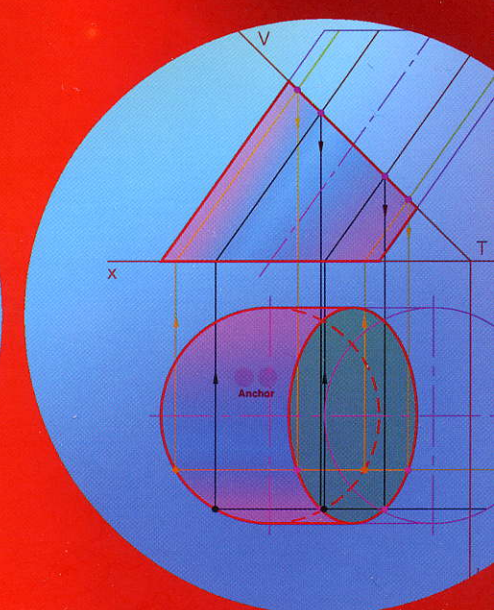
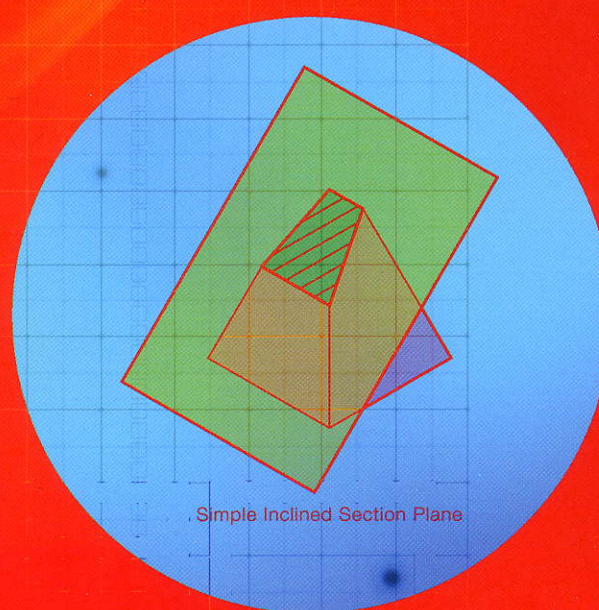
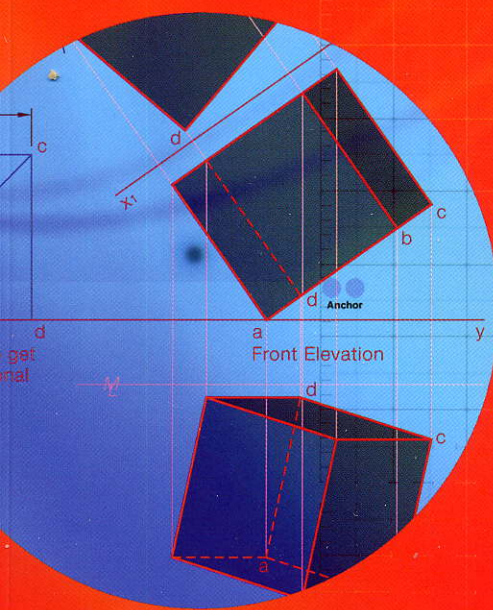


# GRAPHICS IN DESIGN & COMMUNICATION

1

PLANE AND DESCRIPTIVE GEOMETRY



DAVID ANDERSON



# 2 AREA

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**PROJECTION SYSTEMS**

# 2 Orthographic and Auxiliary Projection

## SYLLABUS OUTLINE

### Areas to be studied:

- Definition of a plane.
- Principal planes of reference.
- Auxiliary views, including second and subsequent auxiliary views.
- True shapes of surfaces and true lengths of lines.

### Learning outcomes

Students should be able to:

#### Higher and Ordinary levels

- Represent three-dimensional objects in logically arranged two-dimensional views.
- Apply their knowledge of reference planes and auxiliary projection planes to solving problems using a first auxiliary view.
- Present drawings in first-angle orthographic conventional views.
- Determine the projections, inclinations, true length and true shape, of lines and planes.

#### Higher Level only

- Apply their knowledge of reference planes and auxiliary projection planes to solving problems using a first auxiliary view and subsequent auxiliary views.
- Present drawings in third-angle orthographic conventional views.
- Determine the projections of lines given the angles of inclination to the principal planes of reference.

## Plane

A plane is a flat surface with no thickness. If two points are selected on a plane and joined with a straight line, then the straight line will lie on the plane along its full length. Planes are considered to have no boundaries, to be limitless. We usually draw edges to the planes to help our visualisation of them.

Fig. 2.1 shows the principal planes of reference. Two planes, one vertical and one horizontal, intersect along the straight line  $xy$ . These planes divide space into four sections: first, second, third and fourth angles. When representing objects we generally place them in the first angle or the third angle and project their image onto the horizontal plane and the vertical plane. This gives first-angle projection and third-angle projection.

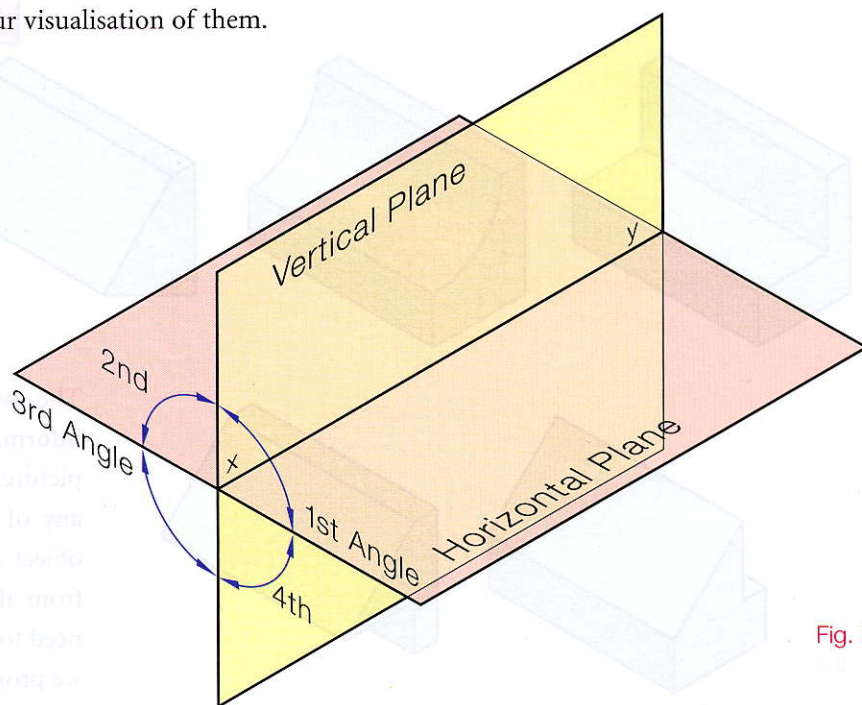


Fig. 2.1



## First-angle Projection

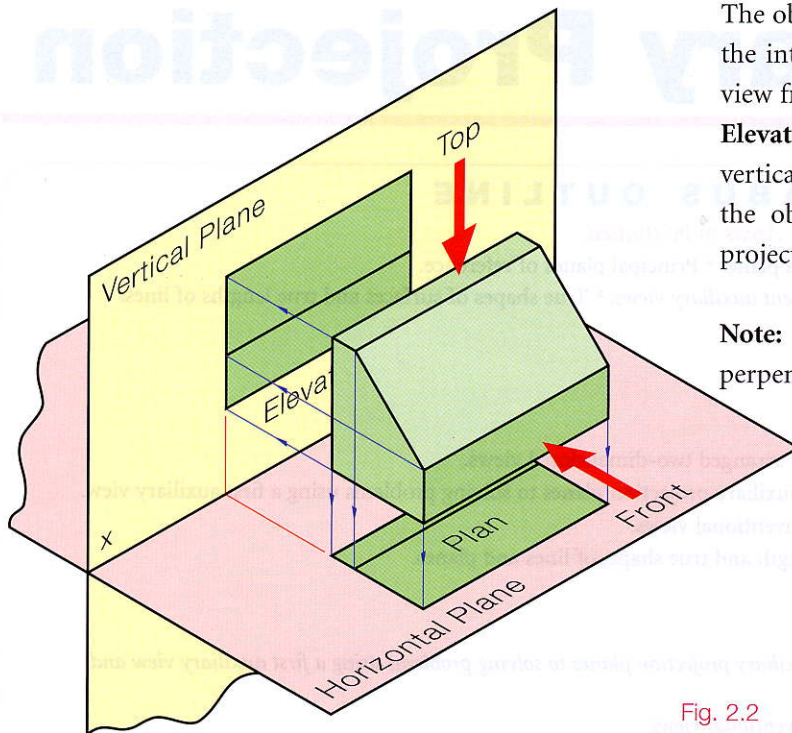


Fig. 2.2

We now fold the horizontal plane down in line with the vertical plane. The plane is hinged about the  $xy$  line. This gives two views of the one object. The elevation is always directly above the plan.

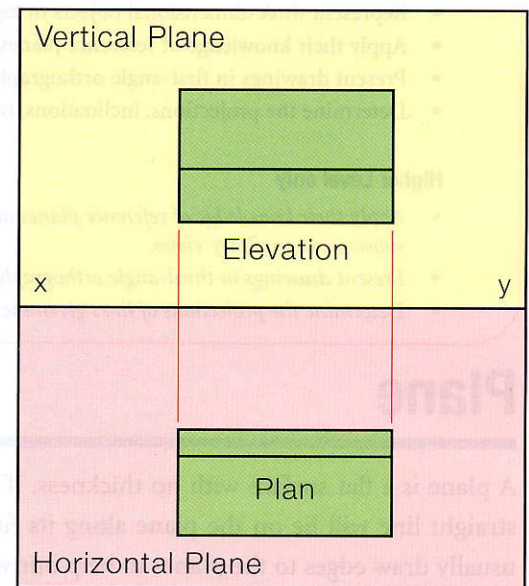


Fig. 2.3

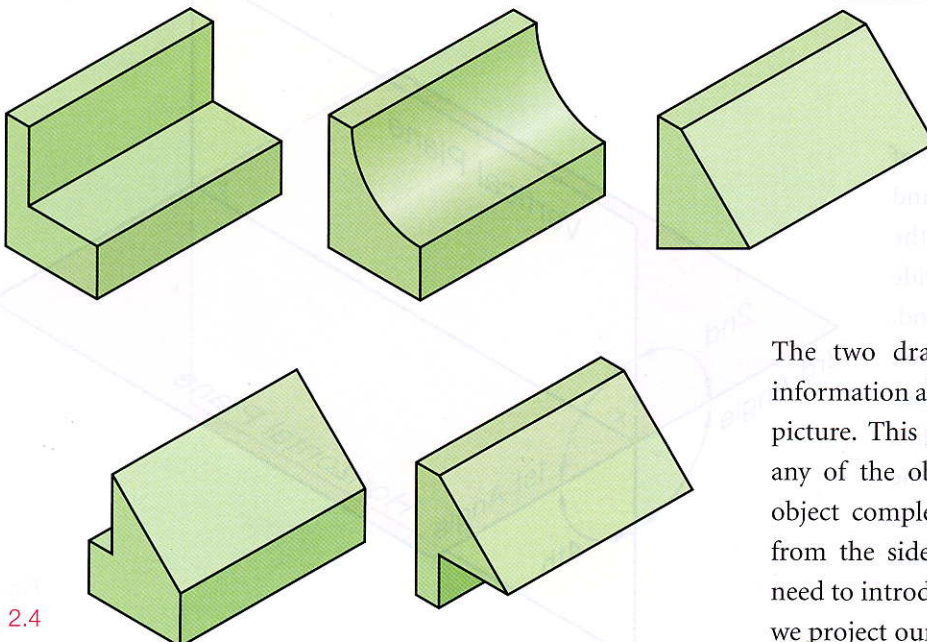


Fig. 2.4

The two drawings together give us a lot of information about the object but not the complete picture. This plan and elevation could represent any of the objects in Fig. 2.4. To represent the object completely we need a third view, a view from the side. When viewing from the side we need to introduce a new vertical plane onto which we project our image.



The plane must be perpendicular to the line of sight (Fig. 2.5). When we consider all three views together we have a complete representation of the object.

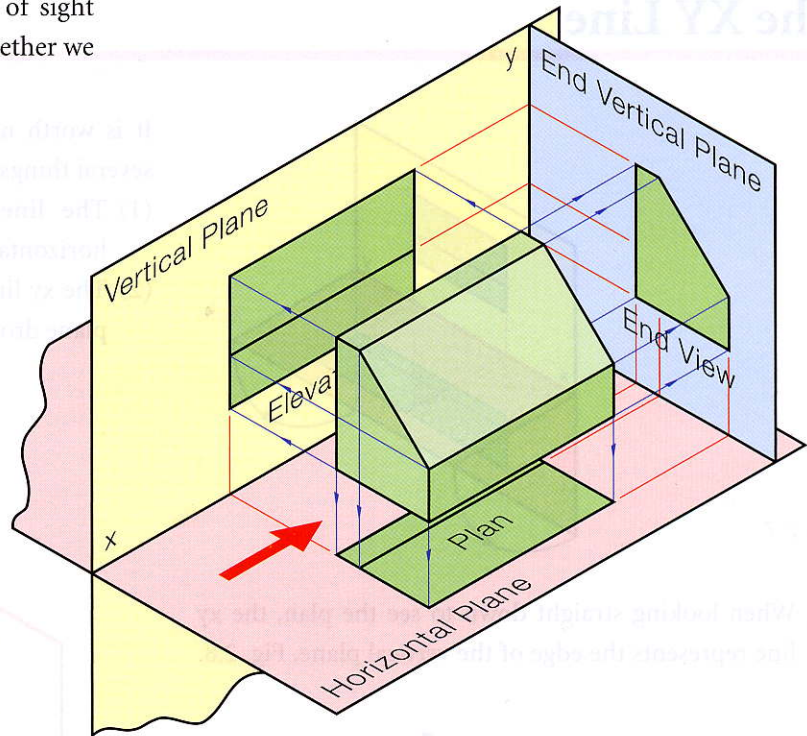


Fig. 2.5

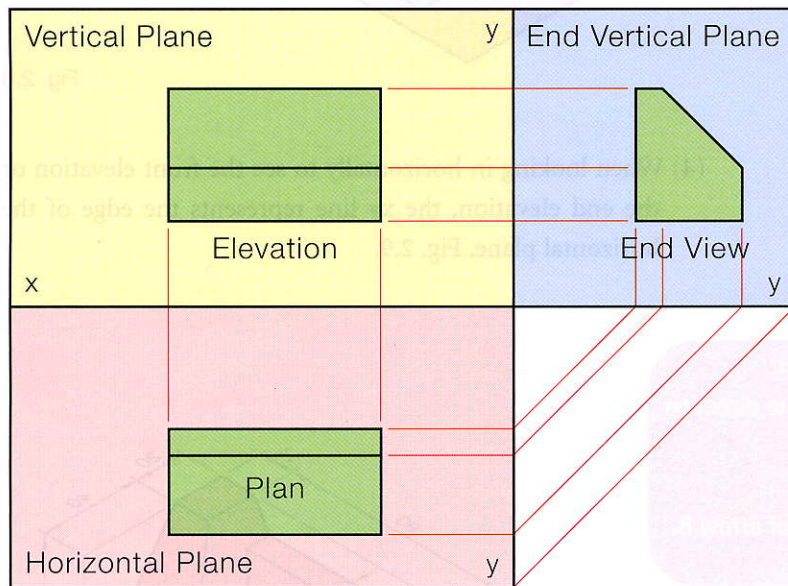


Fig. 2.6

As was mentioned earlier, the vertical planes and horizontal plane are limitless in size. When drawing objects in this format, **orthographic projection**, we dispense with the plane edges and just use the hinge lines, i.e. xy line and yy line. In this example, for clarity, the object was raised up above the horizontal plane. Usually the object is placed on the horizontal plane. This means that the elevations will be on the xy line.

## The XY Line

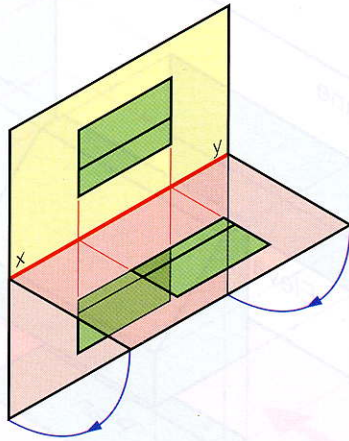


Fig. 2.7

- (3) When looking straight down to see the plan, the xy line represents the edge of the vertical plane, Fig. 2.8.

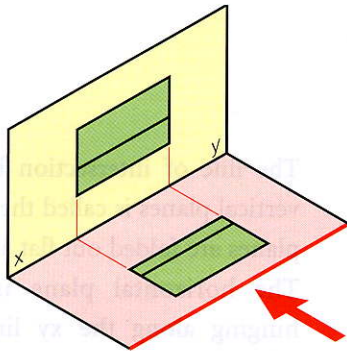


Fig. 2.9

It is worth noting at this stage that the xy line represents several things:

- (1) The line of intersection between the vertical and horizontal planes, Fig. 2.7.
- (2) The xy line is the hinge line about which the horizontal plane drops down in line with the vertical plane, Fig. 2.7.

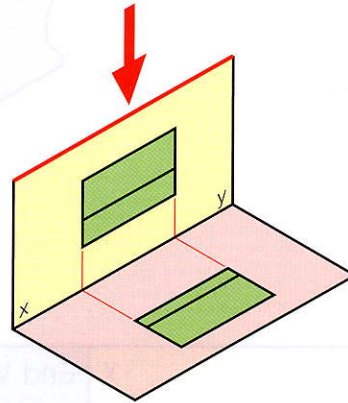


Fig. 2.8

- (4) When looking in horizontally to see the front elevation or the end elevation, the xy line represents the edge of the horizontal plane, Fig. 2.9.

Fig. 2.10 shows a pictorial view of an object.

- (i) Draw a front elevation of the object looking in the direction of arrow A.
- (ii) Project a plan from the front elevation.
- (iii) Project an end elevation looking in the direction of arrow B.

- (1) Draw the xy line first.
- (2) Set up a box that will contain the front elevation on this xy line. The height will be 70 mm and the length will be 104 mm.
- (3) The box for the plan is usually drawn next. The plan will be the same length and directly below the front elevation. The size of the gap between the plan and the xy line is chosen to give a nice drawing layout.

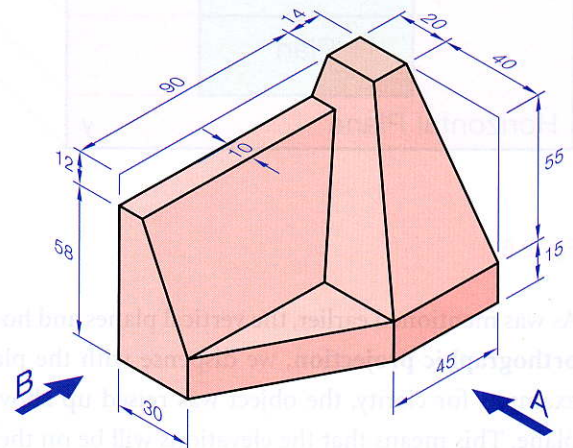


Fig. 2.10



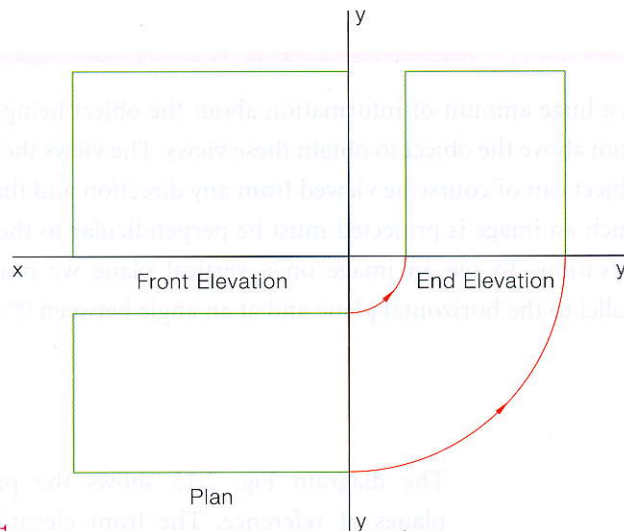


Fig. 2.11

- (5) The details of the three views are built up as shown in Fig. 2.12.

Fig. 2.13 shows a pictorial view of a solid.

- (i) Draw a front elevation looking in the direction of arrow A.
- (ii) Draw an end elevation viewing in the direction of arrow B.
- (iii) Project a plan from the front elevation.

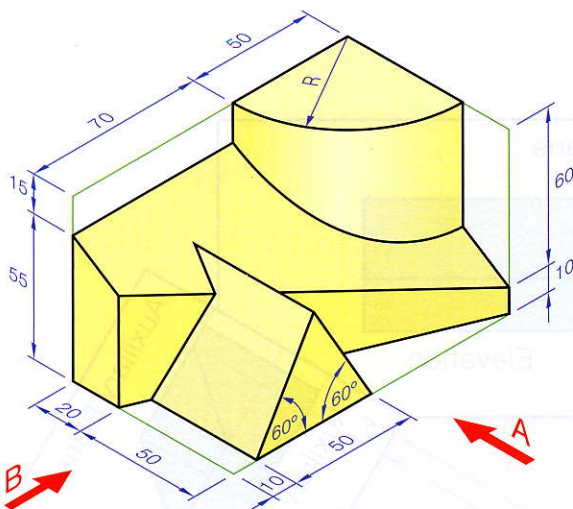


Fig. 2.13

- (2) The views are built up in stages. None of the three drawings can be completed on their own without using information from the other two views. In order to simplify the appearance of the drawing in Fig. 2.14 many of the construction lines have been left out.

- (4) The height of the box for the end elevation is projected across from the front elevation. The intersection between the xy line and the yy line gives the centre for the arcs swung up from the plan, Fig. 2.11.

These arcs represent the end vertical plane as it swings around into place.

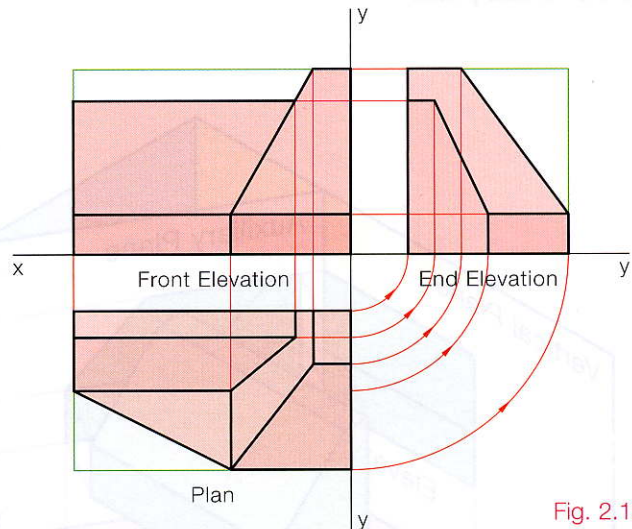


Fig. 2.12

- (1) Since arrow A points to the left the front elevation is drawn on the left. Start with the xy line and the three boxes to contain the views in their correct positions. Arrow B points to the right so the end view is on the right.

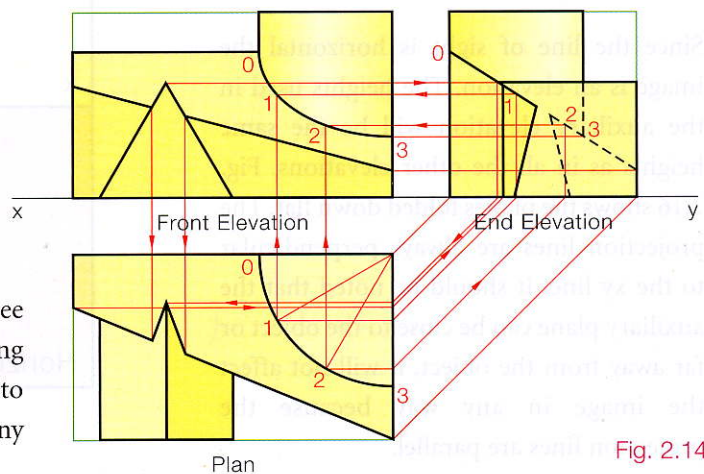


Fig. 2.14



## Auxiliary Elevation

The front elevation, end elevations and plan, together give a huge amount of information about the object being drawn. We looked from in front of the object, from the sides and from above the object to obtain these views. The views themselves were projected onto the principal planes of reference. An object can of course be viewed from any direction and the image projected onto a new plane. Remember, the plane onto which an image is projected must be perpendicular to the line of sight. An image projected onto a vertical plane is an elevation. To see an image on a vertical plane we must view horizontally. An auxiliary elevation, therefore, is a view parallel to the horizontal plane and at an angle between  $0^\circ$  and  $90^\circ$  to the vertical plane.

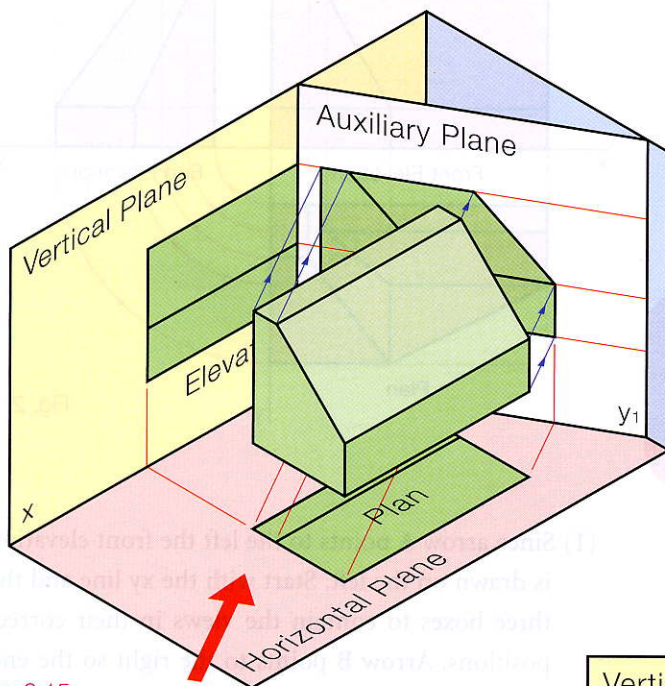


Fig. 2.15

The diagram Fig. 2.15 shows the principal planes of reference. The front elevation and plan are projected in the normal way. To view in the direction of the arrow we must introduce a new vertical plane, as shown, perpendicular to the line of sight. This auxiliary plane intersects the horizontal plane along a line which we call the  $x_1y_1$  line. The view of the object may now be projected onto this plane.

Since the line of sight is horizontal the image is an elevation. The heights used in the auxiliary elevation will be the same heights as in all the other elevations. Fig. 2.16 shows the planes folded down flat. The projection lines are always perpendicular to the  $xy$  line. It should be noted that the auxiliary plane can be close to the object or far away from the object, it will not affect the image in any way because the projection lines are parallel.

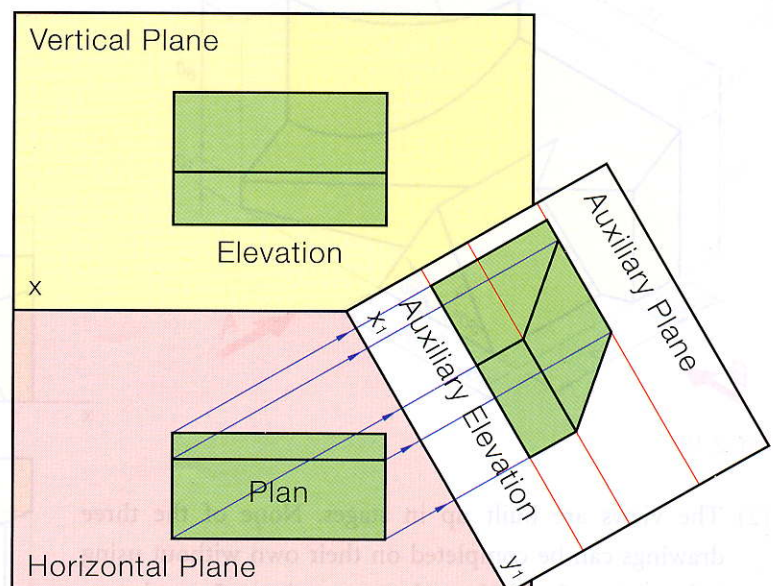


Fig. 2.16



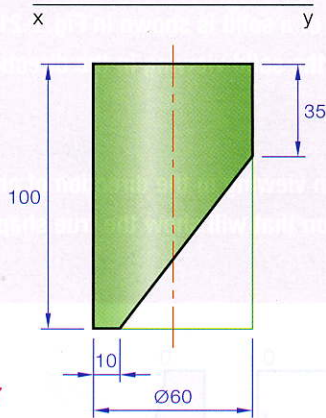


Fig. 2.17

- (1) Draw the plan.
- (2) The elevation will be a circle.
- (3) Divide the elevation into twelve parts and index.
- (4) Project these divisions down to the cut surface of the plan.
- (5) The cut surface is an edge view in plan. View perpendicular to an edge view and we see the true shape. Draw the  $x_1y_1$  parallel to the cut surface.
- (6) Project points up from the plan and take the heights of each point from the elevation.

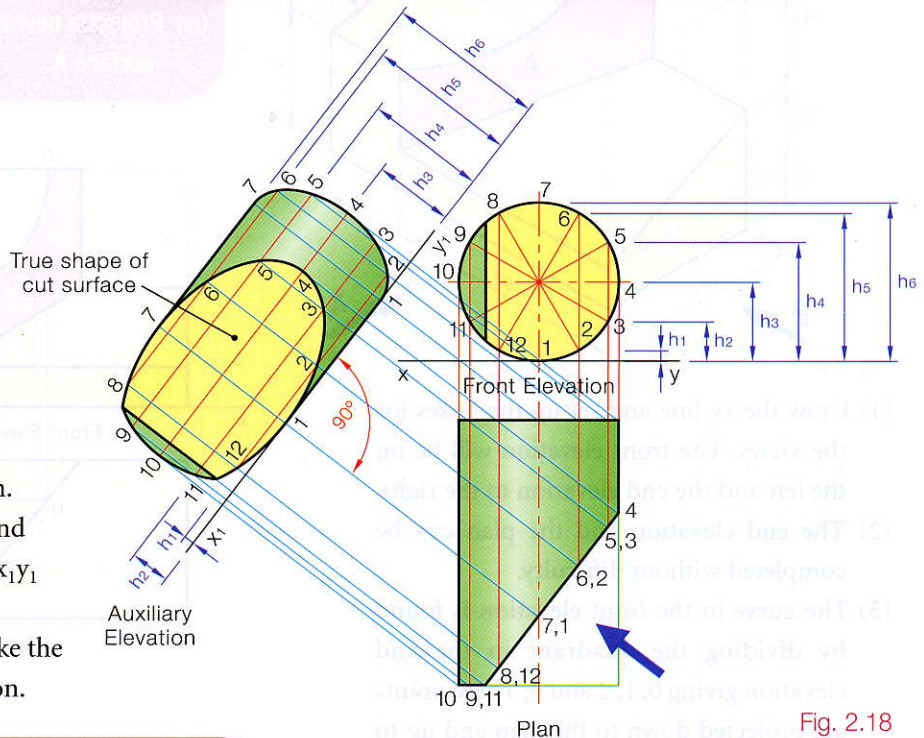


Fig. 2.18

Fig. 2.19 shows the plan and elevation of an object which has been cut by a vertical plane as shown.

- (i) Draw the given views.
- (ii) Draw an auxiliary elevation of the object that will show the cut surface as a true shape.

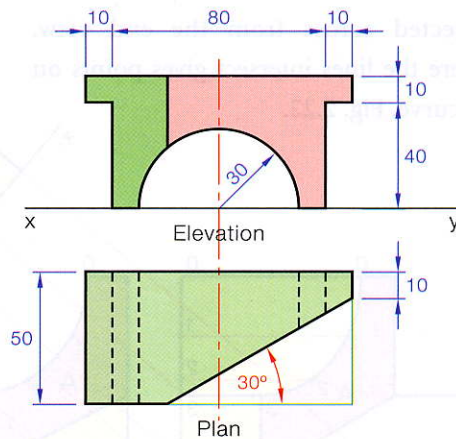


Fig. 2.19

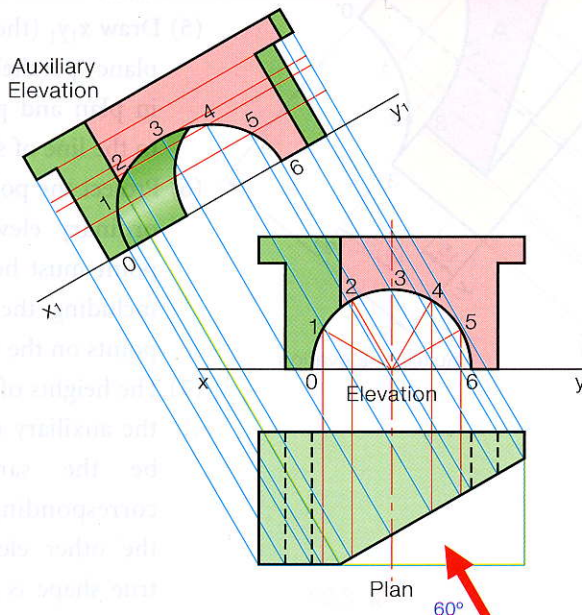


Fig. 2.20

- (1) Divide the semicircle into six and index the points.
- (2) The cut surface is at  $30^\circ$  in the plan so the  $x_1y_1$  will be at  $30^\circ$  and the viewing angle at  $60^\circ$ .
- (3) Project the points from the plan and take heights from the elevation.



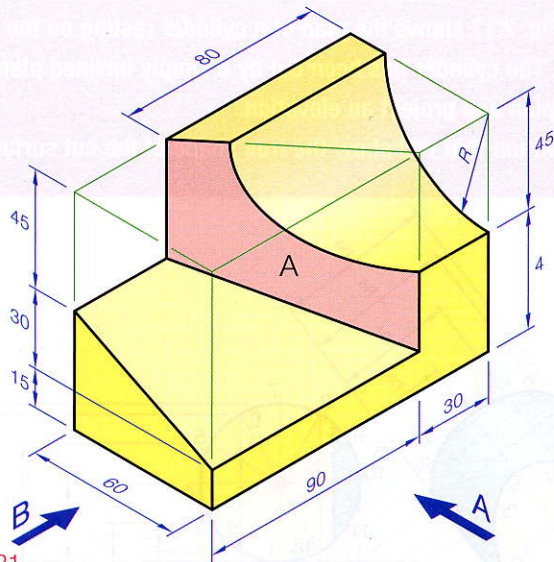


Fig. 2.21

- (1) Draw the xy line and set up the boxes for the views. The front elevation will be on the left and the end elevation to the right.
- (2) The end elevation and the plan can be completed without difficulty.
- (3) The curve in the front elevation is found by dividing the quadrant in the end elevation giving 0, 1, 2 and 3. These points are projected down to the plan and up to the front elevation. They are then projected across from the end view. Where the lines intersect gives points on the curve, Fig. 2.22.

- A pictorial drawing of a solid is shown in Fig. 2.21.
- (i) Draw an elevation of the solid viewing in the direction of arrow A.
  - (ii) Project a plan.
  - (iii) Draw an end elevation viewing in the direction of arrow B.
  - (iv) Project a new elevation that will show the true shape of surface A.

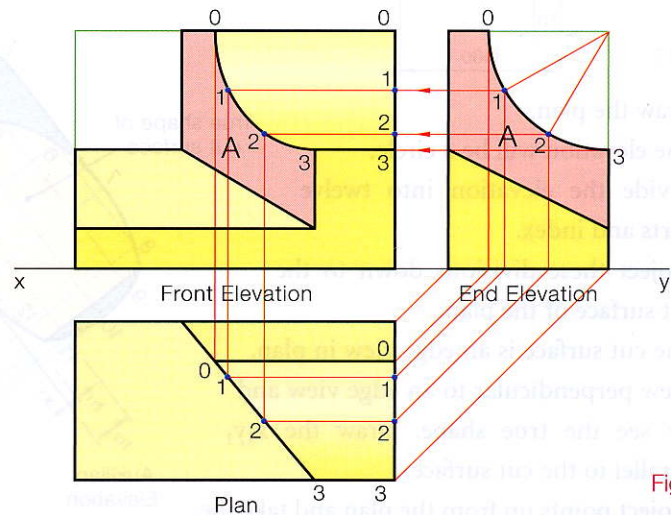


Fig. 2.22

- (4) In order to see the surface A as a true shape we must view it straight on. Surface A is seen as an edge view in the plan. We view perpendicular to this edge view.

- (5) Draw  $x_1y_1$  (the new vertical plane) parallel to surface A in plan and perpendicular to the line of sight.

- (6) Project the points as for an ordinary elevation. Every point must be brought up including the two sets of points on the curves.

- (7) The heights of all points on the auxiliary elevation will be the same as the corresponding points on the other elevations. The true shape is found in the auxiliary.

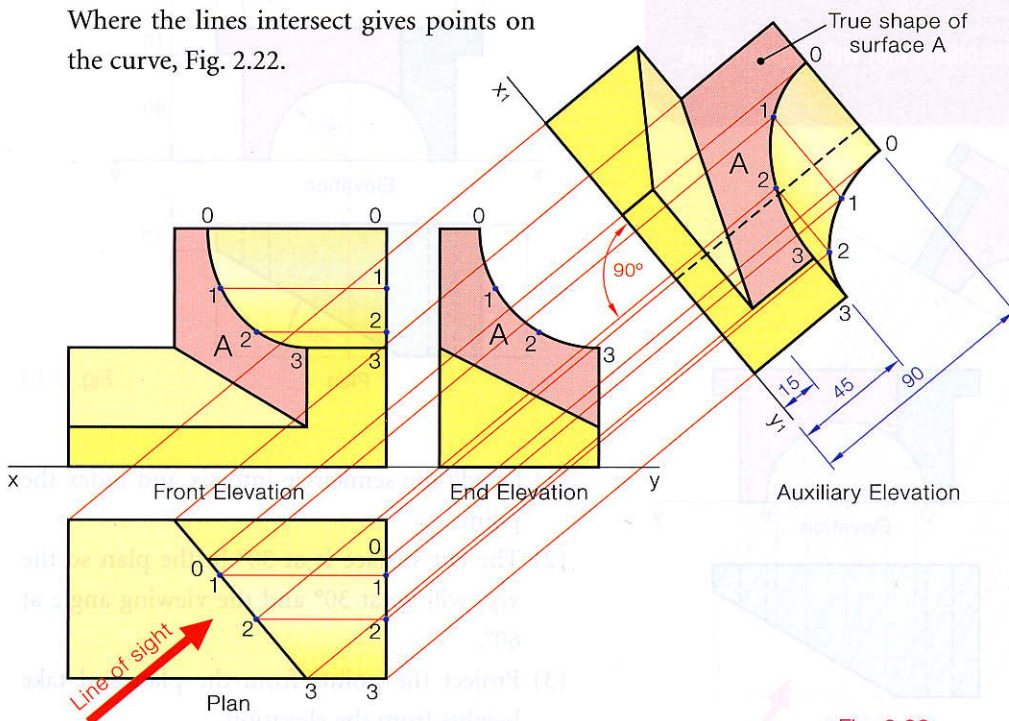


Fig. 2.23



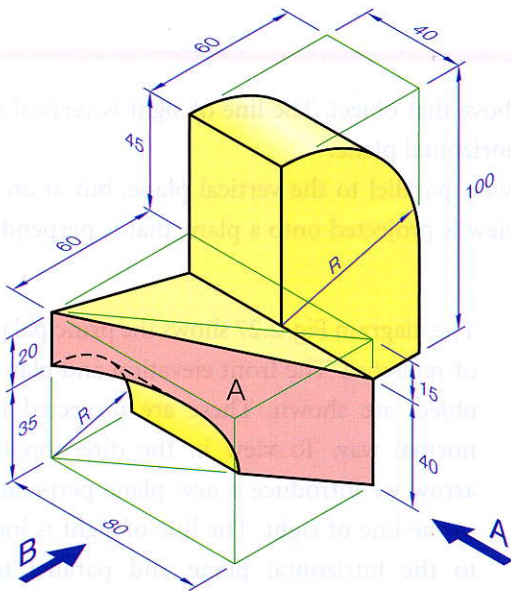


Fig. 2.24 shows a pictorial drawing of an object.

- (i) Draw an elevation of the solid viewing in the direction of arrow A.
- (ii) Project a plan from the elevation.
- (iii) Project an end elevation viewing in the direction of arrow B.
- (iv) Project a new elevation of the solid that will show the true shape of surface A.

Fig. 2.24

- (1) The front elevation will be on the left because we are viewing from the right. The end elevation will be to the right of the front elevation. Draw the  $xy$  line and draw the boxes for the various views.
- (2) The curve in the end view is found by dividing the quadrant in the front elevation and projecting points 0, 1, 2 and 3 round through the views.
- (3) Surface A is seen as an edge view in the plan, so to see a true shape of A perpendicular to the edge view,  $x_1y_1$  is drawn perpendicular to the line of sight.

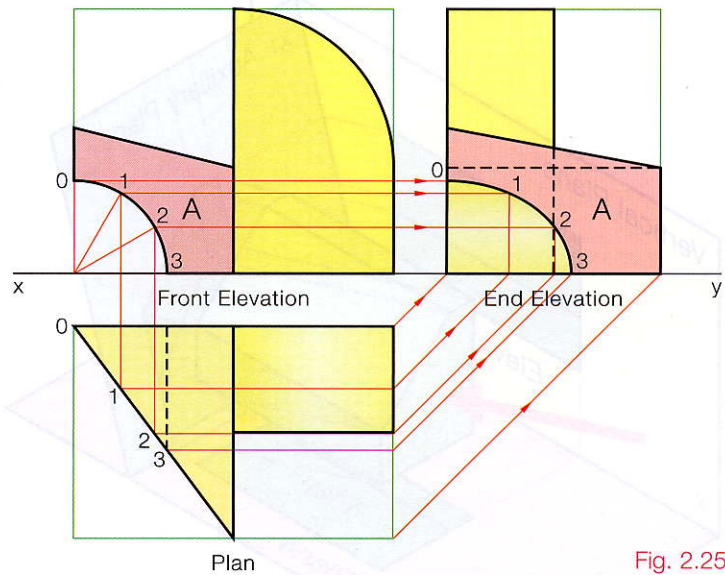


Fig. 2.25

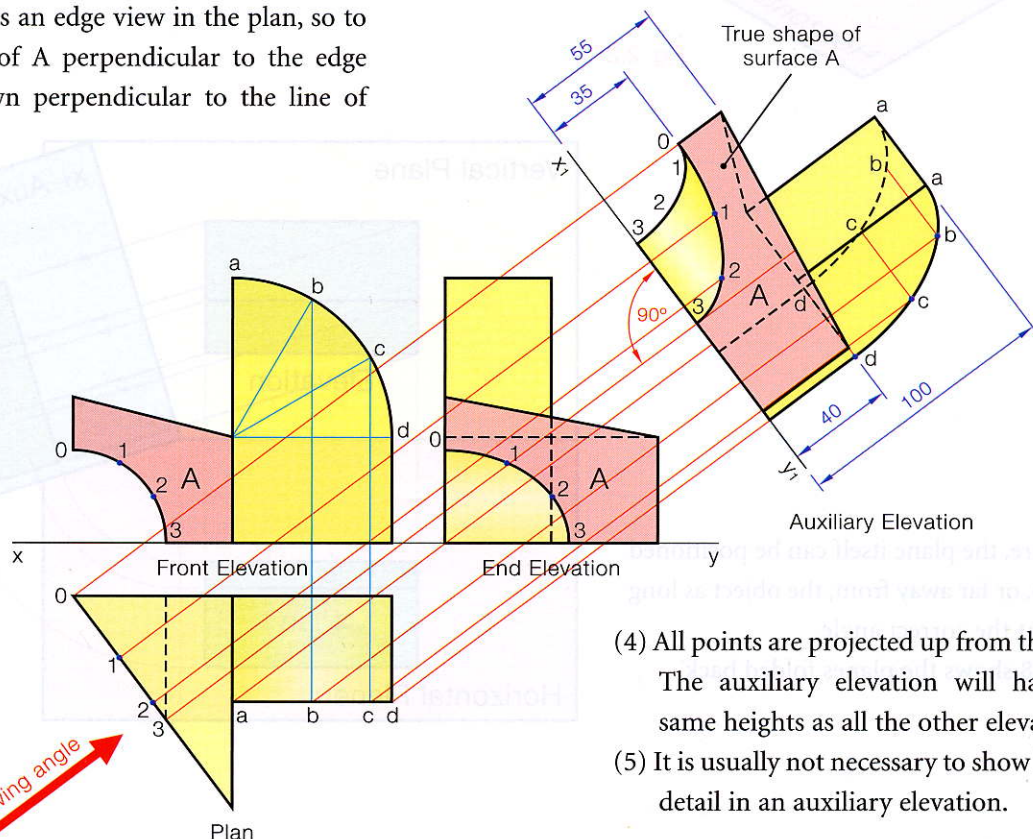


Fig. 2.26

- (4) All points are projected up from the plan. The auxiliary elevation will have the same heights as all the other elevations.
- (5) It is usually not necessary to show hidden detail in an auxiliary elevation.



## Auxiliary Plans

When viewing an object to see its plan we are viewing from directly above that object. The line of sight is vertical and is therefore parallel to the vertical plane. The view is projected onto the horizontal plane.

When viewing an object to get an auxiliary plan we continue to view parallel to the vertical plane, but at an angle between  $0^\circ$  and  $90^\circ$  to the horizontal plane. As is always the case, the view is projected onto a plane that is perpendicular to the line of sight.

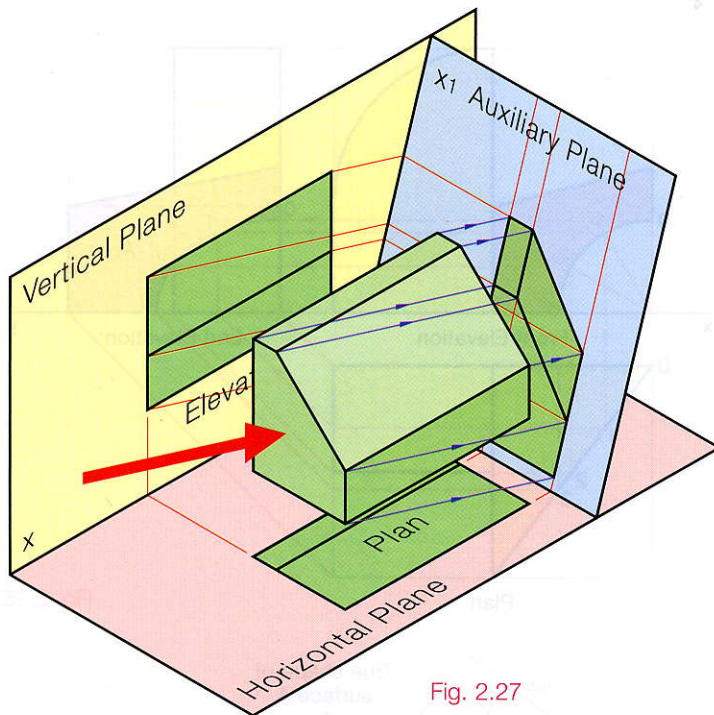


Fig. 2.27

The diagram Fig. 2.27 shows the principal planes of reference. The front elevation and plan of an object are shown. These are projected in the normal way. To view in the direction of the arrow we introduce a new plane perpendicular to the line of sight. The line of sight is inclined to the horizontal plane and parallel to the vertical plane. The auxiliary plane is inclined to the horizontal plane and perpendicular to the vertical plane. The  $x_1y_1$  is the line of intersection between the auxiliary plane and the vertical plane. Since the line of sight is parallel to the vertical plane, the auxiliary plan which is produced will be the same distance from the  $x_1y_1$  as the ordinary plan is from the  $xy$  line.

As before, the plane itself can be positioned close to, or far away from, the object as long as it is at the correct angle.

Fig. 2.28 shows the planes folded back.

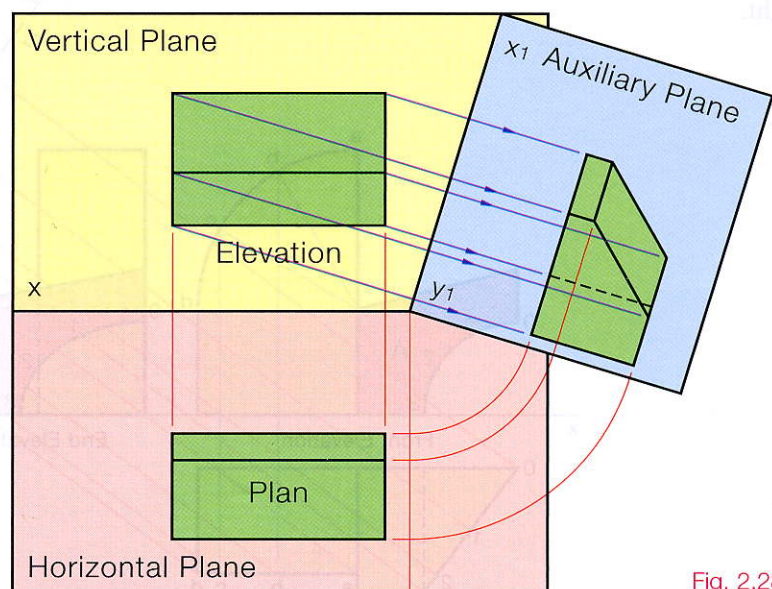


Fig. 2.28



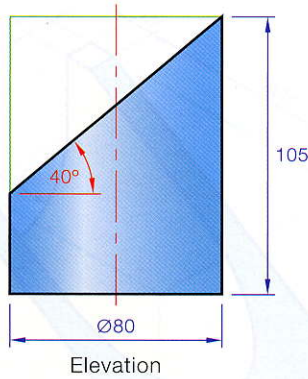


Fig. 2.29

The elevation of a cylinder that has been cut by a simply inclined plane is shown in Fig. 2.29.

- (i) Draw the given elevation and project a plan and an end elevation.
- (ii) Project a new plan of the cylinder that will show the true shape of the cut surface.

- (1) Draw the elevation. The plan, even though the cylinder is truncated, will be a circle.
- (2) Divide the plan into twelve equal divisions.
- (3) Project 0 to 11 up to the sloped surface in elevation and across to the end view.
- (4) Project the same points 0 to 11 from the plan to the end view. The points in the end view are found where the corresponding lines cross, Fig. 2.30.

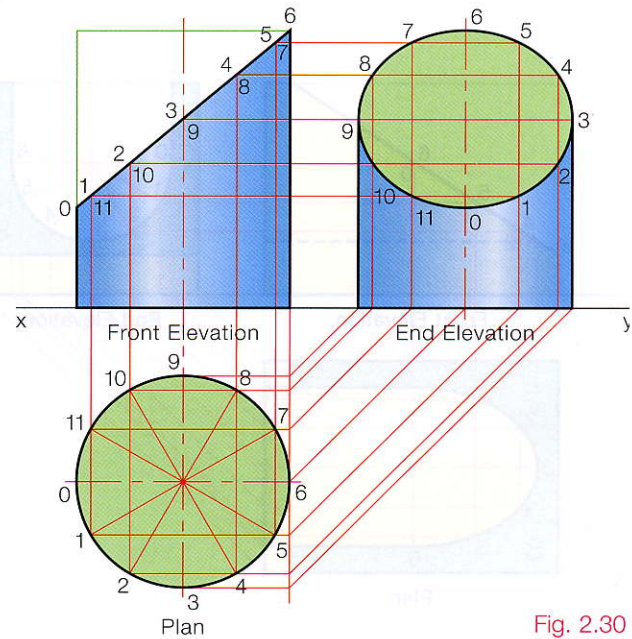


Fig. 2.30

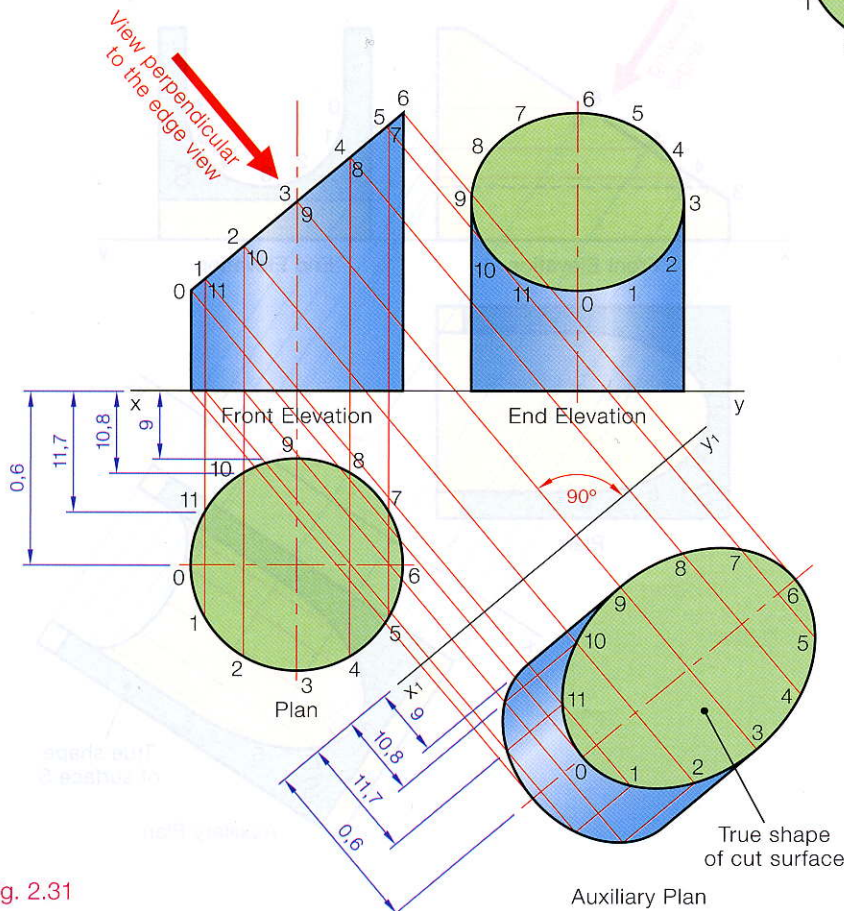


Fig. 2.31

- (5) The front elevation shows the edge view of the cut surface. To see the true shape we view perpendicular to the edge view.
- (6) The  $x_1y_1$  will be drawn parallel to the edge view, which will be perpendicular to the line of sight.
- (7) Project down the points from the elevation.
- (8) The distance from the  $xy$  line to point 9 is taken on the compass and measured on the appropriate line from the  $x_1y_1$  line. This locates point 9 in the auxiliary plan.
- (9) All other points are found in the same way, Fig. 2.31.



The pictorial drawing of a solid is shown in Fig. 2.32.

- (i) Project a front elevation of the solid, viewing in the direction of arrow A.
- (ii) Draw an end elevation, viewing in the direction of arrow B.
- (iii) Project a plan.
- (iv) Project a new plan of the solid showing the true shape of surface S.

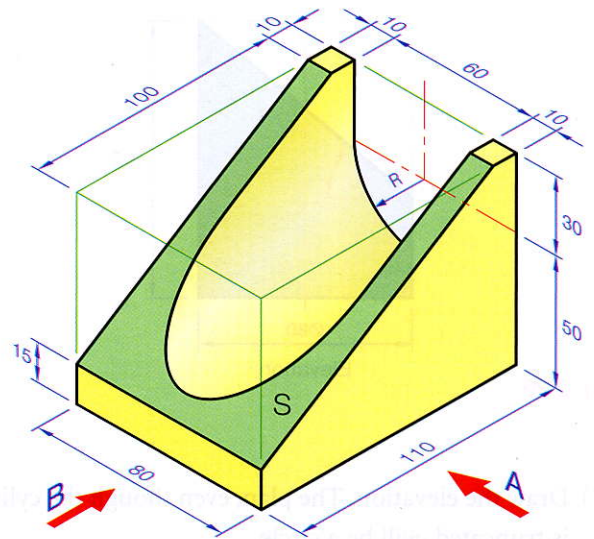


Fig. 2.32

Always think through the problem before starting to plan how the various components of the solution will be positioned on the sheet.

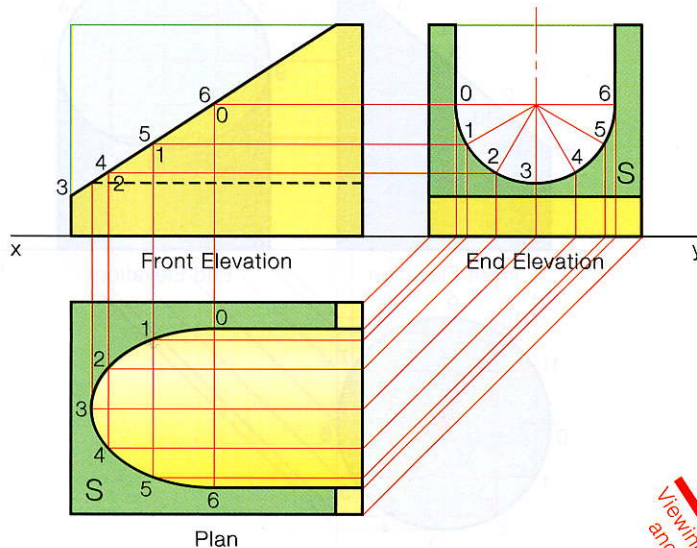


Fig. 2.33

- (1) Front elevation will be on the left and the end elevation to its right, Fig 2.33.
- (2) Both elevations need to be completed before the plan. The curve in the plan is found in the usual way.
- (3) To see the true shape of a surface we view perpendicular to the edge view.
- (4) The construction is clearly shown from the illustration Fig. 2.34.

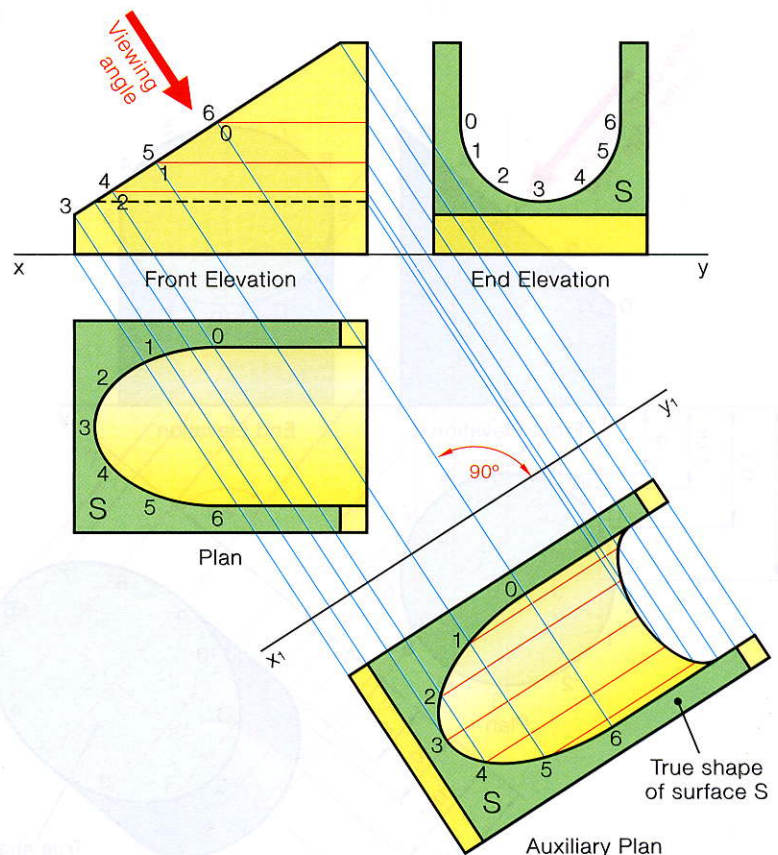


Fig. 2.34



Fig. 2.35 shows a pictorial view of a solid.

- (i) Draw the front elevation of the solid looking in the direction of arrow A.
- (ii) Project a plan from the front elevation.
- (iii) Project an end elevation viewing in the direction of arrow B.
- (iv) Construct an auxiliary plan that will show the true shape of surface S.

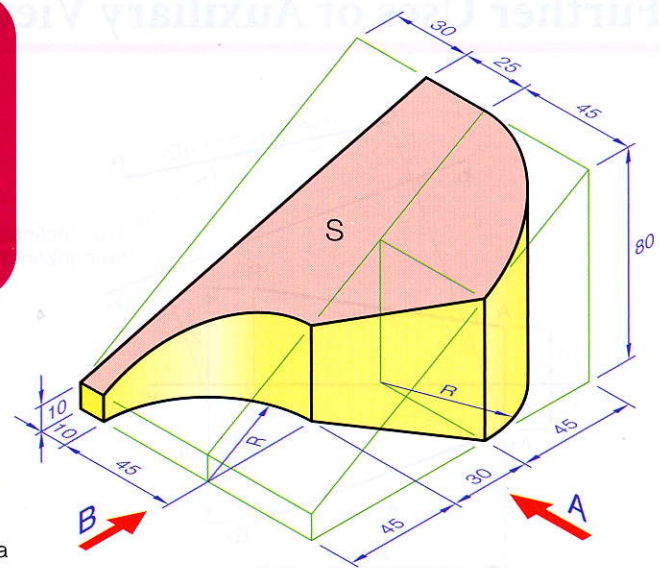


Fig. 2.35

- (1) Draw the front elevation, end elevation and plan. Ensure that you leave enough space for the auxiliary plan which will be down on the right.
- (2) Surface S appears as an edge view in the front elevation. View perpendicular to this edge view to see surface S as a true shape.

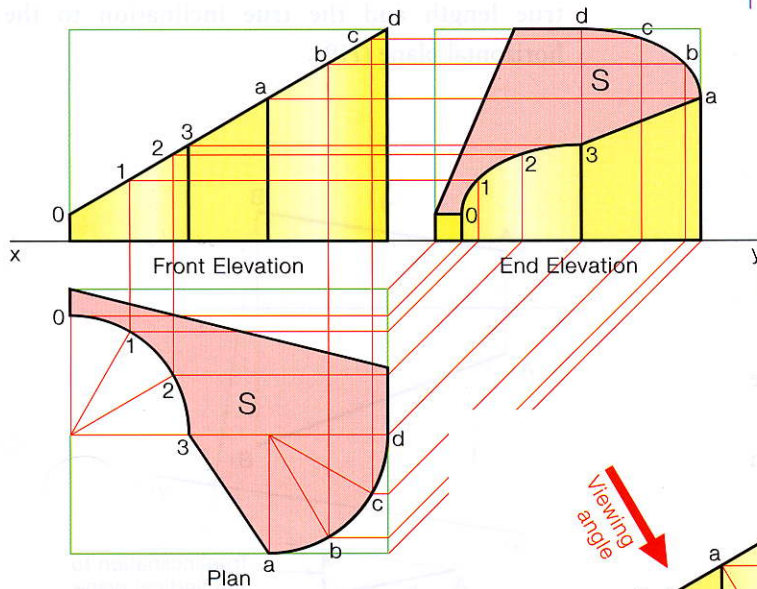


Fig. 2.36

- (3) From each point in the elevation, project lines in the direction of the viewing angle.
- (4) Draw the  $x_1y_1$  line perpendicular to the lines of projection.
- (5) For any point on the plan we take the distance the point is from the  $xy$  line and transfer this distance from  $x_1y_1$  to locate the point in the auxiliary plan, see Fig. 2.37.

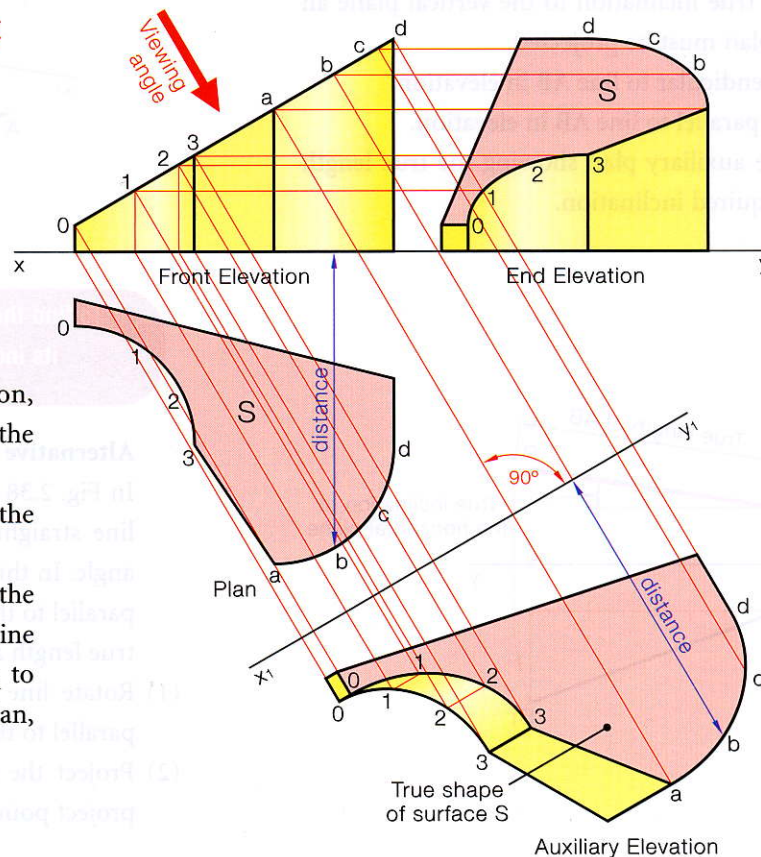


Fig. 2.37



## Further Uses of Auxiliary Views

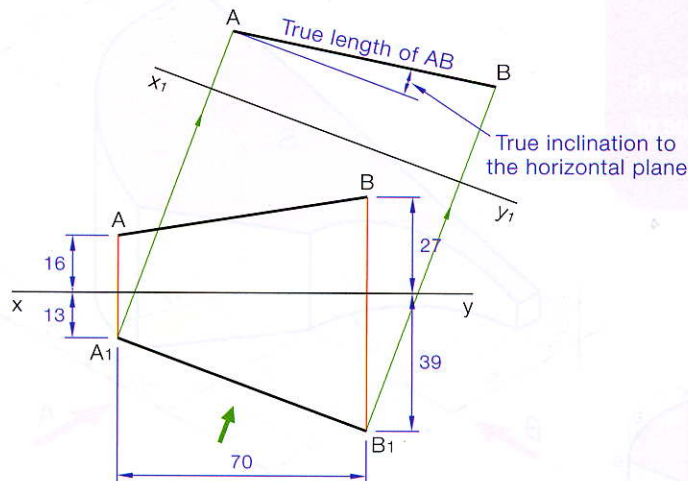


Fig. 2.38

**To find the true length of a line AB and to find its inclination to the vertical plane.**

- (1) Set up the plan and elevation to the same measurements as Fig. 2.38.
- (2) To see the true inclination to the vertical plane an auxiliary plan must be projected.
- (3) View perpendicular to line AB in elevation.
- (4) Draw  $x_1y_1$  parallel to line AB in elevation.
- (5) Project the auxiliary plan showing the true length and the required inclination.

**To find the true length of a line AB and to find its inclination to the horizontal plane.**

- (1) To see the true inclination of the line to the horizontal plane we use an auxiliary elevation. We view perpendicular to the plan of the line.
- (2) Draw the  $x_1y_1$  line parallel to the line in plan.
- (3) Project the new elevation which shows both the true length and the true inclination to the horizontal plane (HP).

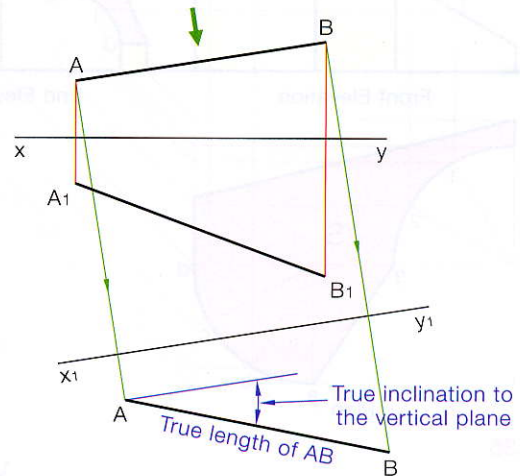


Fig. 2.39

**To find the true length of a line AB and to find its inclination to the horizontal plane.**

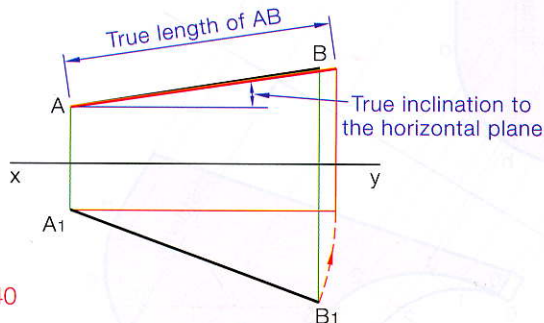


Fig. 2.40

### Alternative Method

In Fig. 2.38 we moved our line of sight to view the line straight on and thus see its true length and angle. In this method we rotate the line so that it is parallel to the vertical plane and hence will show its true length and true angle in elevation.

- (1) Rotate line AB in plan about point A until it is parallel to the  $xy$  line.
- (2) Project the end of the new line to elevation and project point B in elevation across to meet it.



To find the true length of a line AB and to find its inclination to the vertical plane.

### Alternative Method

Similar construction as Fig. 2.40 except the line is rotated in elevation giving the true length and angle in plan.

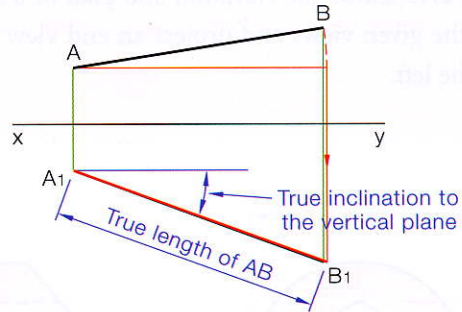


Fig. 2.41

To find the inclination of a plane abcd to the horizontal plane.

Lines dc and ab are horizontal, and are therefore seen as true lengths in the plan. Project an auxiliary elevation, viewing in the direction of the true lengths. The plane abcd will appear as an edge view in the auxiliary and the angle can be seen.

### Note:

- (1) A line on a plane parallel to the xy line in elevation will appear as a true length in plan.  
The converse is also true.
- (2) A line on a plane parallel to the xy line in plan will appear as a true length in elevation.
- (3) When a line on a plane appears as a true length, viewing along the true length will show an edge view of that plane.

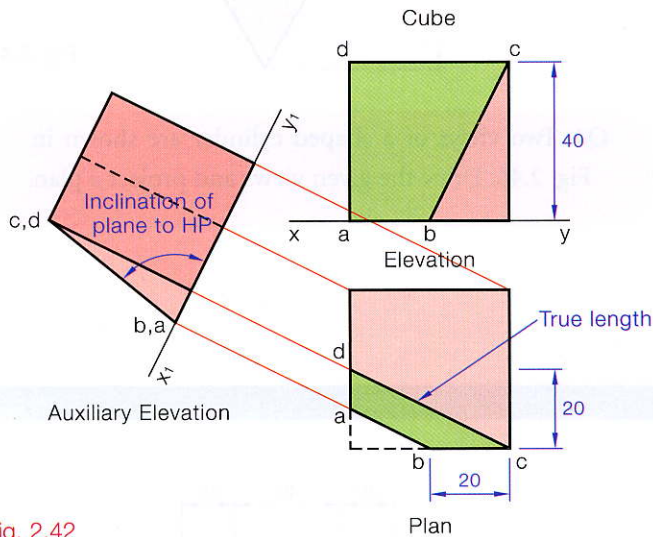


Fig. 2.42

## Activities

Q1. Fig. 2.43 (2.44) shows the plan and elevation of an object. Make a pictorial drawing showing the planes of reference and demonstrating how the views are projected.

Q2. Make a pictorial drawing showing the planes in question one rebatted in line with the vertical plane. Show the elevation and plan in their respective positions on the planes.

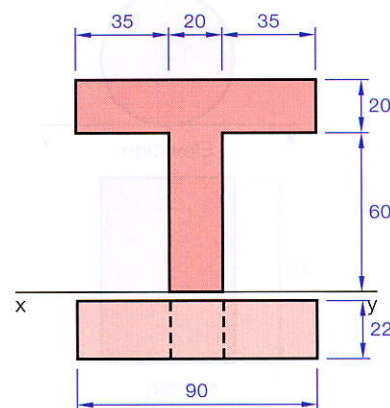


Fig. 2.43 (2.44)



Q3. Fig. 2.45 shows the elevation and plan of a solid. Draw the given views and project an end view looking from the left.

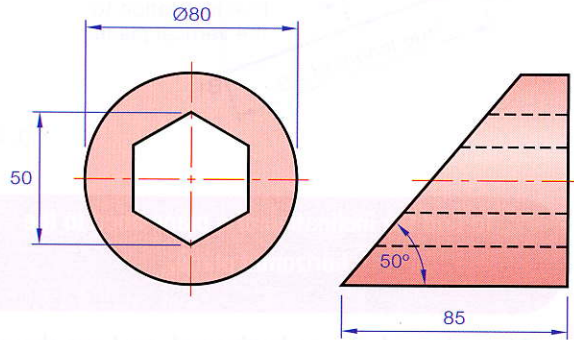


Fig. 2.46

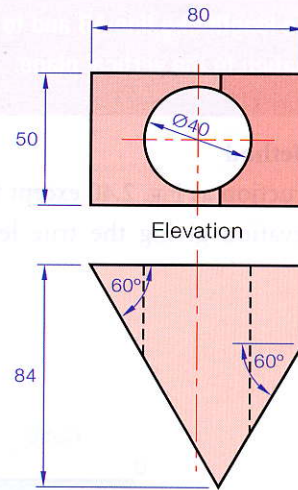


Fig. 2.45

Q4. Two views of a shaped cylinder are shown in Fig. 2.46. Draw the given views and project a plan.

#### AUXILIARY ELEVATIONS

Q5. Given the plan and elevation in Fig. 2.47. Draw the given views and project an auxiliary elevation in the direction of arrow A.

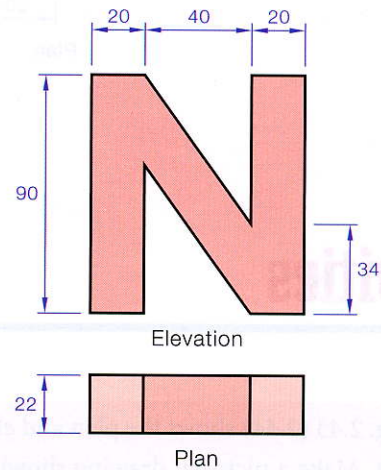


Fig. 2.47

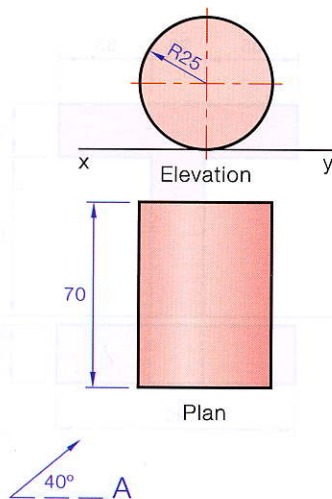


Fig. 2.48

Q6. The diagram Fig. 2.48 shows the plan and elevation of a cylinder lying on the horizontal plane. Draw the given views and project an auxiliary elevation of the solid viewing in the direction of arrow A.



Q7. The diagram Fig. 2.49 shows the plan, elevation and end view of a solid. Draw the given views and project an auxiliary elevation in the direction of the arrow A.

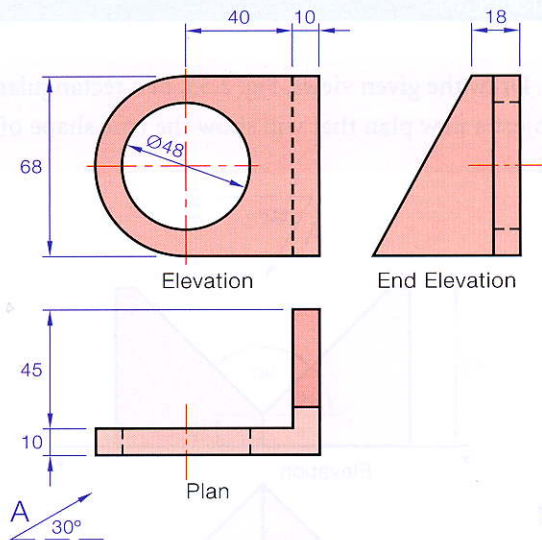


Fig. 2.49

Q8. The drawings in Fig. 2.50 show the projections of a shaped block with a hole drilled through the centre. Project the given views and project a new elevation of the solid which includes the true shape of surface A.

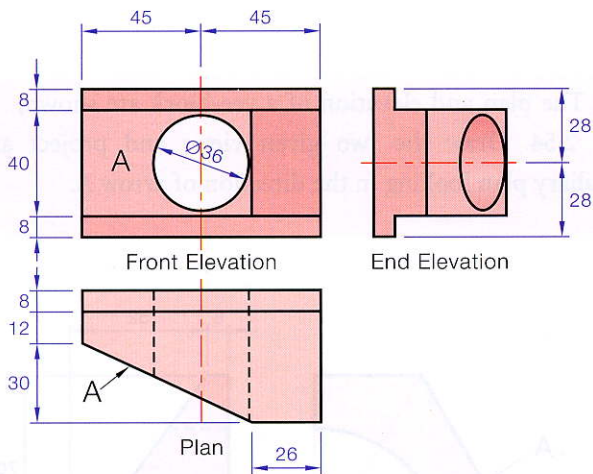


Fig. 2.50

Q9. The diagram Fig. 2.51 shows the front elevation, end elevation and plan of a shaped solid. Draw the given views and project an auxiliary elevation in the direction of the arrow.

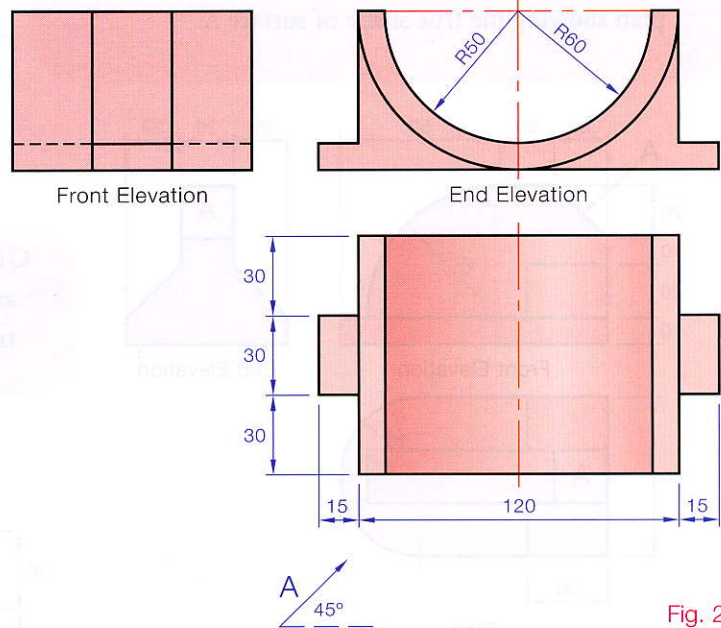


Fig. 2.51

Q10. Draw the given views Fig. 2.52 and project an end elevation. Project a new elevation viewing in the direction of arrow A.

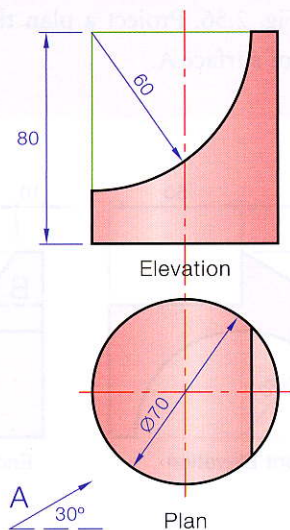


Fig. 2.52



## AUXILIARY PLANS

Q11. Draw the given views, Fig. 2.53, of a rectangular-based pyramid. Project a new plan that will show the true shape of surface A.

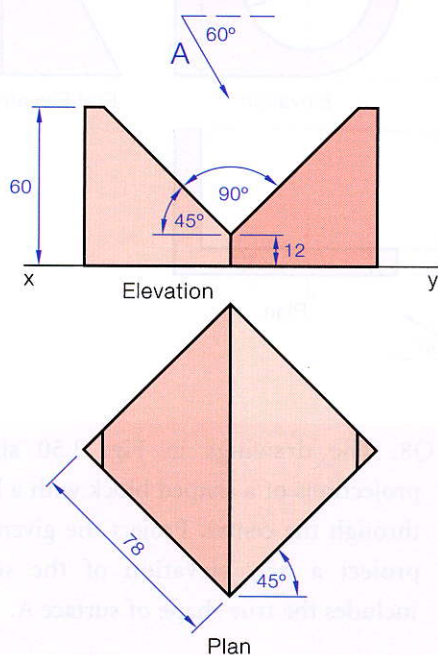


Fig. 2.54

Q13. Fig. 2.55 shows the front and end elevations of a solid. Project a plan of the solid. Project an auxiliary plan showing the true shape of surface A.

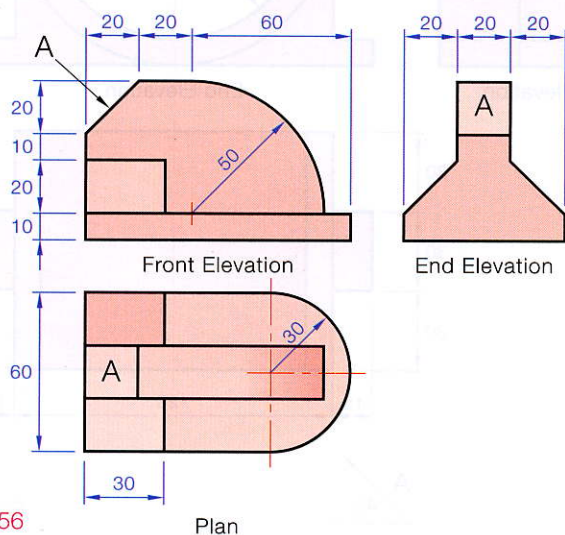


Fig. 2.56

Q15. Given the front elevation and plan, Fig. 2.57. Draw the given views and project a plan. Project a new plan of the solid showing the true shape of surface B.

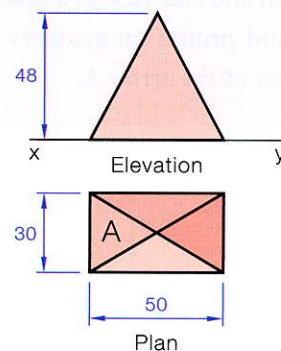


Fig. 2.53

Q12. The plan and elevation of a vee-block are shown, Fig. 2.54. Draw the two given views and project an auxiliary plan looking in the direction of arrow A.

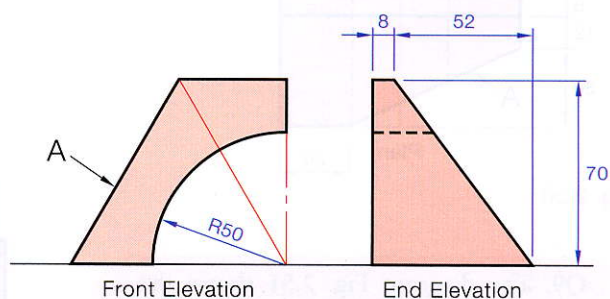


Fig. 2.55

Q14. Draw the given front elevation, end elevation and plan, Fig. 2.56. Project a plan that will show the true shape of surface A.

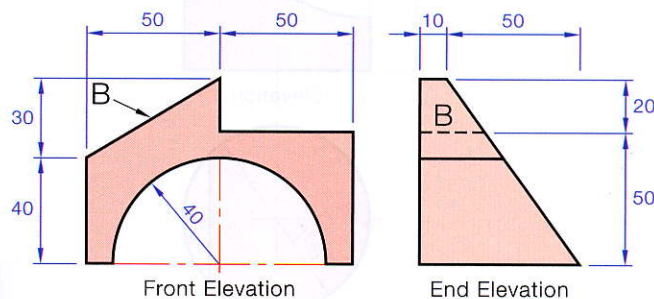


Fig. 2.57



## LINES IN SPACE

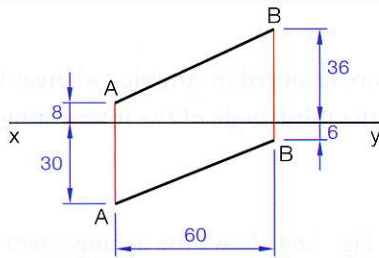


Fig. 2.58

Q17. Given the plan and elevation of a line CD, Fig. 2.59. Find the true length of the line and its inclination to the horizontal plane using the rotation method.

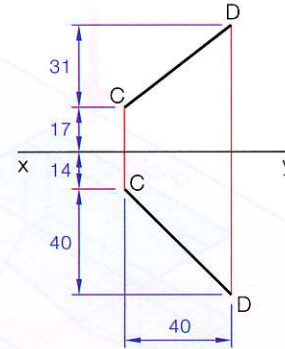


Fig. 2.59

Q18. Given the plan of a line AB and the elevation of one end of the line, A, Fig. 2.60. The true inclination of this line is to be  $30^\circ$  to the horizontal plan. Draw the elevation.

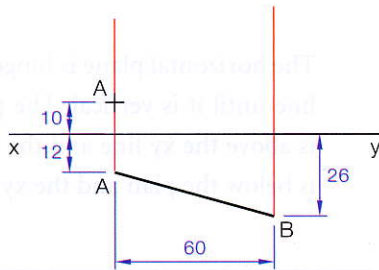


Fig. 2.60

Q19. Given the elevation of a line CD and the plan of one end of the line, C, Fig. 2.61. If the true length of this line is to be 80 mm, complete the plan.

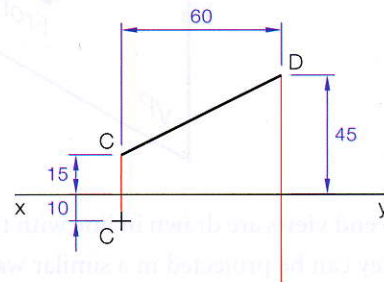


Fig. 2.61

Q20. Given that a line AB has a true length of 70 mm. End A rests somewhere on line A in plan and  $A_1$  in elevation. End B rests somewhere on line B in plan and  $B_1$  in elevation. Find the elevation and plan of the line.

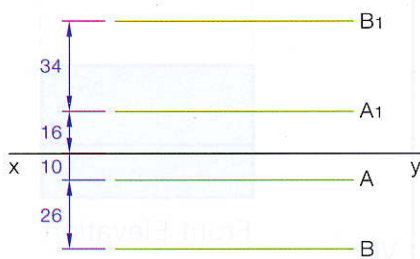


Fig. 2.62

Q21. Given the elevation of a line CD and the elevation of one end of the line, C, Fig. 2.63. If the true length of the line is 76 mm, complete the elevation.

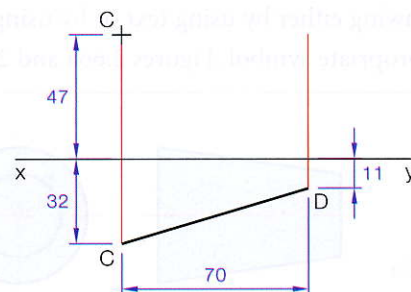


Fig. 2.63



## Third-angle Projection

Third-angle projection is the system of orthographic projection which is more favoured in America whereas first-angle projection is more favoured in Europe. The object to be drawn is placed in the third angle of the intersecting reference planes. The views are found by looking **through** the planes.

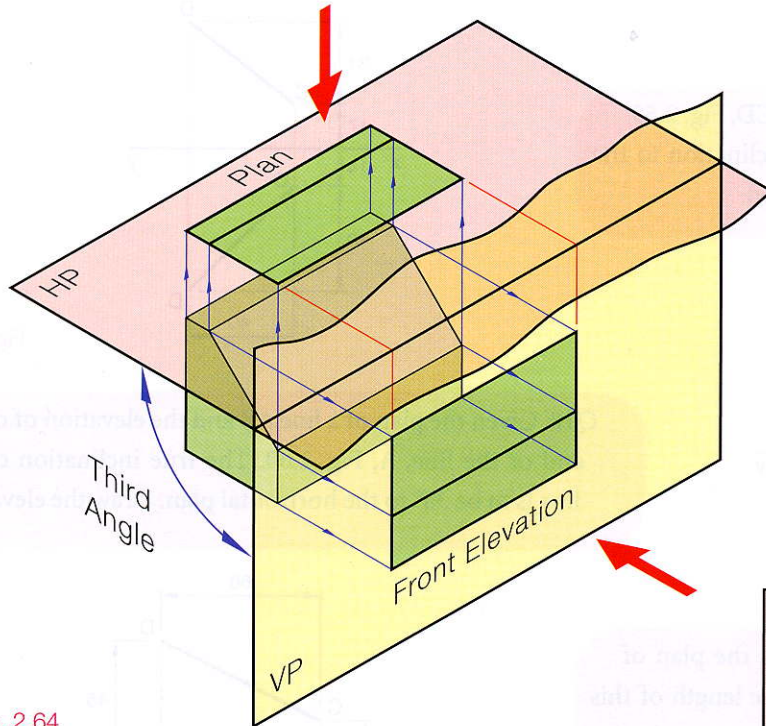


Fig. 2.64

The two end views are drawn in line with the front elevation, Fig. 2.65. They can be projected in a similar way as was done in first-angle projection. It should be noted that we are viewing through the plane and then projecting the image back onto the same plane. The end elevation on the left, therefore, will be the view from the left and the end elevation to the right of the front elevation will be the view from the right of the object.

### Symbol

The angle of projection must be stated on a drawing either by using text or by using the appropriate symbol, Figures 2.66a and 2.66b.

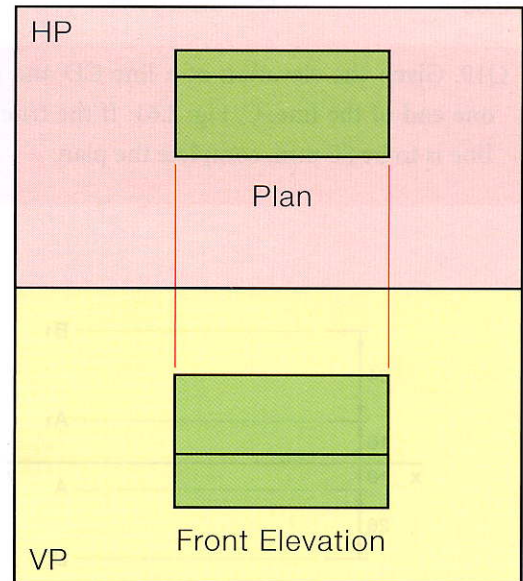
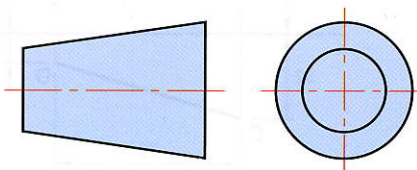
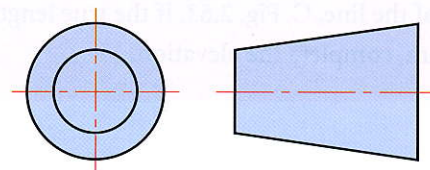


Fig. 2.65

Fig. 2.66a



First-angle Projection



Third-angle Projection

Fig. 2.66b



- (i) A front elevation looking in the direction of arrow A.
- (ii) A plan.
- (iii) An end elevation projected from the other two views.

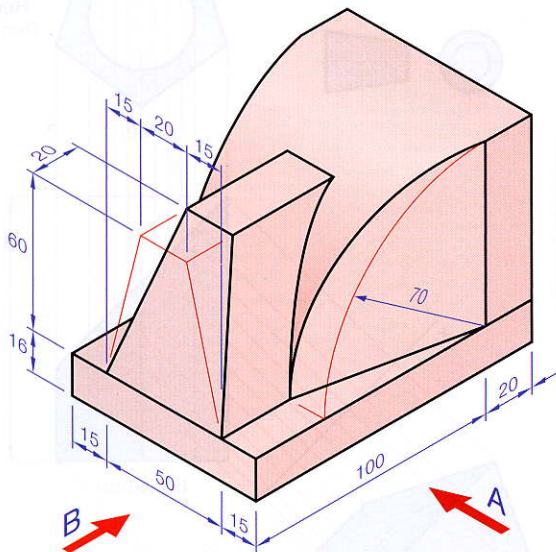


Fig. 2.70 shows a possible arrangement for the  $xy$  line and the  $yy$  line. The construction of the views themselves should be straightforward.



- (i) A front elevation viewing in the direction of arrow A.
- (ii) A plan.
- (iii) An end elevation viewed in the direction of arrow B.

