Locate the centre of the new sphere by the use of locii. Draw the locus of points which are equidistant from the circumference of sphere A and the x_1y_1 line.

(4) Similarly for sphere B and the x_1y_1 line.
(5) The locii intersect giving a point which is equidistant from the sphere A, the sphere B and the x_1y_1 line.
(6) Draw the sphere and project back through the views.
(7) The points of contact are also found in the auxiliary and projected back through the views, Fig. 4.62.

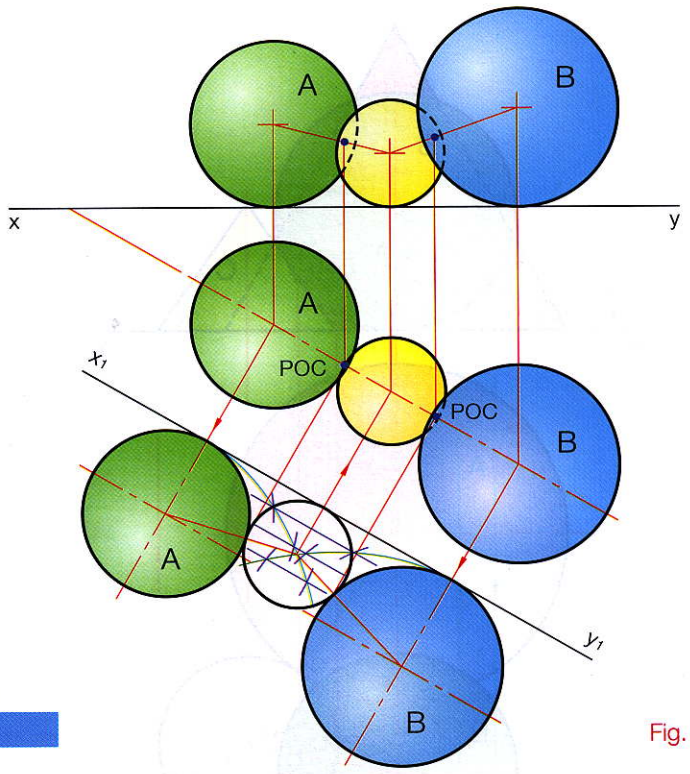


Fig. 4.62

PROBLEM 6

The diagram Fig. 4.63 shows the plan of a right cone A having an altitude of 90 mm, resting on the horizontal plane.

(i) Draw the plan and elevation of the cone and show the projections of a sphere B which rests on the horizontal plane and makes contact with the cone at point P.

(ii) Show the projections of another right cone C resting on the horizontal plane. This cone is to make contact with sphere B, 15 mm above the horizontal plane, and to touch cone A at the point Q.

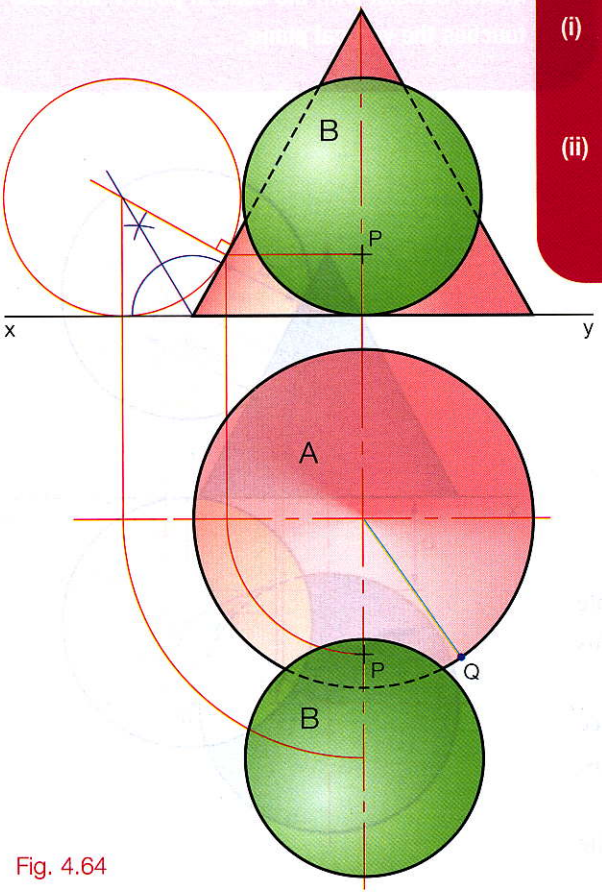


Fig. 4.64

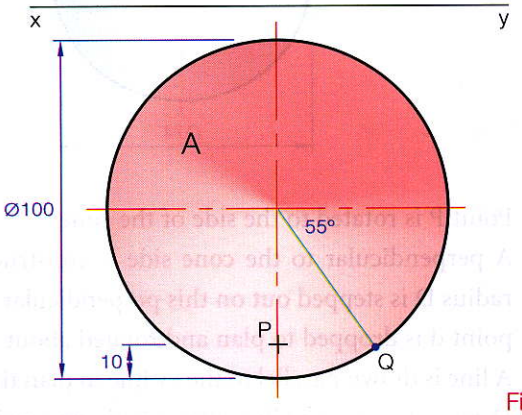


Fig. 4.63

(1) Draw the plan and elevation of the cone.
(2) Find sphere B in the usual way, Fig. 4.64.

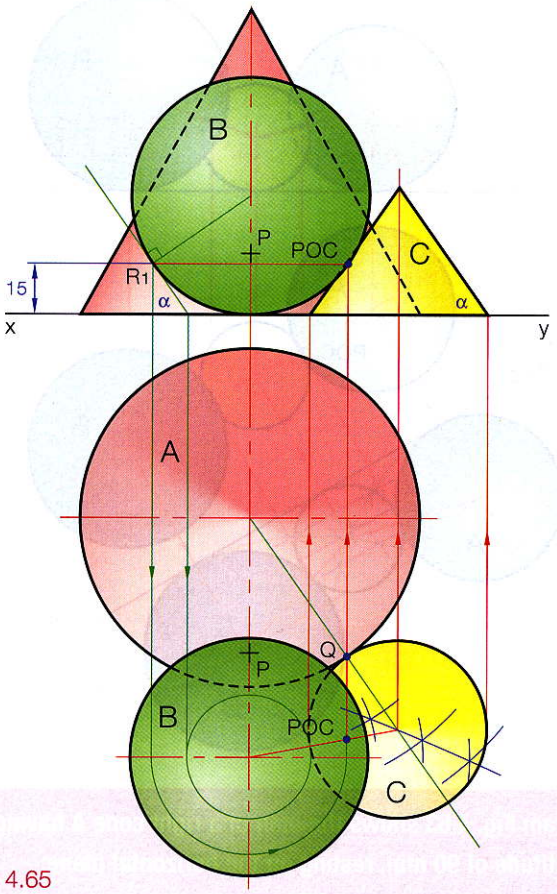


Fig. 4.65

PROBLEM 7

The plan of a right cone is shown in Fig. 4.66. The cone has an altitude of 70 mm. Also shown is a point P on the cone's surface.

(i) Draw the projections of the cone and point P.

(ii) Draw the elevation and plan of a sphere that makes contact with the cone at point P and also touches the vertical plane.

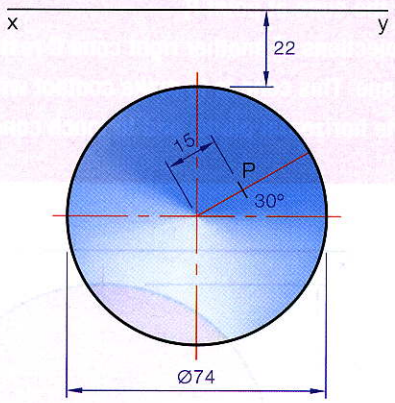


Fig. 4.66

(1) Point P is rotated to the side of the cone.

(2) A perpendicular to the cone side is constructed. A possible radius D is stepped out on this perpendicular to point d. This point d is dropped to plan and rotated about the cone.

(3) A line is drawn parallel to the xy line in plan the same distance D away. The arc and line intersect giving a point on the locus.

(4) Repeat for larger distances forming the locus.

(5) The centre of the required sphere will be on this locus and the line extended from the cone centre through P, Fig. 4.67.

(3) The angle of the cone C can be found by constructing a tangent at the height of the point of contact, on the circumference of sphere B, at point R_1.

(4) The base of cone C projects underneath sphere B and is rotated.

(5) The base circle of cone C must touch this rotated point and point Q and is found by using a locus as shown in Fig. 4.65.

(6) Once the base circle is found, the point of contact is obtained by joining the centres in plan and by dropping point R_1 from elevation and rotating.

(7) The elevation of cone C is found by projection, having its base angle matching that of the tangent at R_1.

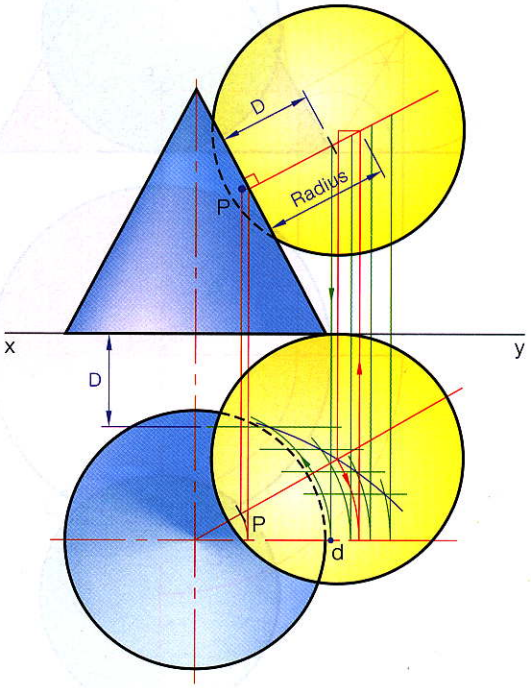


Fig. 4.67

Tangent Planes to Solids

PROBLEM 1

To draw a plane tangential to a sphere at a given point P on its surface.

(1) In plan join the sphere centre to P.
(2) Project an auxiliary view having x_1y_1 parallel to cP.
(3) Locate point P in the auxiliary. It will be on the circumference.
(4) Draw the edge view of the tangent plane in auxiliary.
(5) Find the traces.

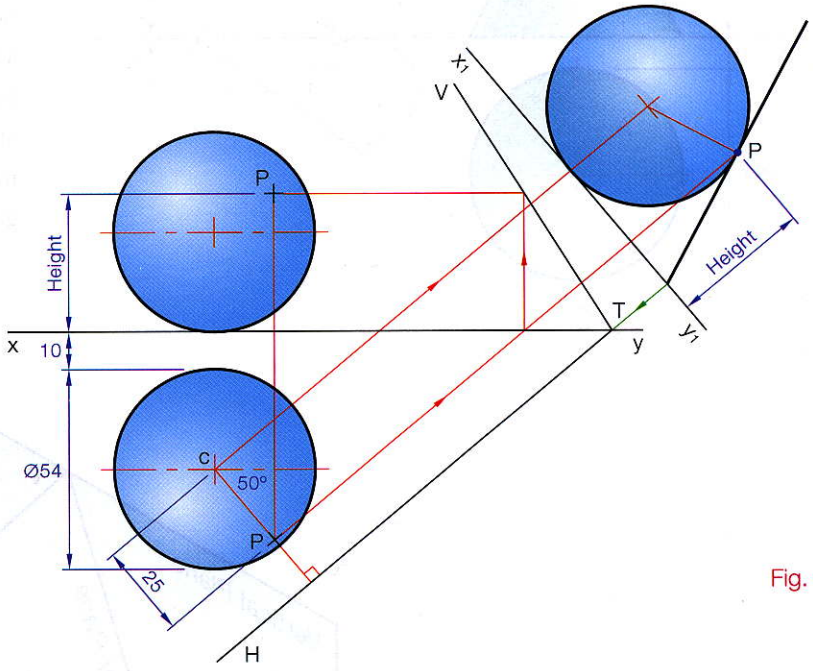


Fig. 4.68

PROBLEM 2

To draw the traces of a plane tangential to a cone at a given point P on its surface.

(1) In plan, draw the generator from the cone apex through point P to the base of the cone. The horizontal trace will be perpendicular to this.
(2) Find P in elevation and the vertical trace in the usual way.

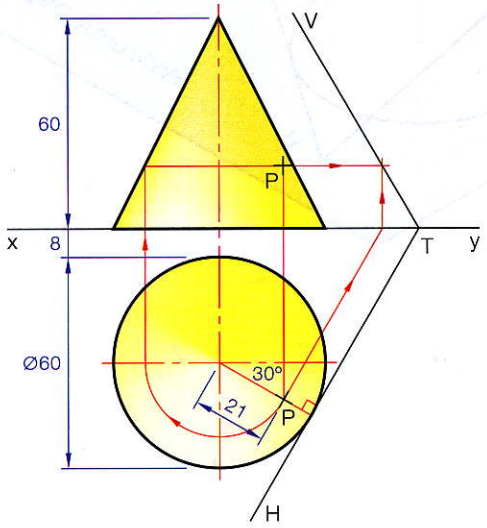


Fig. 4.69

PROBLEM 3

To draw the traces of a plane that is tangential to the cone A and that contains point P.

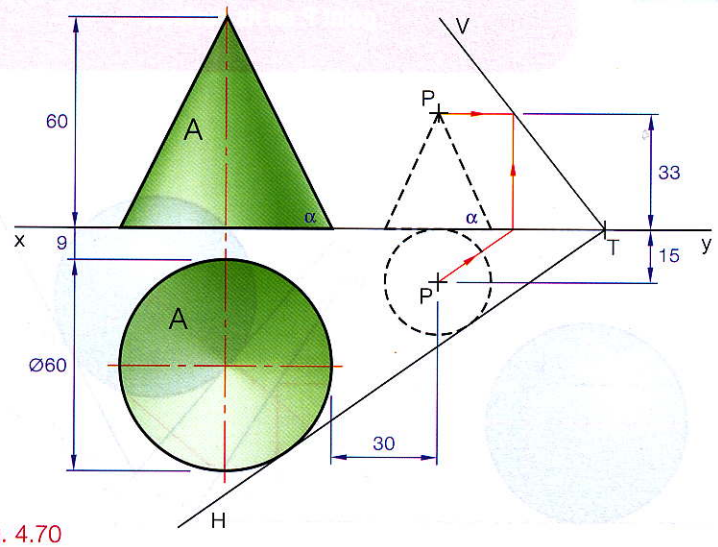


Fig. 4.70

(1) Set up the question.

(2) Draw the plan and elevation of a cone, having the same base angle as cone A and having point P as its apex.

(3) The horizontal trace will be tangential to the two base circles. The vertical trace is found as before.

The tangent plane's horizontal trace will be tangential to the base circles of the cones. It can also be seen from the pictorial that the plane makes contact with the cones along a whole generator, Fig. 4.71.

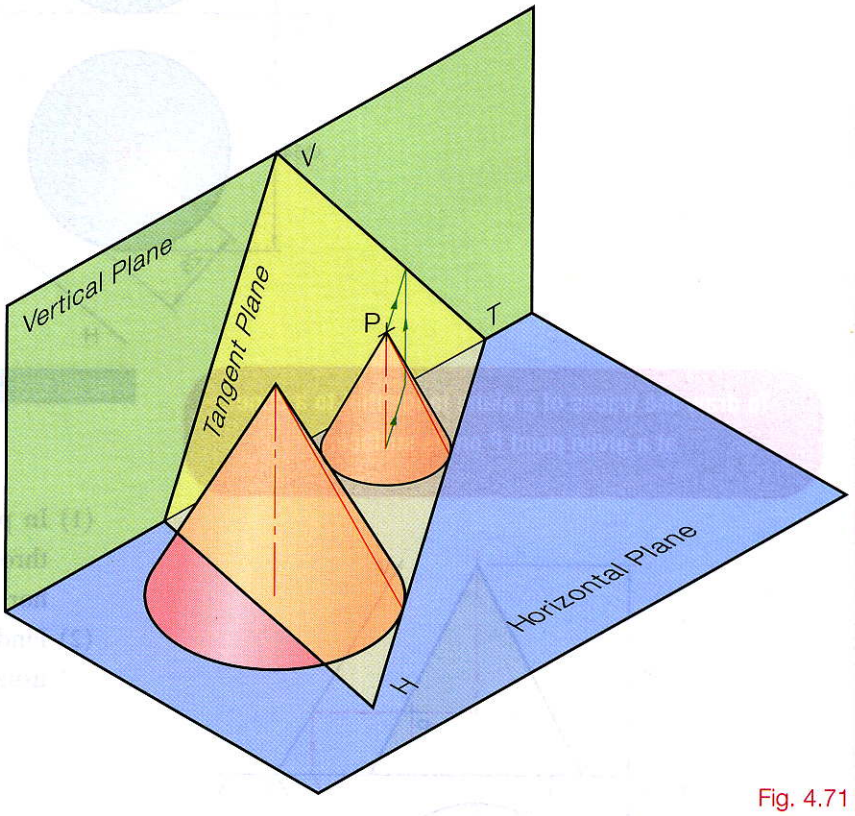


Fig. 4.71

HIGHER LEVEL

PROBLEM 4

To determine the traces of a plane which shall be tangential to a given cone and a given sphere.

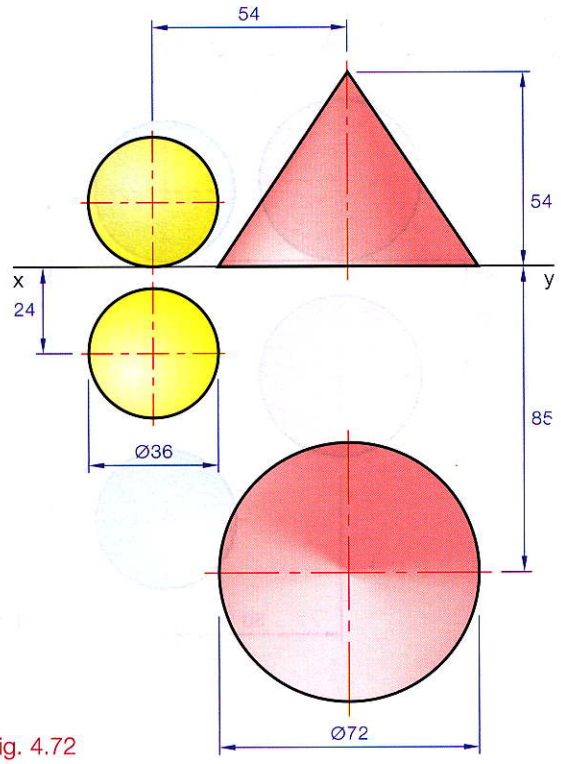


Fig. 4.72

A cone is placed over the sphere having the same base angle as the given cone. This cone's generators are tangential to the sphere.

The problem is now similar to the previous one and is solved in the same way, Fig 4.73.

It should be noted that there are four possible solutions to this problem. These are shown in Figures 4.74 and 4.75.

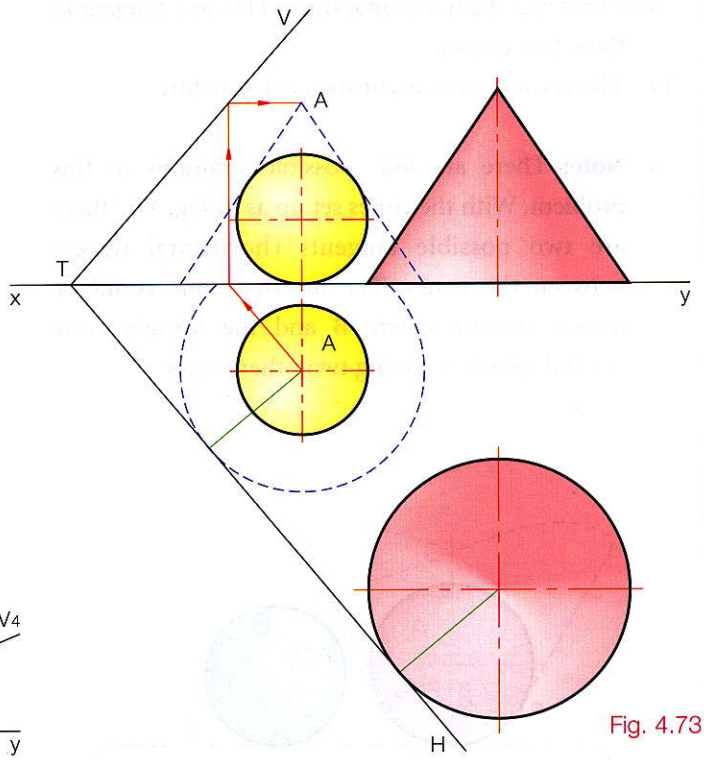


Fig. 4.73

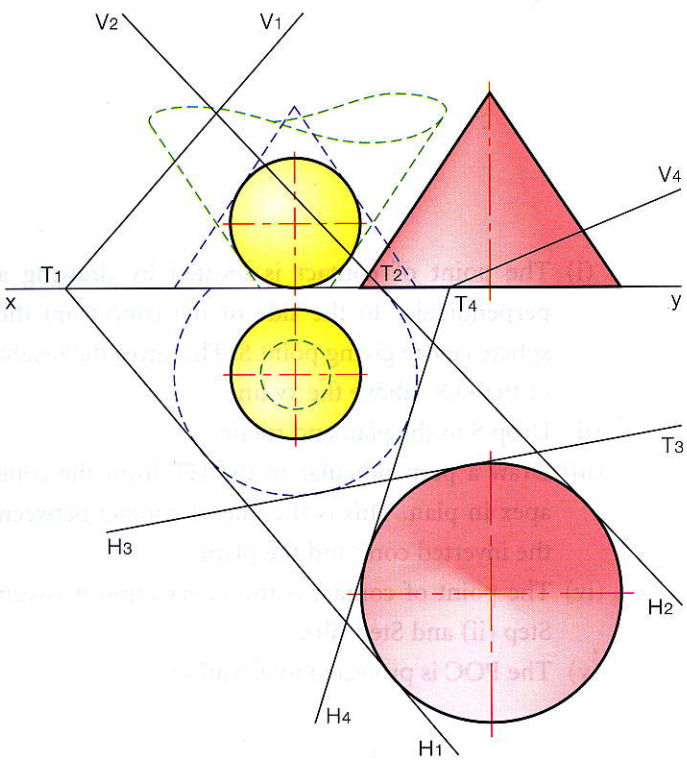


Fig. 4.74

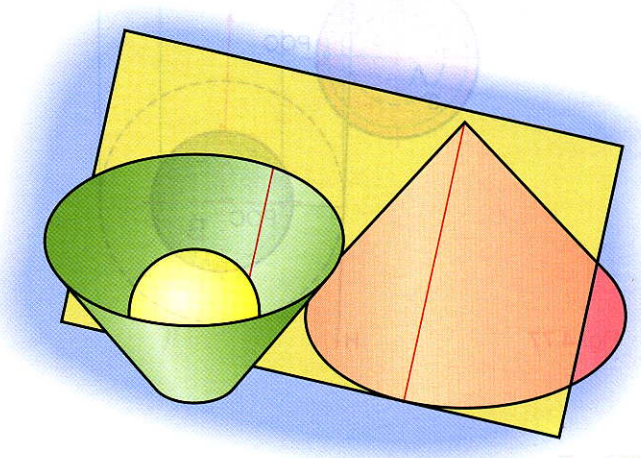


Fig. 4.75

Given the plan and elevation of two spheres A and B. Determine the traces of a plane that is tangential to both solids, makes an angle of 60° to the horizontal plane and passes between the spheres. Find the points of contact.

PROBLEM 5

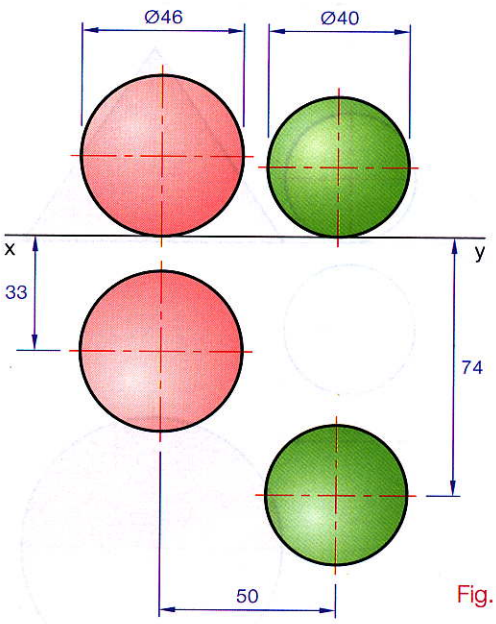


Fig. 4.76

(1) Construct a 60° base angle cone over one of the spheres. Now place an inverted 60° base angle cone to envelop the other sphere.
(2) Find the circles in plan where these cones make contact with the horizontal plane.
(3) Construct the horizontal trace (HT) as a tangent to these two circles.
(4) The vertical trace is constructed as before.

Note: There are four possible solutions to this problem. With the cones set up as in Fig. 4.77 there are two possible tangents (horizontal traces) between the cones. The inverted cone could be drawn around sphere B and the upright cone around sphere A, giving two other traces.

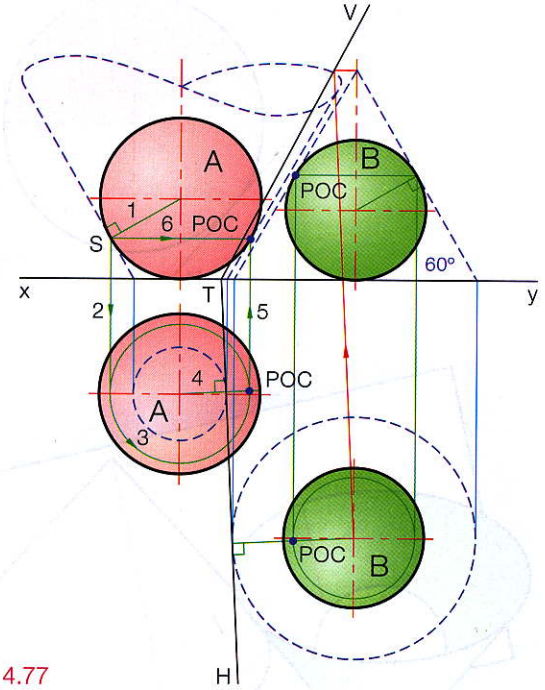


Fig. 4.77

(i) The point of contact is located by drawing a perpendicular to the side of the cone from the sphere centre giving point S. This gives the height of the POC above the xy line.
(ii) Drop S to the plan and rotate.
(iii) Draw a perpendicular to the HT from the cone apex in plan. This is the line of contact between the inverted cone and the plane.
(iv) The point of contact is the intersection between Step (ii) and Step (iii).
(v) The POC is projected to elevation.

Activities

Q1. Fig. 4.78 shows the elevation of a cone and a sphere. The cone is in contact with the sphere A at point P.
Draw the elevation and plan of the solids in contact and determine the exact position of point P.

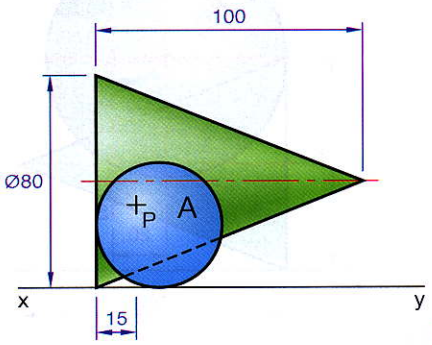


Fig. 4.78

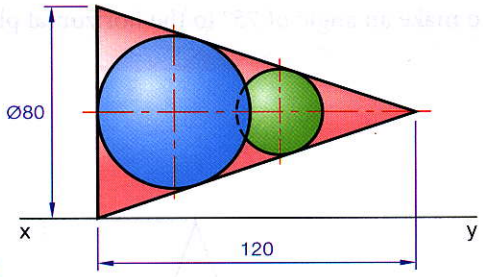


Fig. 4.79

Q2. Fig. 4.79 shows the elevation of a square-based pyramid and two spheres. The solids are in contact with each other.
Draw the elevation and plan of the solids showing all points of contact.

Q3. Fig. 4.80 shows the elevation of a sphere A and a cone B. The two solids are in contact.
(i) Draw the plan and elevation of the sphere and cone in contact.
(ii) Draw the projections of another sphere C of 40 mm diameter that is in contact with the cone and sphere and has its centre on line S–S.

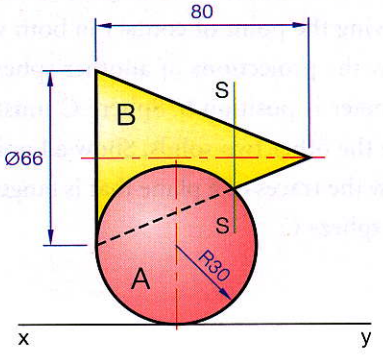


Fig. 4.80

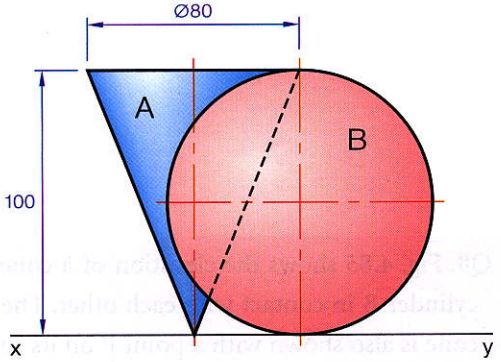


Fig. 4.81

Q4. Fig. 4.81 shows the elevation of a right cone A and a sphere B. Both solids are in contact.
(i) Draw the plan and elevation of the solids.
(ii) Draw the projections of the smallest possible sphere that touches the cone A, the sphere B and the horizontal plane.

Q5. Fig. 4.82 shows the elevation of a sphere A in contact with a cone B.
(i) Draw the plan and elevation of the two solids showing the point of contact clearly.
(ii) Draw the projections of a second sphere C of diameter 30 mm which touches both solids. Sphere C makes contact with sphere A at a point 44 mm above the horizontal plane.

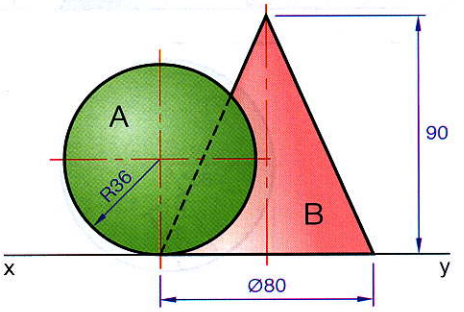


Fig. 4.82

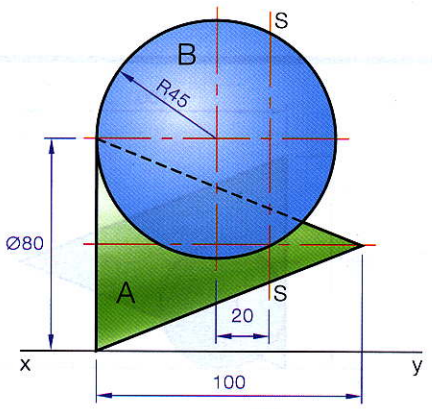


Fig. 4.83

Q6. The diagram Fig. 4.83 shows a right cone A and a sphere B, which are in contact with each other.

(i) Draw the front elevation, end elevation and plan of the solids showing the point of contact in all views.

(ii) Draw the projections of another sphere C of 36 mm diameter and having its centre on line S–S. Sphere C must make contact with the other two solids. Show all points of contact.

(iii) Draw the traces of a plane that passes through the centre of sphere B and the apex of cone A. The plane is to make an angle of 75° to the horizontal plane.

Q7. Fig. 4.84 shows the elevation of a right cone A in contact with a sphere B.

(i) Draw the elevation and plan of the two solids showing the point of contact in both views.

(ii) Draw the projections of another sphere C of 30 mm diameter is position S. Sphere C must make contact with the other two solids. Show all points of contact.

(iii) Draw the traces of a plane that is tangential to cone A and sphere C.

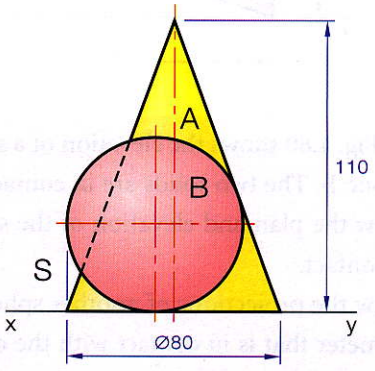


Fig. 4.84

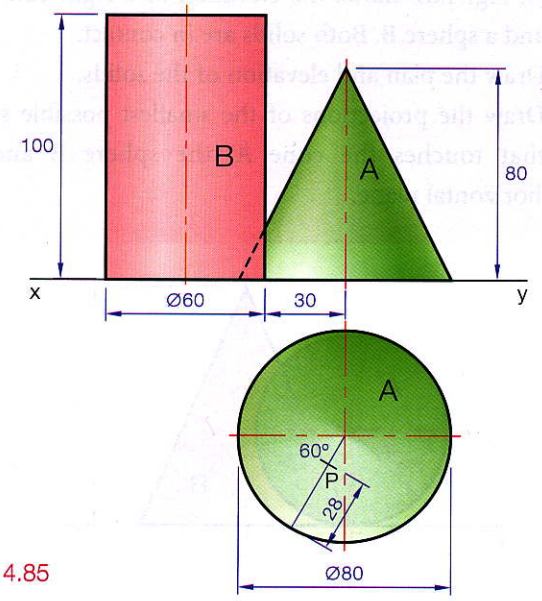


Fig. 4.85

Q8. Fig. 4.85 shows the elevation of a cone A and a cylinder B in contact with each other. The plan of the cone is also shown with a point P on its surface.

(i) Draw the plan and elevation of both solids.

(ii) Draw the projections of a sphere C which makes contact with the cone A at point P and which also makes contact with the cylinder.

(iii) Draw the traces of the plane that passes through the lowest point of sphere C and is tangential to cone A.