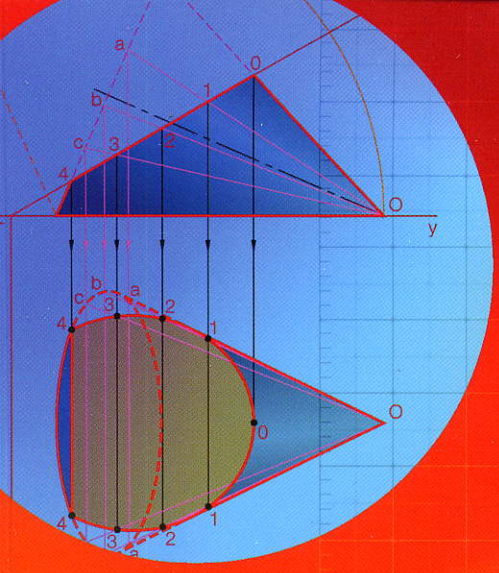
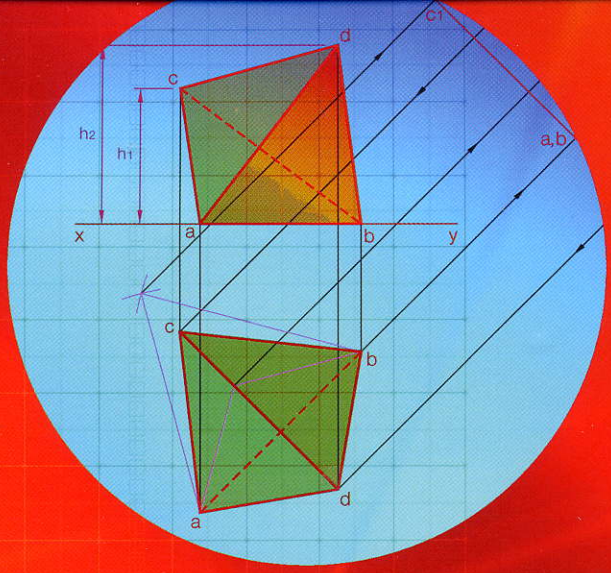
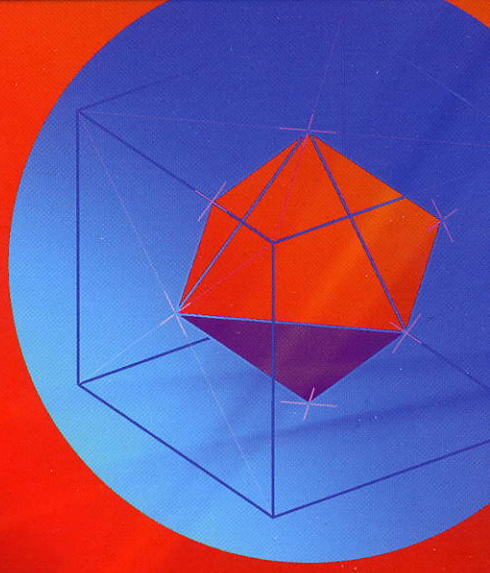
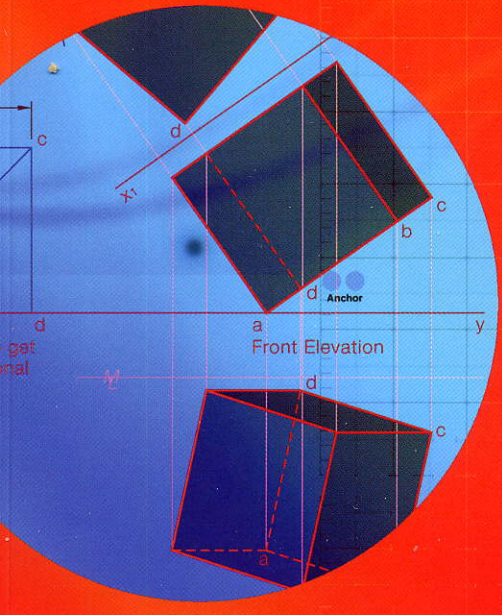


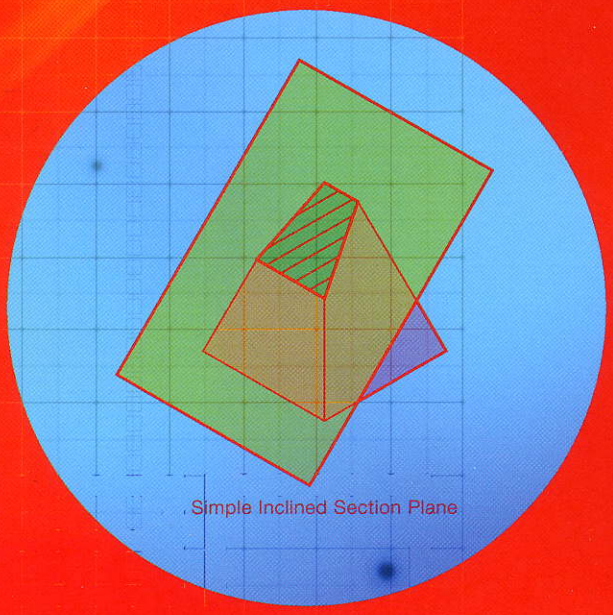


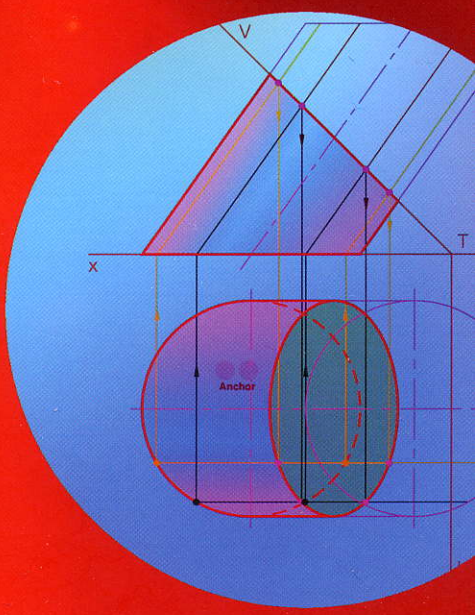


GRAPHICS IN DESIGN & COMMUNICATION 1

PLANE AND DESCRIPTIVE GEOMETRY







DAVID ANDERSON

9 The Oblique Plane

SYLLABUS OUTLINE

Areas to be studied:

• Definition of planes, simply inclined and oblique. • Determination of oblique and *tangent* planes. • True shape and inclinations of planes to principal planes of reference. • Intersection of oblique planes, lines and *dihedral angle.* • Sectioning of right solids by oblique planes. • *Treatment of planes as laminar surfaces given rectangular coordinates.* • *Properties and projections of skew lines.* • *Spatial relationships between lines and planes.*

Learning outcomes

Students should be able to:

Higher and Ordinary levels

- Distinguish between simply inclined and obliquely inclined plane surfaces.
- Determine the angle of inclination between given planes and the principal planes of reference.
- Determine the true length and inclination of given lines.
- Establish the true shape of an obliquely inclined plane.
- Determine the line of intersection between two planes.
- Determine the projections and true shape of sections of solids resulting from simply inclined and oblique cutting planes.

Higher level only

- *Construct obliquely inclined planes given the angles of inclination to the principal planes of reference and to include a given line or point.*
- *Establish the dihedral angle between two intersecting planes.*
- *Display knowledge of the relationships between planes and lines.*
- *Understand the concept of a laminar surface defined by spatial coordinates.*
- *Solve a variety of problems involving the intersection, inclination and positioning of laminar plane surfaces.*
- *Define the concept of skew lines and their use in solving practical problems.*
- *Establish various spatial relationships between skew lines and other lines and planes, including distance, inclination and direction.*

A plane is usually represented by its lines of intersection with the principle planes of reference. These lines are called the **traces** of the plane. The line of intersection between the plane and the vertical plane is called the **vertical trace** and the line of intersection between the plane and horizontal plane is called the **horizontal trace.**

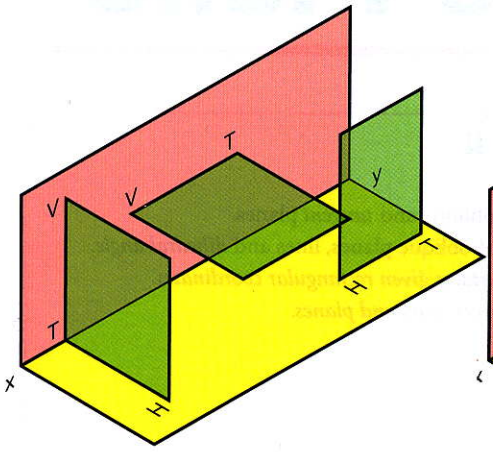


Fig. 9.1

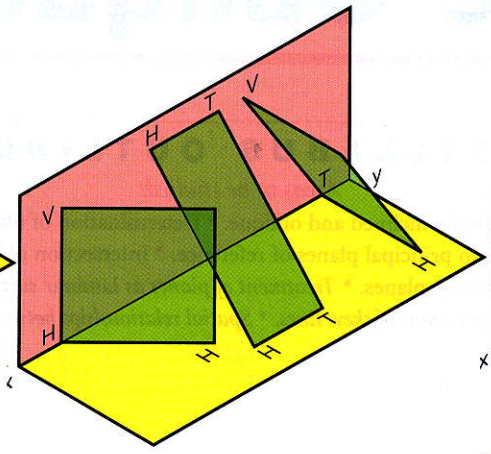


Fig. 9.2

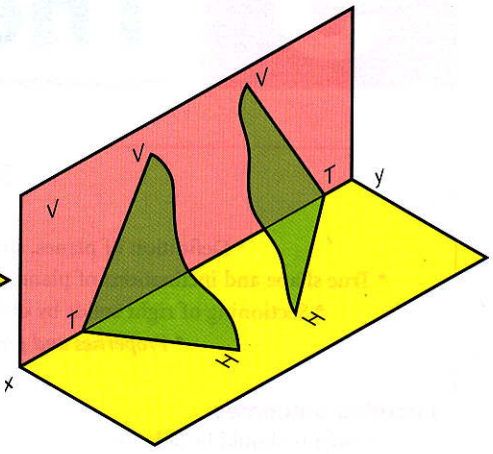


Fig. 9.3

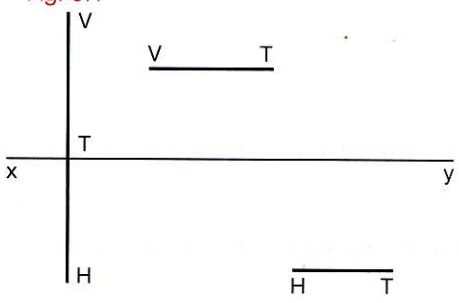


Fig. 9.1a

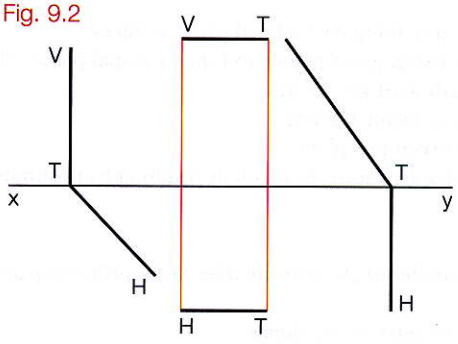


Fig. 9.2a

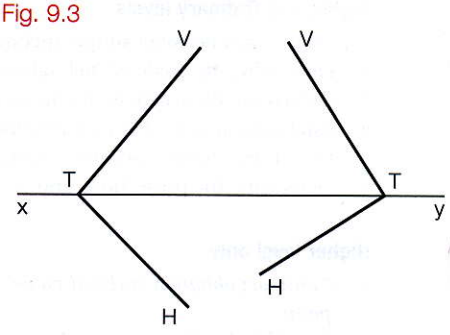


Fig. 9.3 a

Given the plan of a point P on an oblique plane VTH. Find the elevation of this point. Fig. 9.4

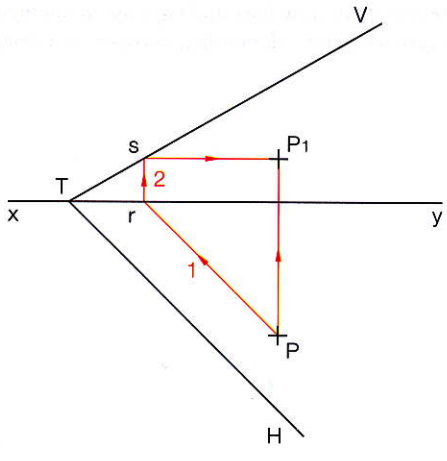


Fig. 9.4

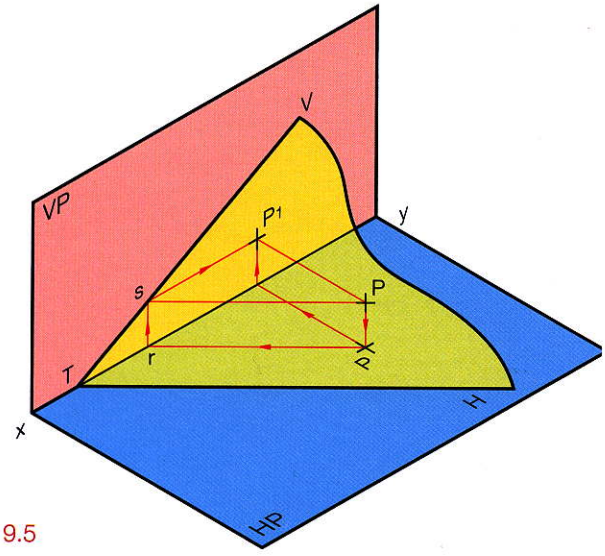


Fig. 9.5

(1) Draw a line parallel to the horizontal trace from point P to hit the xy line at r.
(2) Erect a perpendicular at r to hit the vertical trace at s.
(3) From s, project horizontally (parallel to the xy line).
(4) Project vertically from P in plan to intersect this horizontal thus locating P_1 in elevation, Fig. 9.5.

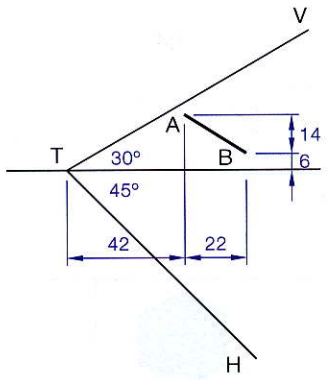


Fig. 9.6

Lines drawn parallel to the vertical trace in elevation will be parallel to the xy line in plan.

Lines drawn parallel to the horizontal trace in plan will be parallel to the xy line in elevation.

Given the plan of a lamina ABC on an oblique plane VTH. In plan AB = 32 mm, BC = 55 mm and AC = 44 mm. Find the elevation. Fig. 9.8

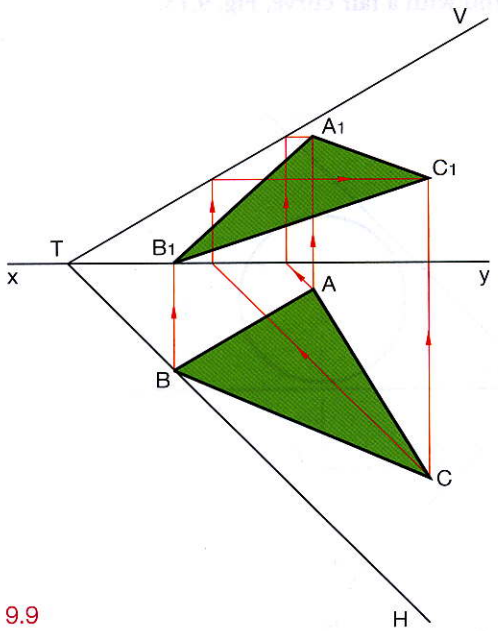


Fig. 9.9

Given the elevation of a line AB on an oblique plane VTH. Find its plan. Fig. 9.6

(1) From A, project parallel to the VT line to meet the xy line at r.
(2) Project vertically down from r to s on the HT line.
(3) Project s parallel to the xy line.
(4) Drop a perpendicular from A in elevation to find A_1.
(5) Point B is done in a similar manner, Fig. 9.7.

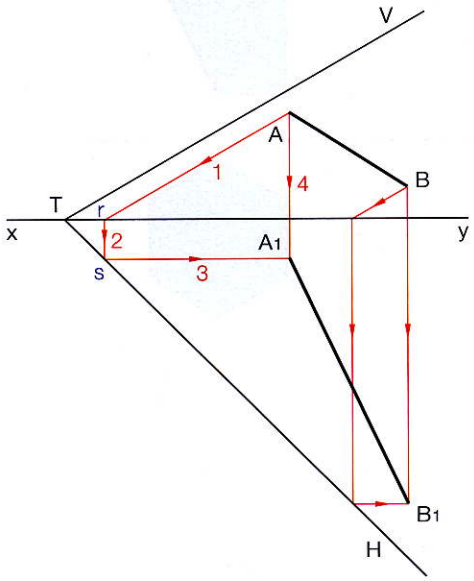


Fig. 9.7

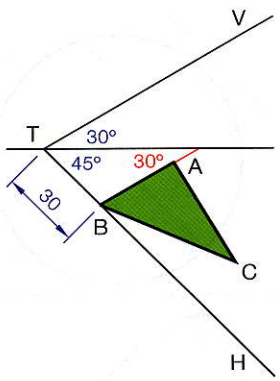


Fig. 9.8

(1) Each point is brought up as in previous examples and found in elevation.
(2) Index each point and join finding $A_1B_1C_1$ in elevation, Fig. 9.9.

Point B is on the HT. Since the horizontal trace is the line of intersection between the oblique plane and the horizontal trace, every point on the HT will also be on the horizontal trace.

Given the plan of a figure resting on the oblique plane VTH. Find its elevation. Fig. 9.10

The construction is the same as in previous examples.

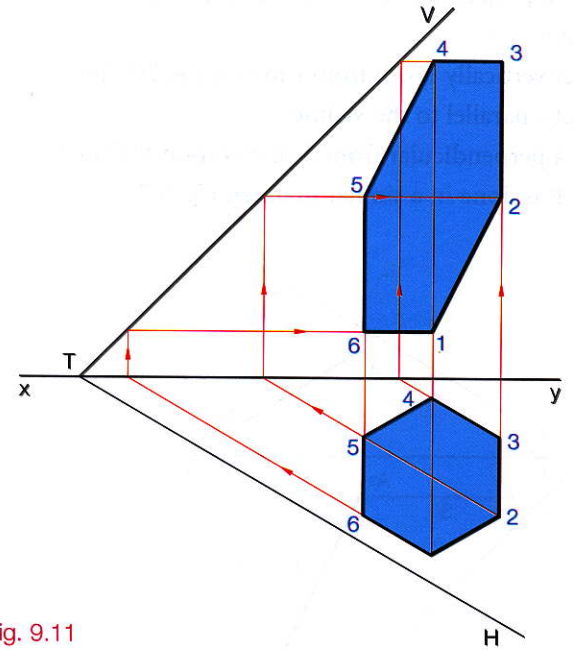


Fig. 9.11

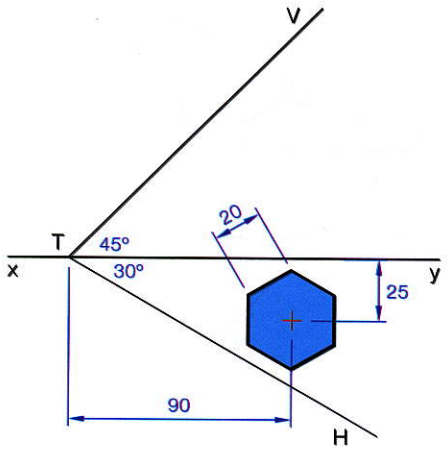


Fig. 9.10

Given the elevation of a figure resting on the oblique plane VTH. Find its plan. Fig. 9.12

Since this figure is not made up of straight lines we must choose points on the curve. In Fig. 9.13 we have chosen eight points. Once the points are found in plan they are joined freehand with a fair curve, Fig. 9.13.

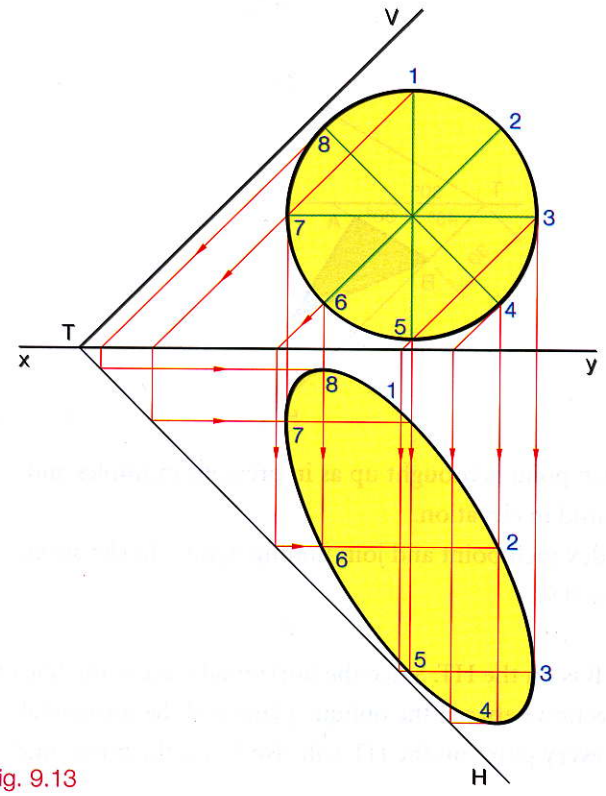


Fig. 9.13

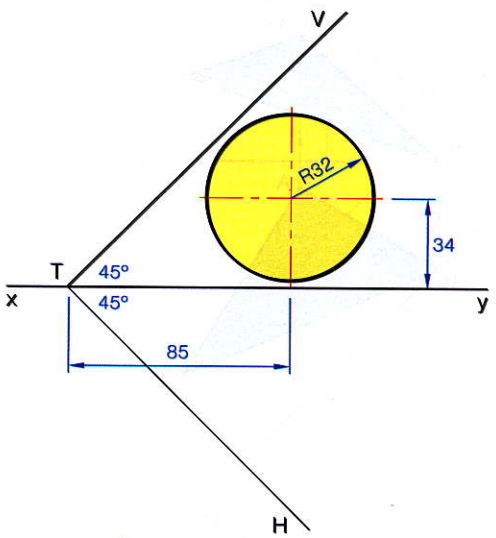


Fig. 9.12

True Inclination of an Oblique Plane to the VP and the HP

To find the true inclination of an oblique plane to the HP. Fig. 9.14

To find the true angle, insert a half cone underneath the plane and tangential to it. The base angle of this cone will be the true angle of the plane to the horizontal plane.

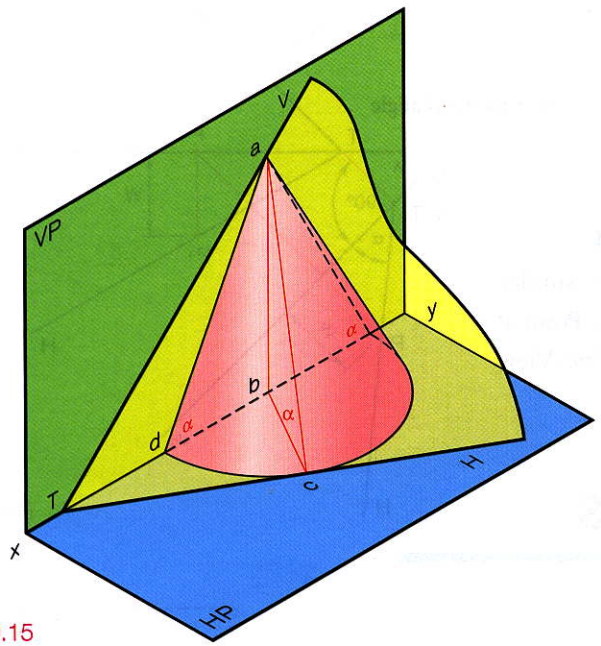


Fig. 9.15

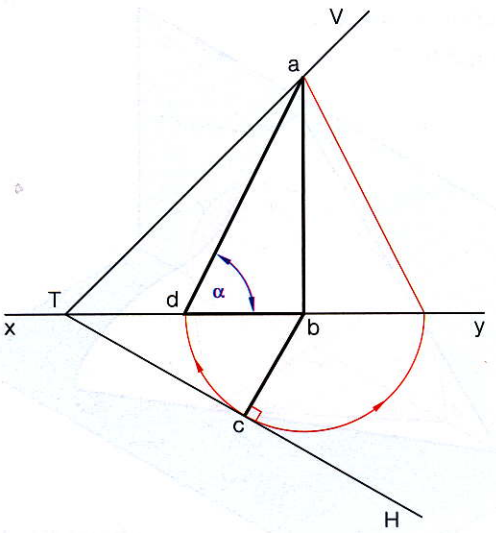


Fig. 9.14

(1) Draw a vertical line ab in elevation.
(2) From b on the xy line, draw a perpendicular to the HT finding c.
(3) Line bc equals the radius of the cone. Draw the half cone in plan.
(4) Draw the elevation of the cone. The base angle of the cone is the true angle of the plane to the horizontal plane, Fig. 9.15.

Alternative Method

The true angle an oblique plane makes with the horizontal plane can be found by using an auxiliary elevation. Fig. 9.16

> **Note:** The point P on the oblique plane is usually chosen at position A as it reduces the amount of projection involved.

(1) Choose any point P on the plane in plan and project it to elevation.
(2) Take an auxiliary elevation viewing along the horizontal trace.
(3) x_1y_1 is drawn perpendicular to the line of sight.
(4) The horizontal trace projects as a point view.
(5) Point P is projected to the auxiliary elevation and its height h is taken from the elevation.
(6) The oblique plane is seen as an edge view showing its true angle to the horizontal plane.

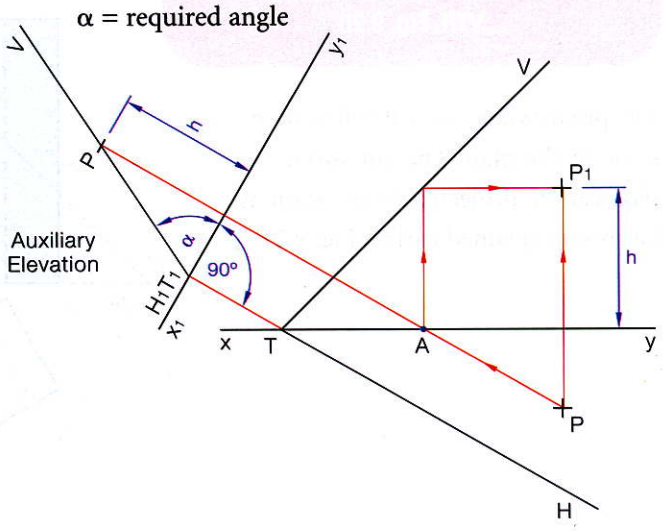


Fig. 9.16

To find the true inclination of an oblique plane to the vertical plane. Fig. 9.17

The construction is similar to Figures 9.14 and 9.15.

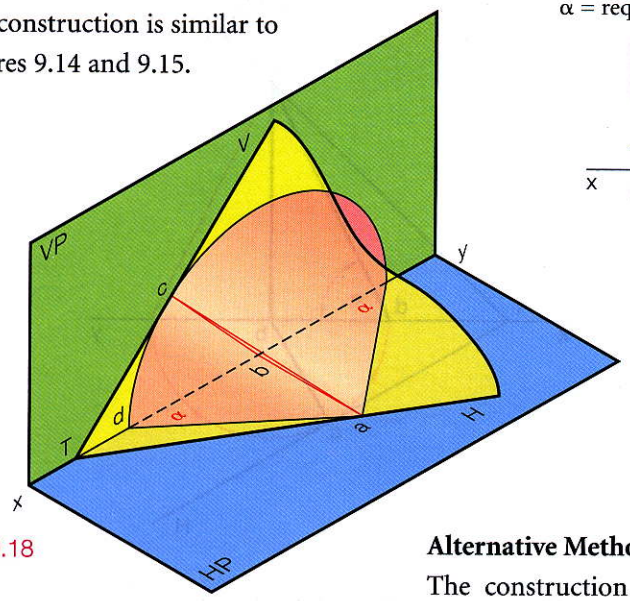


Fig. 9.18

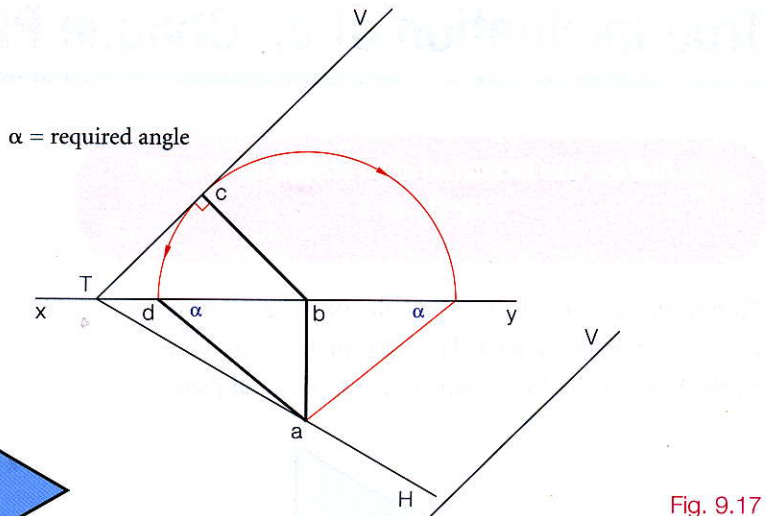


Fig. 9.17

Alternative Method

The construction is similar to that for Fig. 9.16. Point P is chosen on the xy line. View along the VT line.

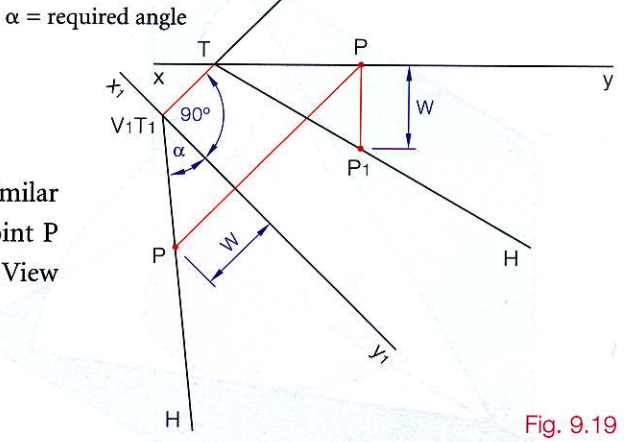


Fig. 9.19

Solids Cut by Oblique Planes

Given the plan and elevation of a square-based prism and the traces of a plane VTH. Show the projections of the prism when it is cut by the oblique plane VTH. Fig. 9.20

The prism when cut will still look the same in the plan. The cut surface in plan can be projected to elevation as has been explained earlier, Fig. 9.21.

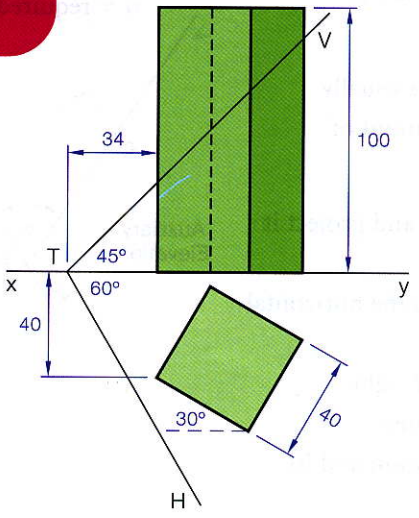


Fig. 9.20

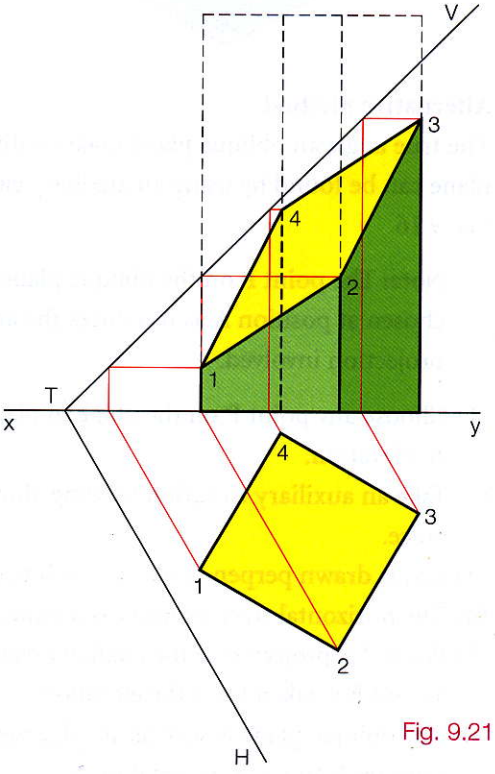


Fig. 9.21

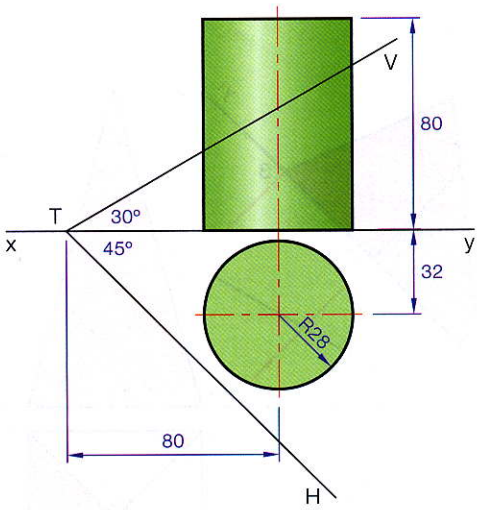


Fig. 9.22

Given the plan and elevation of a cylinder and the traces of a plane VTH. Show the projections of the cylinder when it is cut by the oblique plane VTH. Fig. 9.22

As in the last example, the plan of the solid is unchanged after it has been cut by the plane. Divide the plan up into a number of divisions. Index the points and project each point to elevation as usual, Fig. 9.23.

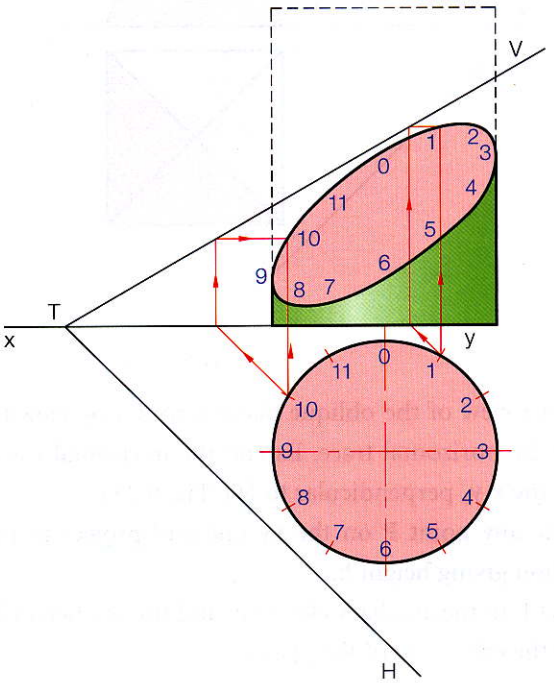


Fig. 9.23

Given the plan and elevation of a square-based pyramid and the traces of a plane VTH. Draw the projections of the solid after it has been cut by the oblique plane. Fig. 9.24

The plan does not show the cut surface in this case. This type of problem is solved by getting an edge view of the oblique plane and hence determining the cut points on the solid.

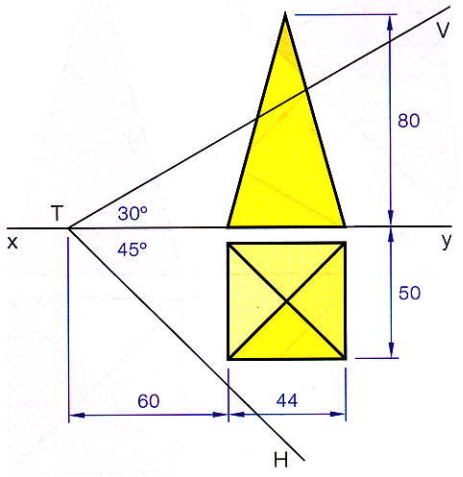


Fig. 9.24

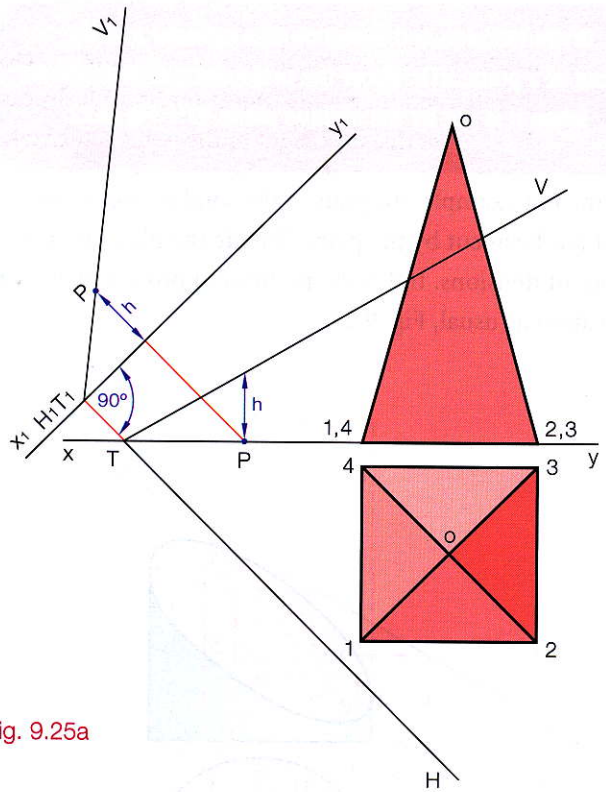


Fig. 9.25a

(1) An edge view of the oblique plane is found by viewing along the horizontal trace. Extend the horizontal trace. Draw the x_1y_1 perpendicular to HT, Fig. 9.25a.

(2) Choose any point P on the xy line and project to the elevation giving height h.

(3) Project P to the auxiliary elevation and use the height h. Draw the edge view of the plane.

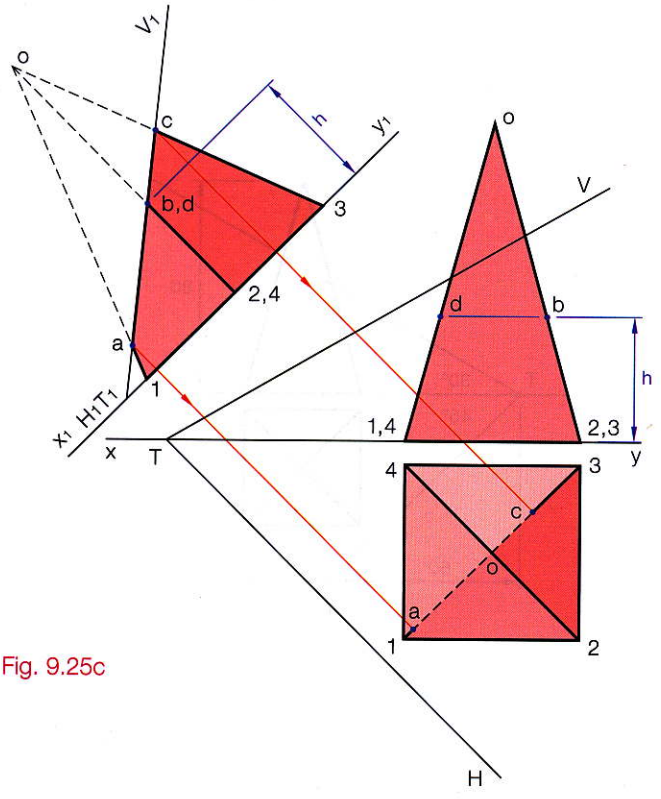


Fig. 9.25c

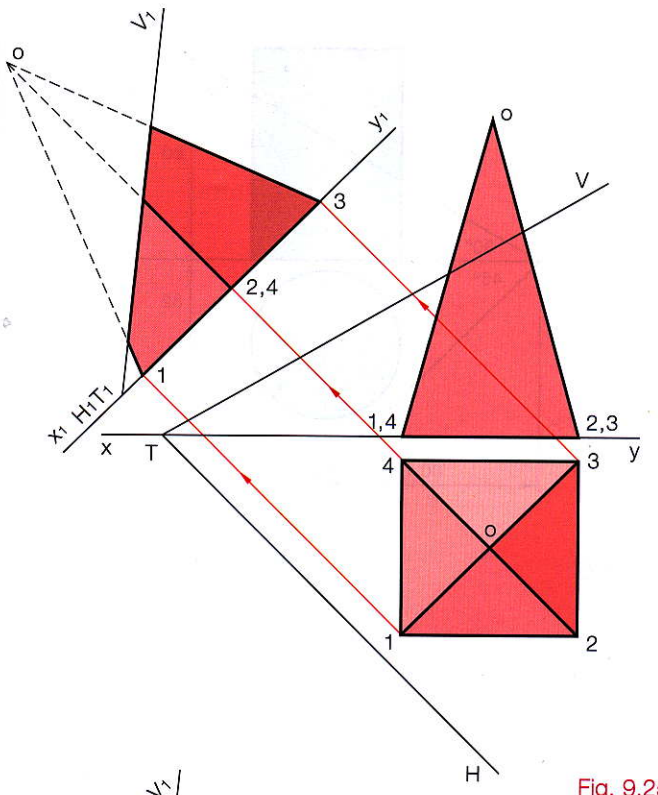


Fig. 9.25b

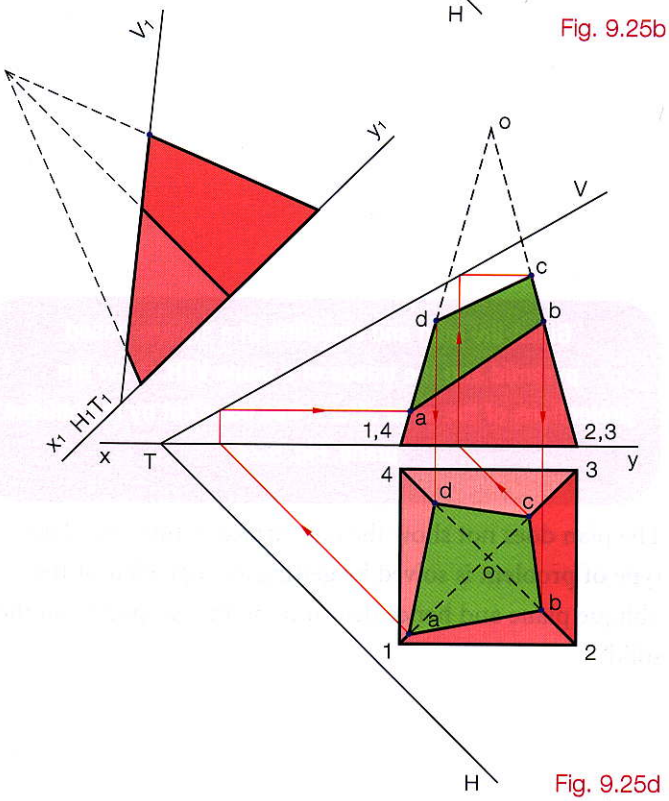


Fig. 9.25d

(4) Project the auxiliary elevation of the pyramid, Fig. 9.25b.

(5) The auxiliary elevation shows the cut points a, b, c and d. Points a and c are projected to plan, the heights of b and d are used to find them in elevation, Fig. 9.25c.

(6) Complete the plan and elevation by projection, Fig. 9.25d.

Shown in Fig. 26 is a regular hexagonal-based pyramid which is to be cut by the plane VTH. Draw the plan and elevation of the cut solid. Fig. 9.26

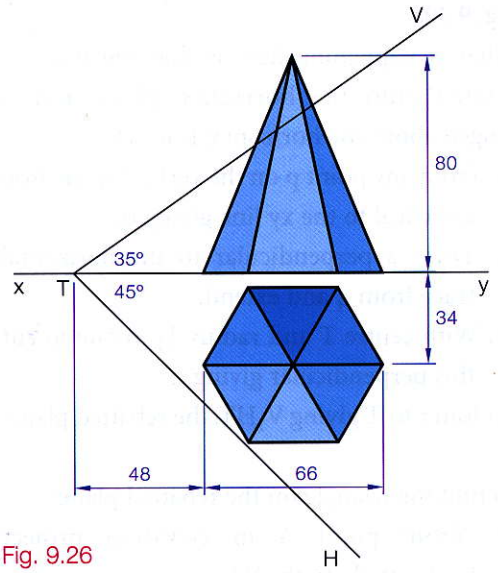


Fig. 9.26

(1) Draw the given plan and elevation and the traces of the plane VTH.
(2) Index the corners.
(3) Project the auxiliary elevation to find the points on the cut surface.
(4) Project to plan and up to the elevation, Fig. 9.27.

Some points, when projected down from the auxiliary elevation to the plan, do not show a clear intersection, e.g. the point on line 5,0. In cases like this the height of the point can be taken from the auxiliary and used in the elevation to find the point which may then be projected to plan.

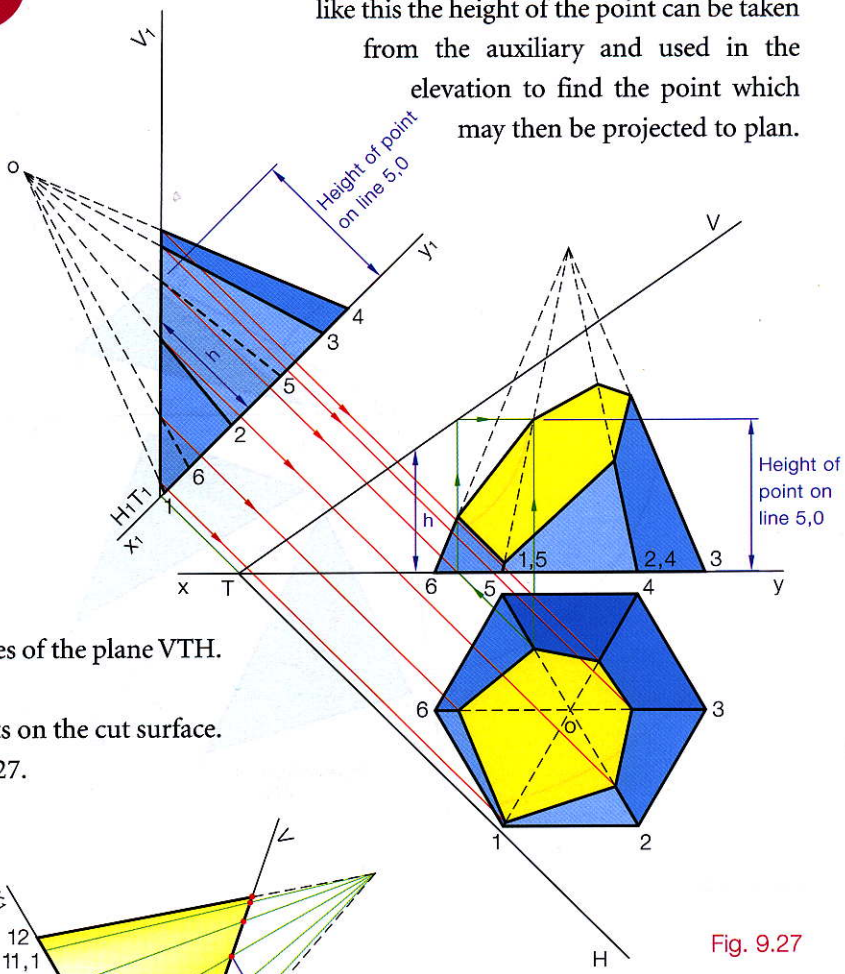


Fig. 9.27

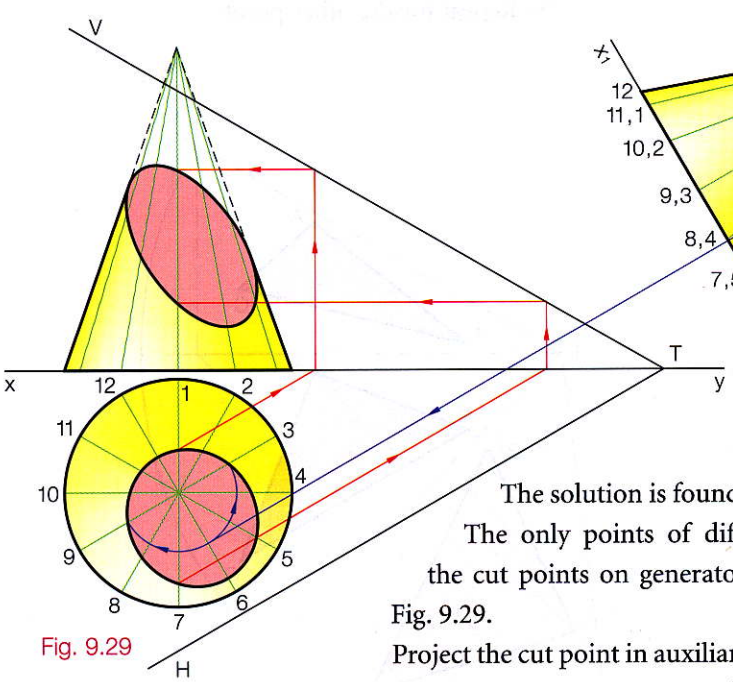


Fig. 9.29

The solution is found as before. The only points of difficulty are the cut points on generator 9 and 3, Fig. 9.29.

Project the cut point in auxiliary across to the side of the cone, generator 6. Project to plan on generator 6 and rotate onto generator 9 and 3. The cut points on generators 1 and 7 in plan must be projected around via the xy line and VT as shown.

Shown in Fig. 9.28 is the plan and elevation of a cone which is to be cut by the plane VTH. Draw the plan and elevation of the cut solid.

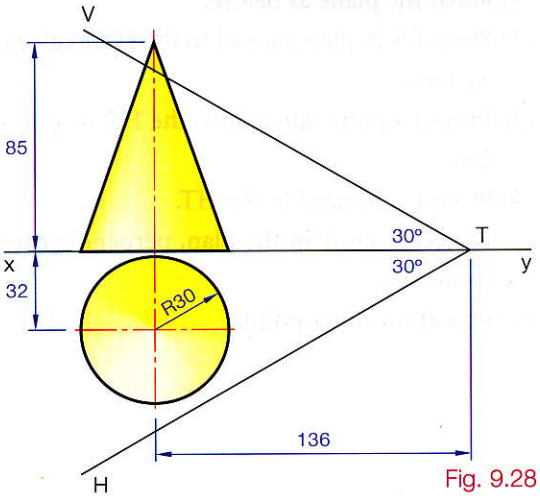


Fig. 9.28

To Find the True Shape of an Object on an Oblique Plane

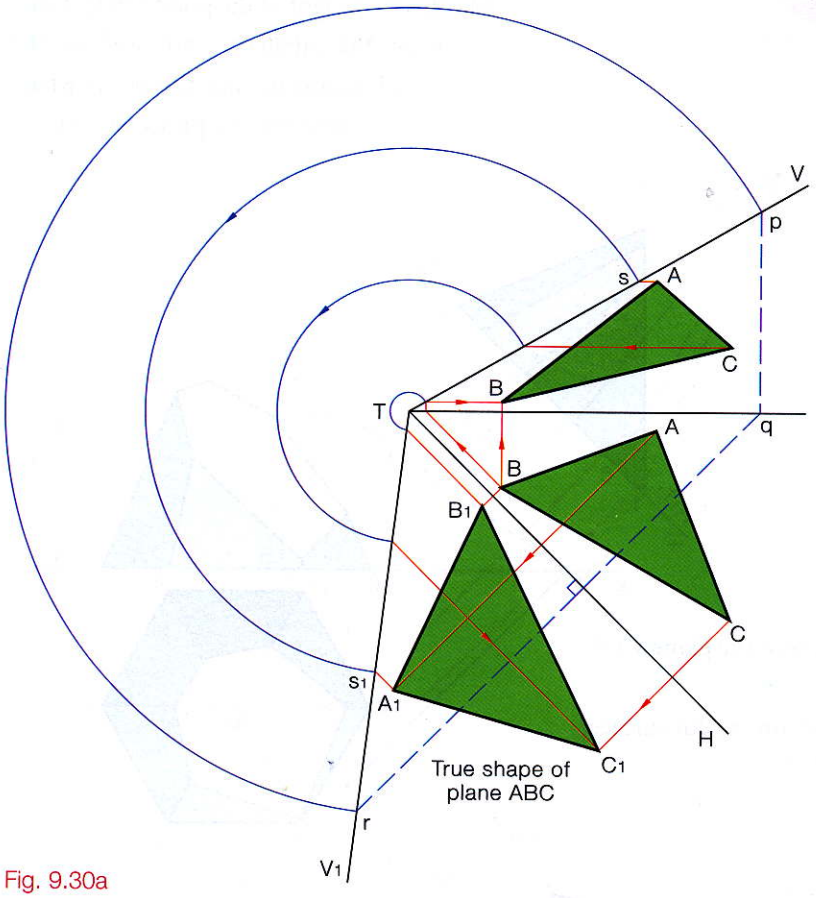


Fig. 9.30a

Method 1: Rebatment of the Plane

Fig. 9.30a

What is happening here is that the plane is rotated onto the horizontal plane and is hinged about the horizontal trace HT.

(1) From any point p on the vertical trace drop a vertical to the xy line giving q.

(2) Draw a perpendicular to the horizontal trace from q and extend.

(3) With centre T and radius Tp rotate to cut this perpendicular giving r.

(4) Join r to T giving V_1HT, the rebatted plane.

To find the triangle on the rebatted plane:

(1) From point A in elevation project horizontally to the VT giving s.

(2) With T as centre rotate point s to V_1T giving s_1.

(3) Project s_1 parallel to the HT.

(4) From A in plan project across perpendicular to the HT to locate A_1.

(5) Repeat for the other points.

Method 2: Rebatment of the Plane

Fig. 9.30b

This is very similar to Method 1.

(1) Rebat the plane as before.

(2) Project A in plan parallel to the HT to give point s on the xy line.

(3) Project s perpendicular to the HT to give s_1 on the V_1T line.

(4) Project s_1 parallel to the HT.

(5) Project from A in the plan, perpendicular to the HT to locate A_1.

(6) Repeat for other points.

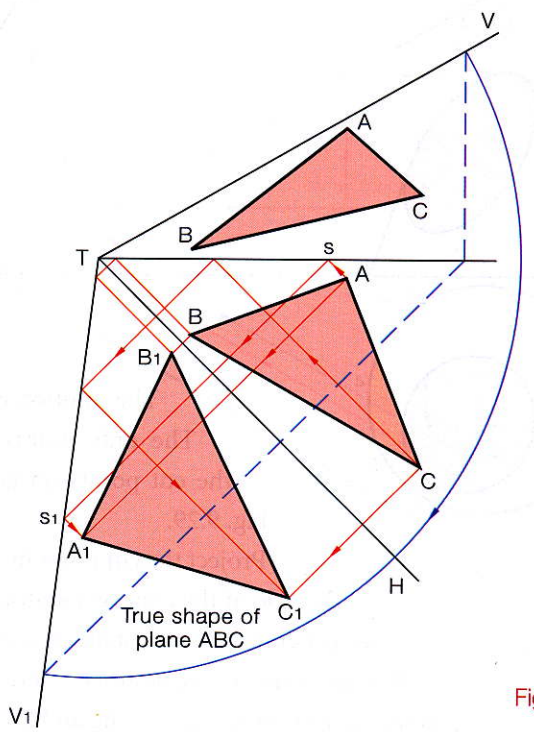


Fig. 9.30b

Method 3: Auxiliary Elevation and Rebatment

Fig. 9.31

An edge view of the oblique plane is found and the plane can then be easily rebatted onto the horizontal plane.

(1) View along the HT to see the plane as an edge view. Extend the horizontal trace.
(2) Draw x_1y_1 perpendicular to the HT line extended.
(3) Choose any point p on the xy line and project to the auxiliary.
(4) Find the height of point p from the elevation, height h, and measure on the auxiliary.
(5) Draw the plane in the auxiliary V_1TH.
(6) Project the triangular surface ABC to the auxiliary.
(7) Rotate the points A_1B_1C in the auxiliary, about point T, onto the x_1y_1 line.
(8) Project back to plan.
(9) Project from the plan perpendicular to the HT to locate the points A_2B_2 and C_2.

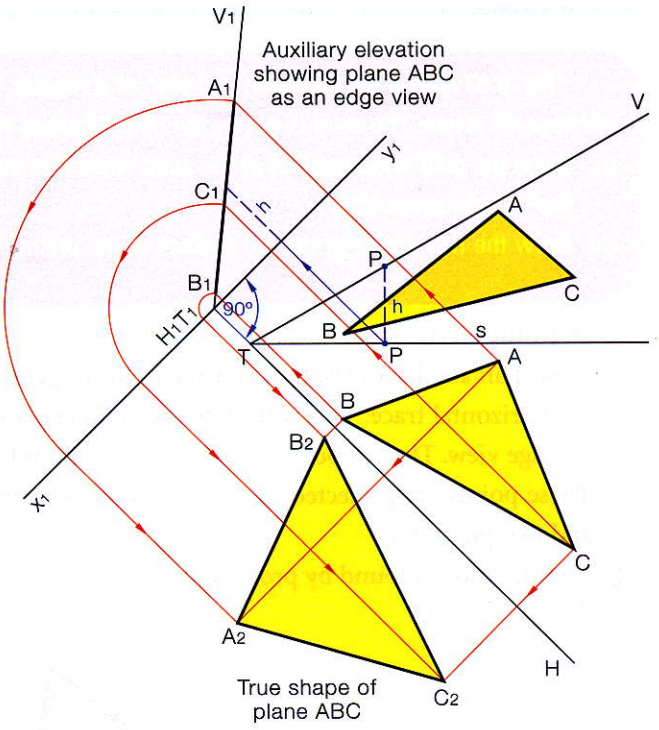


Fig. 9.31

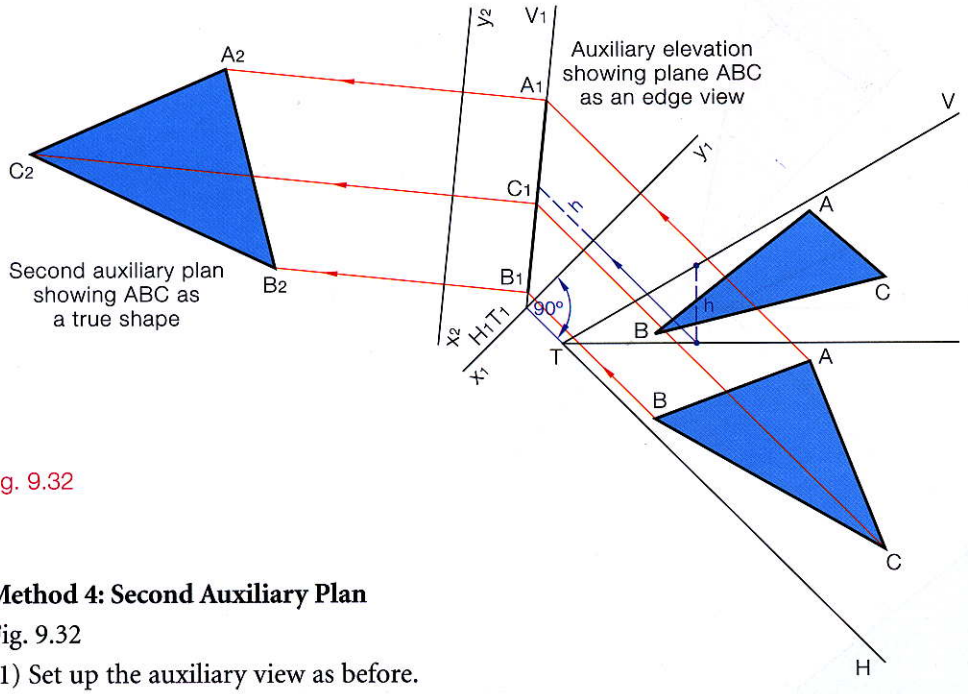


Fig. 9.32

Method 4: Second Auxiliary Plan

Fig. 9.32

(1) Set up the auxiliary view as before.
(2) Draw x_1y_1 parallel to $V_1T_1H_1$.
(3) Project points $A_1B_1C_1$ perpendicular to x_2y_2.
(4) The distance from x_1y_1 to A is taken and used to locate A_2.
(5) Similar for B_2 and C_2.

Worked Examples

Fig. 9.33 shows the elevation and plan of a regular hexagonal prism. The solid is cut by the oblique plane VTH.

(i) Draw the plan and elevation of the solid when it is cut by the oblique plane VTH.

(ii) Draw the true shape of the cut surface of the prism.

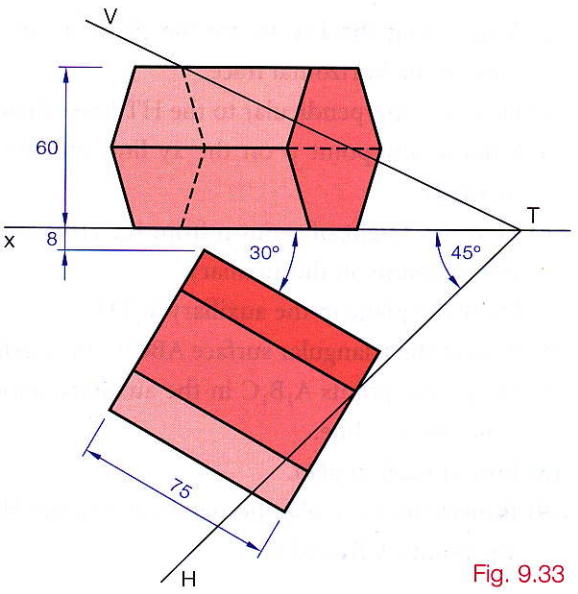


Fig. 9.33

(1) Draw the given views.

(2) Project an auxiliary elevation viewing in the direction of the horizontal trace. This will show the oblique plane as an edge view. The cut points 1, 2, 3 and 4 are found.

(3) These points are projected to plan, giving two point 1's and two point 2's.

(4) The elevation is found by projection.

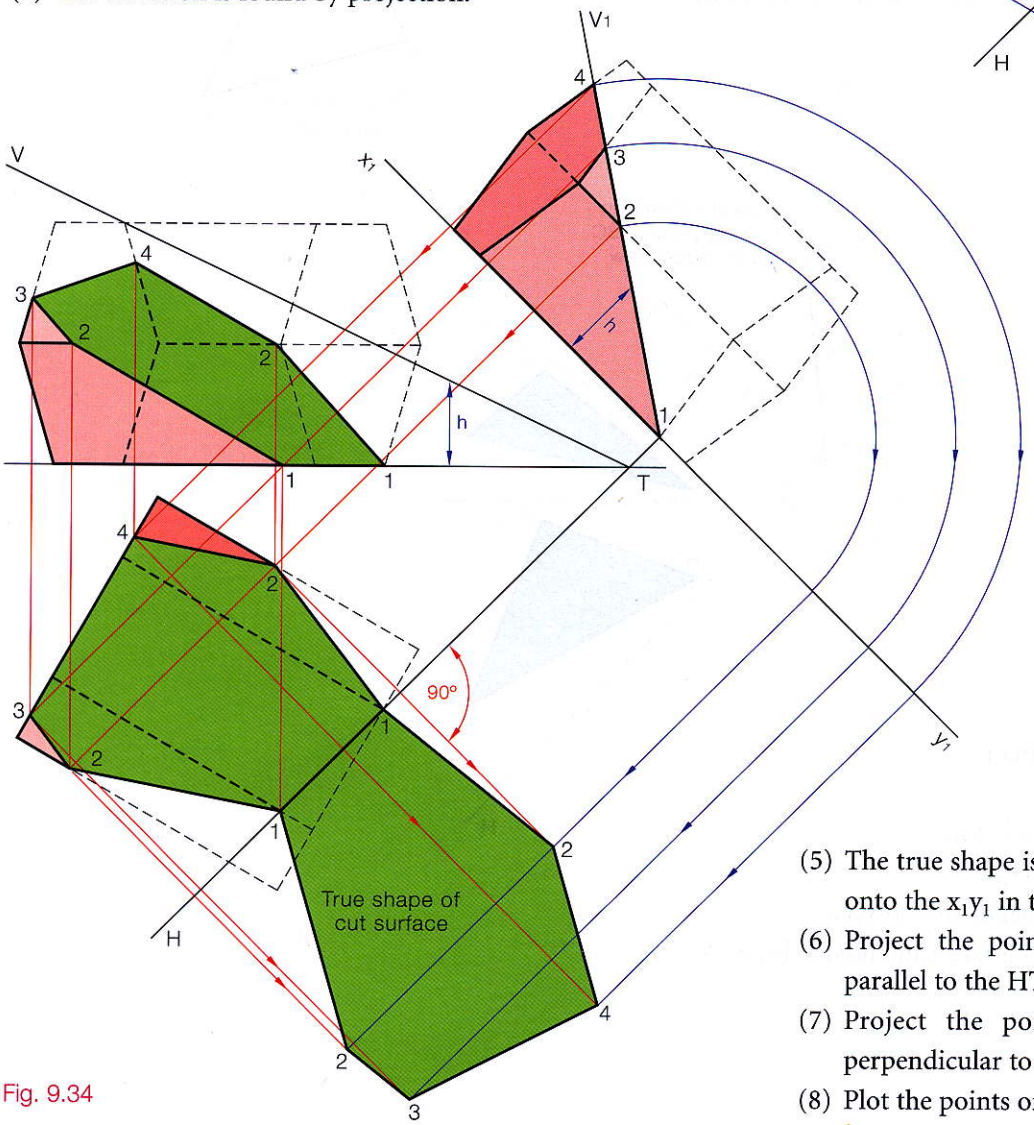


Fig. 9.34

(5) The true shape is found by rebatting the plane onto the x_1y_1 in the auxiliary view.

(6) Project the points down from the auxiliary parallel to the HT.

(7) Project the points across from the plan perpendicular to the HT.

(8) Plot the points on the true shape, Fig. 9.34.

The plan and elevation of a solid which is cut by an oblique plane VTH is shown in Fig. 9.35.

(i) Draw the plan and elevation of the solid when it is cut by the oblique plane VTH.

(ii) Draw the true shape of the cut surface of the solid.

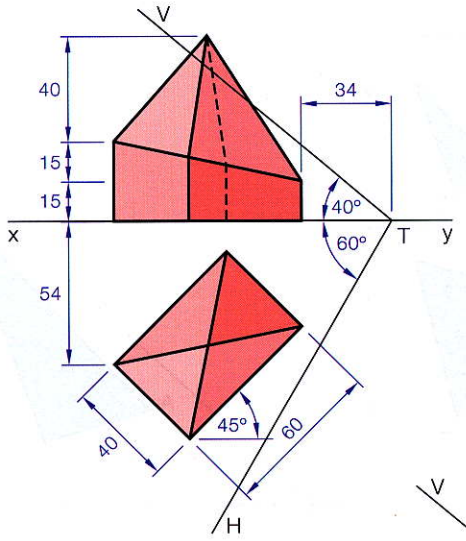


Fig. 9.35

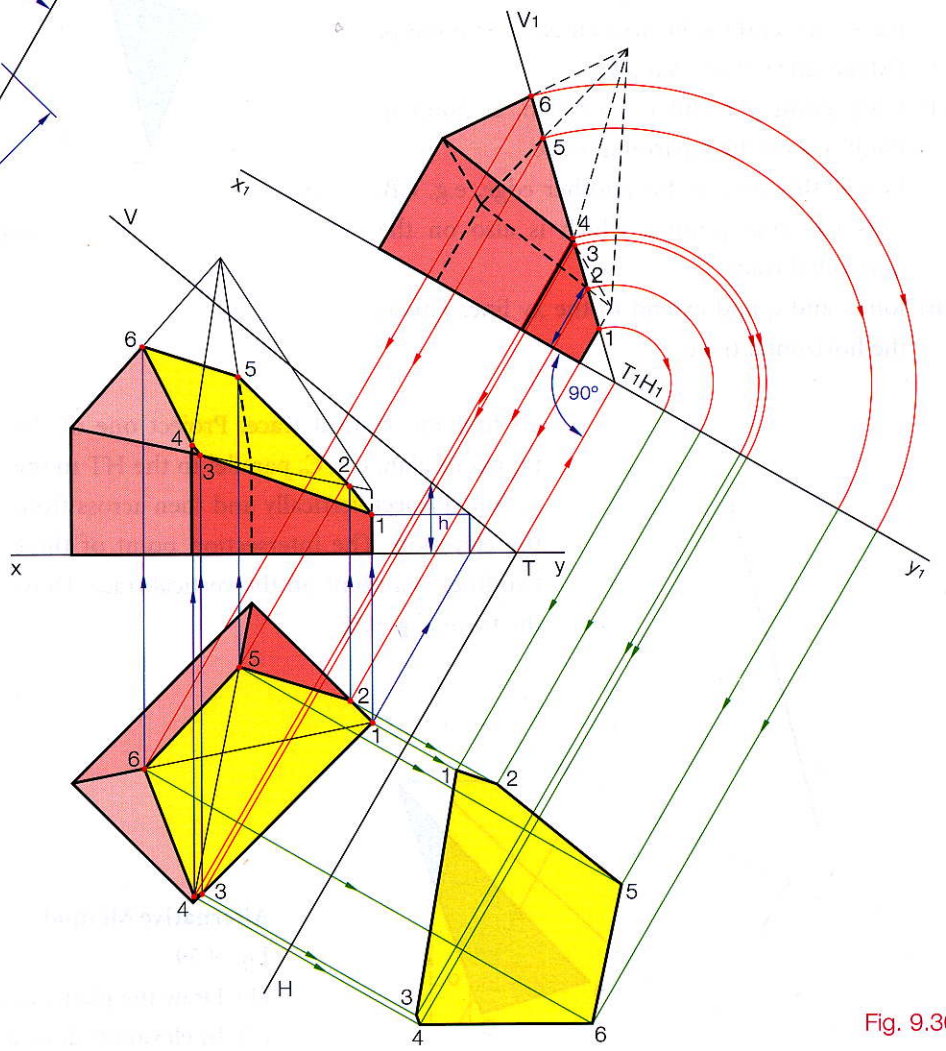


Fig. 9.36

(1) Draw the given views.

(2) Project an auxiliary elevation viewing along HT.

(3) Draw the solid in the auxiliary thus determining six points on the cut surface.

(4) Project the points down from the auxiliary. Point 1 is on a vertical and will be in the corner in plan.

(5) Project all points to elevation. Point 1 being on a vertical, may not be projected directly, but must be brought parallel to the HT to the xy line, vertically to the VT and then horizontally.

(6) The true shape is found by rebatting the oblique plane in the auxiliary, Fig. 9.36.

FINDING THE TRACES OF A PLANE

Given the plan and elevation of a lamina ABC. Find the traces of the plane that contains this lamina. Fig 9.37

(1) Draw the plan and elevation as given.
(2) In elevation extend an edge, e.g. CA, until it hits the xy line and the horizontal plane at point p.
(3) Extend line CA in plan also.
(4) Drop point p to hit CA in plan, at point q. Point q is on the horizontal trace.
(5) Repeat this process for another edge, e.g. CB. This will find point s which is also on the horizontal trace.
(6) Join s and q and extend to the xy line. This is the horizontal trace.

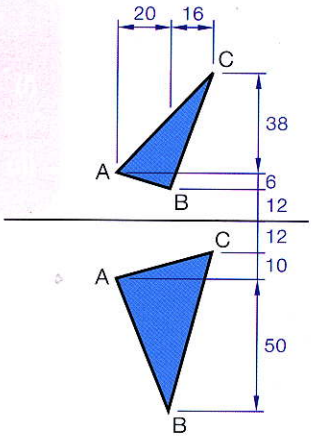


Fig. 9.37

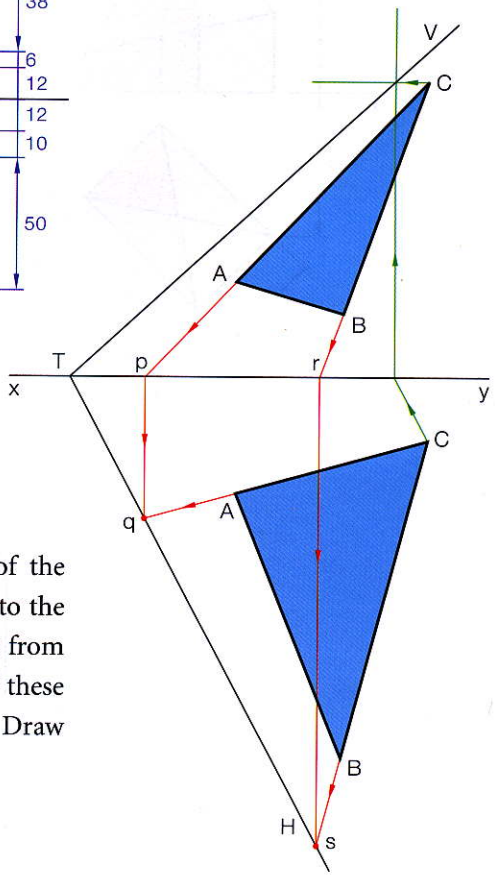


Fig. 9.38

(7) To find the vertical trace. Project one of the points in plan, e.g. C, parallel to the HT to the xy line. Project vertically and then across from the elevation. The intersection point of these two lines is a point on the vertical trace. Draw the trace, Fig. 9.38.

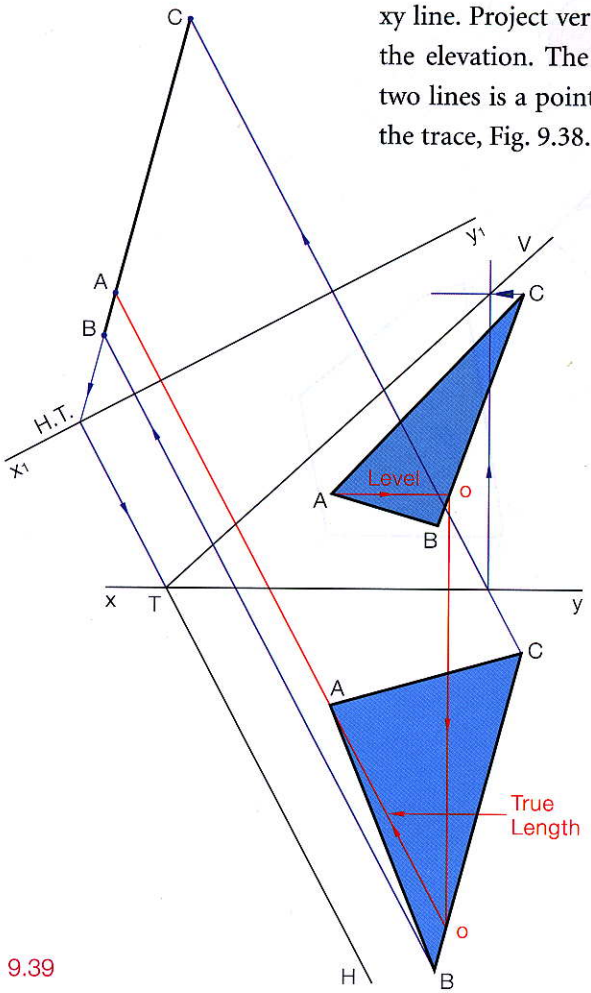


Fig. 9.39

Alternative Method

Fig. 9.39

(1) Draw the plan and elevation.
(2) In elevation draw a level line across the surface. Line AO for example.
(3) Find this line in plan. Point O is projected to plan and is found on edge CB. Join point O to corner A.
(4) View in the direction of OA and project an auxiliary elevation. The lamina projects as an edge view because we are viewing along a true length on its surface.
(5) Extend the edge view in auxiliary to the x_1y_1 line thus giving a point view of the HT. Project the horizontal trace back to plan.
(6) Find the vertical trace as above.

Fig. 9.40 shows the plan and elevation of a solid which is cut by the oblique plane VTH.

(i) Draw the plan and elevation of the solid when it is cut by the oblique plane VTH.

(ii) Draw the horizontal and vertical traces of a plane that contains surface A of the solid.

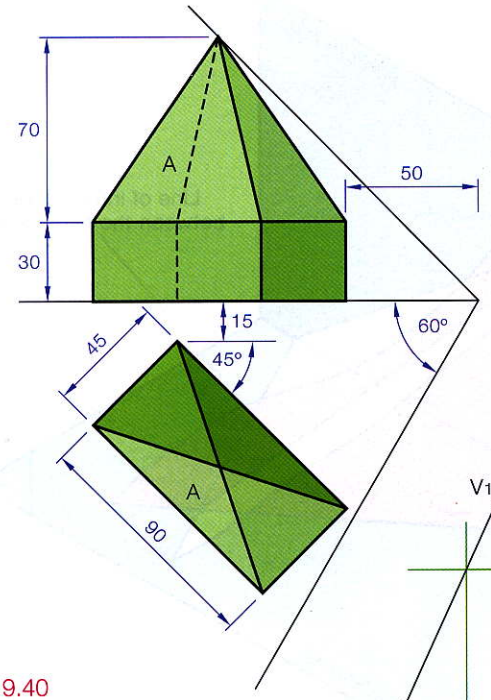


Fig. 9.40

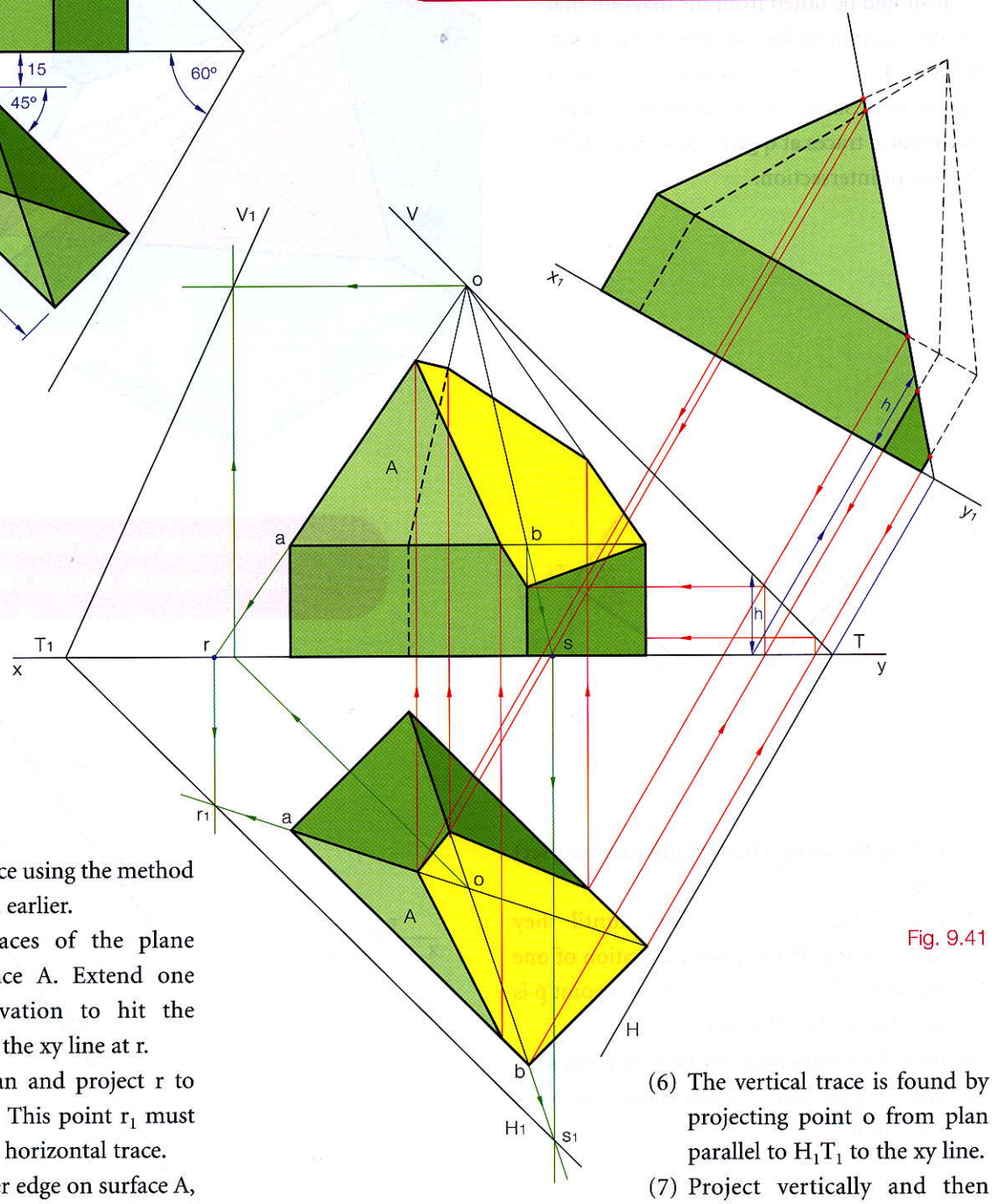


Fig. 9.41

(1) Find the cut surface using the method already explained earlier.

(2) To find the traces of the plane containing surface A. Extend one edge oa in elevation to hit the horizontal plane, the xy line at r.

(3) Extend oa in plan and project r to meet it giving r_1. This point r_1 must be a point on the horizontal trace.

(4) Repeat for another edge on surface A, edge ob. This finds point s on the horizontal trace.

(5) Join s_1 to r_1 and extend to the xy line. This is the horizontal trace.

(6) The vertical trace is found by projecting point o from plan parallel to H_1T_1 to the xy line.

(7) Project vertically and then horizontally across from point o in elevation, giving a point on the V_1T_1 line, Fig. 9.41.

INTERSECTING OBLIQUE PLANES

The diagram, Fig. 9.42, shows a pictorial view of the planes of reference and two intersecting oblique planes.

When two planes intersect the line of intersection will always be a straight line.

It should be noted from the diagram that the intersection of the two horizontal traces HT and H_1T_1 at p gives one end of the line of intersection. Similarly the intersection of the two vertical traces at q gives the other end of the line of intersection.

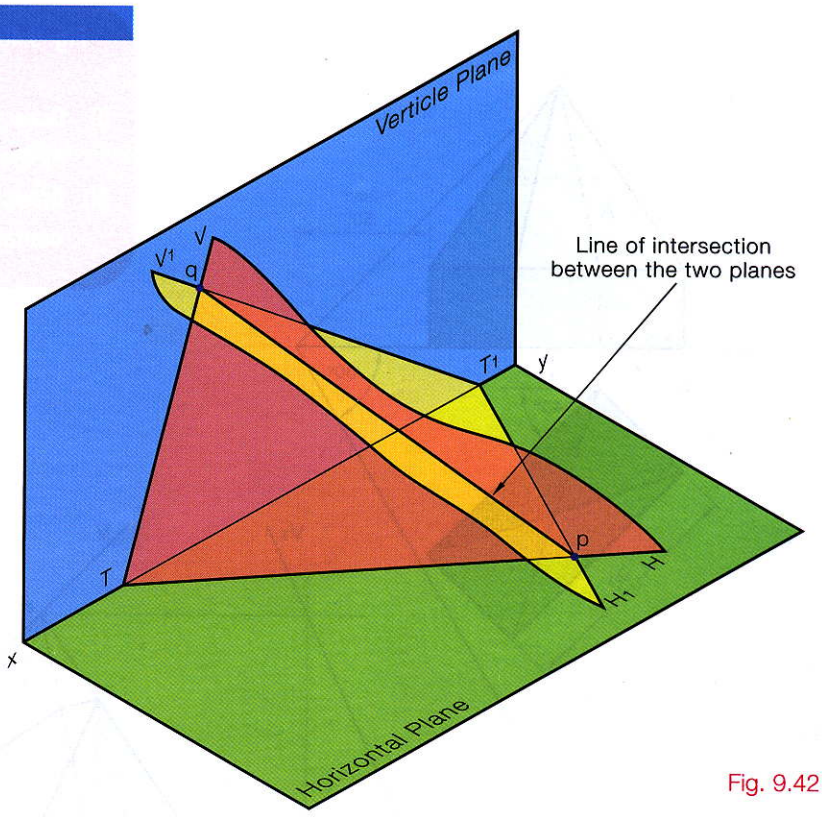


Fig. 9.42

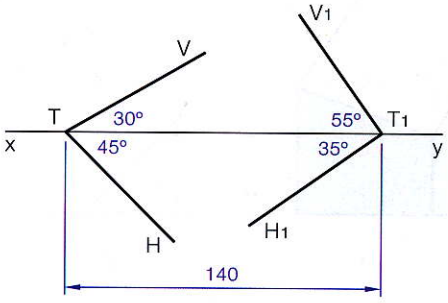


Fig. 9.43a

Given two oblique planes VTH and $V_1T_1H_1$ to find the line of intersection between them. Fig. 9.43a

(1) Extend the vertical traces until they intersect at q.

(2) Extend the horizontal traces until they intersect at p. Point q is the elevation of one end of the line of intersection and point p is the plan of the other end.

(3) Project p to elevation (xy line) and join to q.

(4) Project q to plan (xy line) and join to p, Fig. 9.43b.

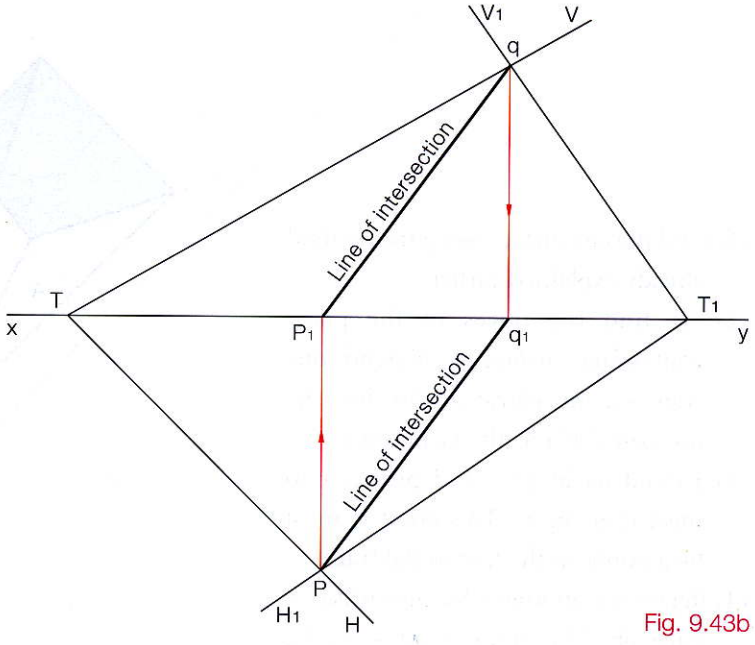


Fig. 9.43b

To find the true length of the line of intersection between two oblique planes.

(1) Find the line of intersection as above.

(2) Consider pq as the hypotenuse of a right-angled triangle that stands vertically underneath the line of intersection. Rotate this triangle until it lies horizontally, thus showing the true length of the line pq, Fig. 9.44.

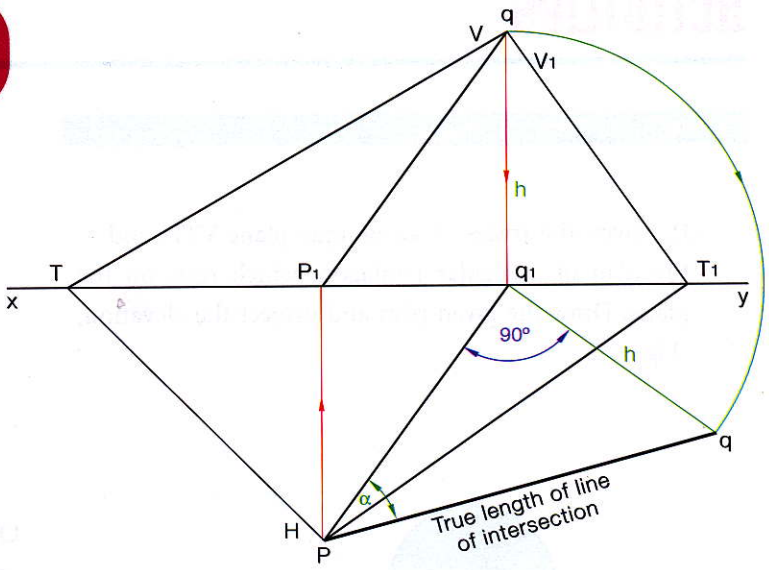


Fig. 9.44

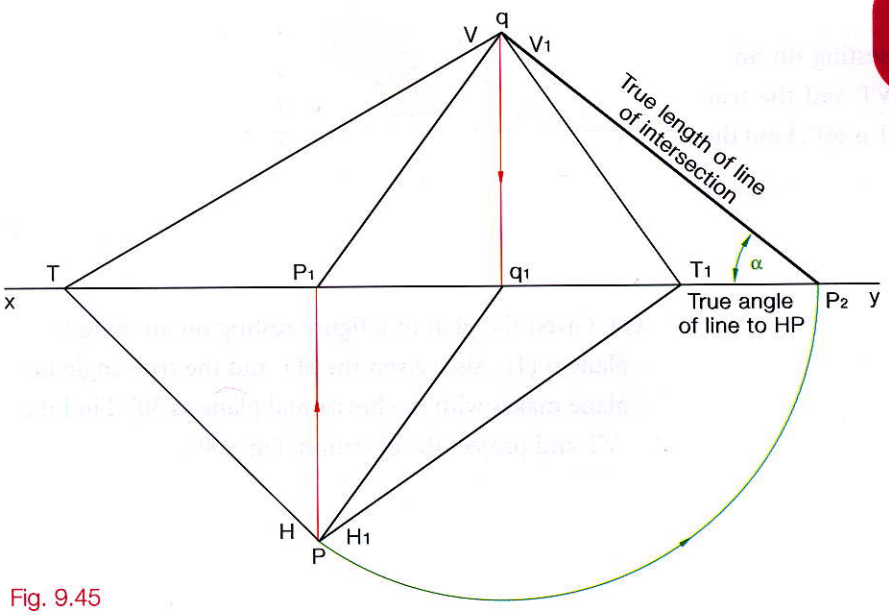


Fig. 9.45

Alternative method of finding the true length of the line of intersection. Fig. 9.45

(1) With the compass on q_1 rotate point p to p_2 on the xy line.

(2) Join p_2 to q giving the true length of the line of intersection.

Note: In both methods the true angle, α, that the line makes with the horizontal plane is also found.

Activities

FIGURES ON OBLIQUE PLANES

Q1. Given the traces of an oblique plane VTH and the plan of a regular pentagon, which rests on the plane. Draw the given plan and project the elevation, Fig. 9.46.

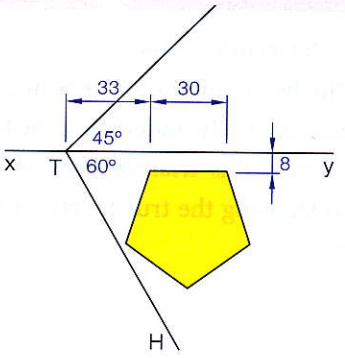


Fig. 9.46

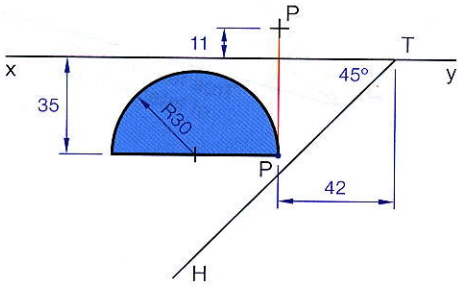


Fig. 9.47

Q2. Given the plan of a figure resting on an oblique plane of which you are given the horizontal trace. Also given is the elevation of the corner P. Find the vertical trace and complete the views, Fig. 9.47.

Q3. Given the elevation of an object resting on an oblique plane VTH. Also, given the VT and the true angle the plane makes with the vertical as 60°. Find the HT and complete the plan, Fig. 9.48

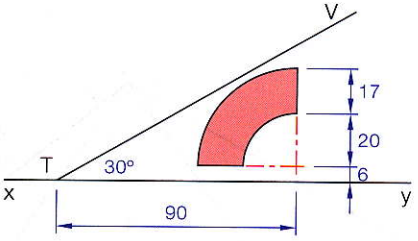


Fig. 9.48

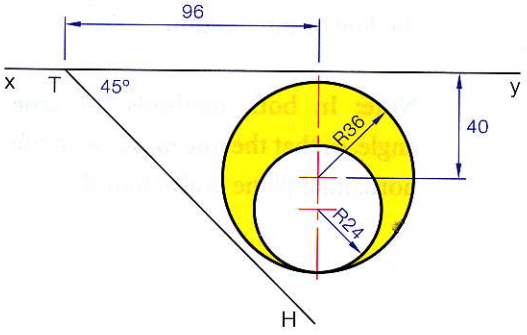


Fig. 9.49

Q4. Given the plan of a figure resting on an oblique plane VTH. Also, given the HT and the true angle the plane makes with the horizontal plane as 50°. Find the VT and project the elevation, Fig. 9.49.

CUT SOLIDS

Q5. Given the plan of a tetrahedron resting on the horizontal plane and the traces of a plane VTH. Draw the projections of the solid when it is cut by the plane, Fig. 9.50.

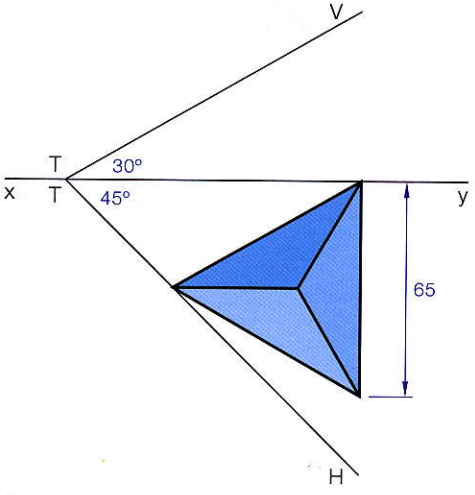


Fig. 9.50

Q6. Given a square-based pyramid which is to be cut by the oblique plane VTH. Draw the plan and elevation of the cut solid, Fig. 9.51.

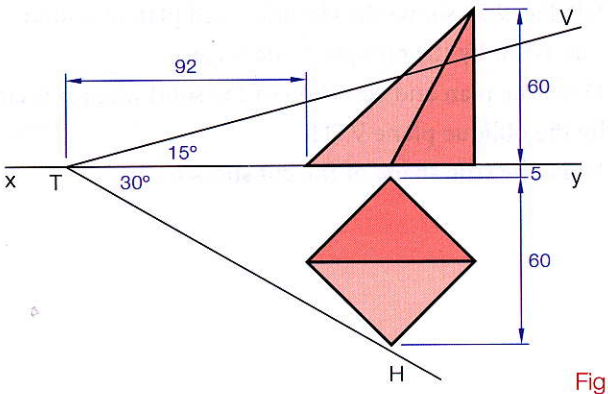


Fig. 9.51

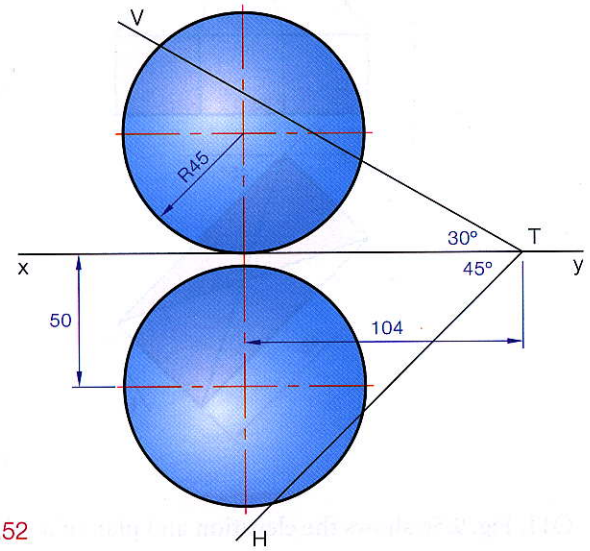


Fig. 9.52

Q7. Given the plan and elevation of a sphere to draw its projections when it is cut by the oblique plane VTH, Fig. 9.52.

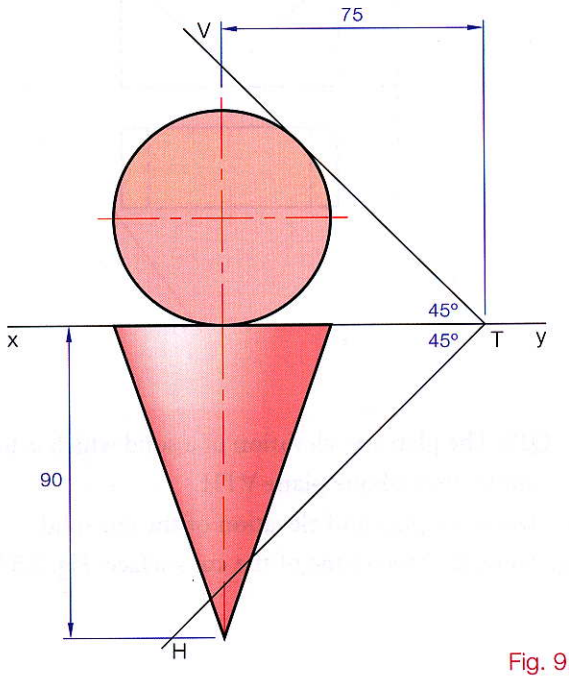


Fig. 9.53

Q8. Given the plan and elevation of a cone having its base resting on the VP. Draw the plan and elevation of this cone when it is cut by the oblique plane VTH, Fig. 9.53.

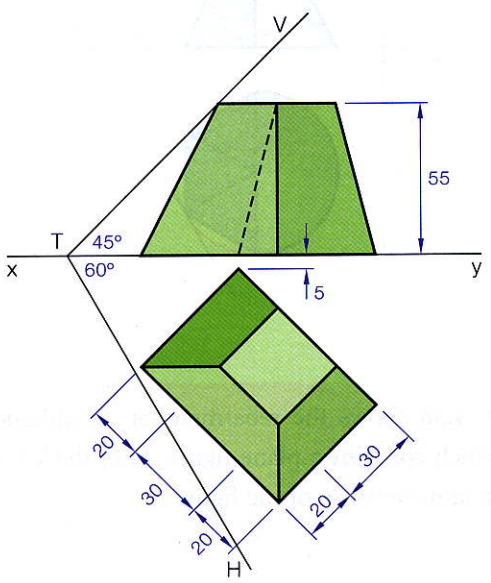


Fig. 9.54

SECTIONS AND TRUE SHAPES

Q9. The elevation and plan of a solid which is to be cut by the oblique plane VTH is shown in Fig. 9.54.

(i) Draw the elevation and plan of the solid when it is cut by the oblique plane VTH.

(ii) Draw the true shape of the cut surface.

Q10. Fig. 9.55 shows the elevation and plan of a solid that is cut by the oblique plane VTH.

(i) Draw the plan and elevation of the solid when it is cut by the oblique plane VTH.

(ii) Draw the true shape of the cut surface.

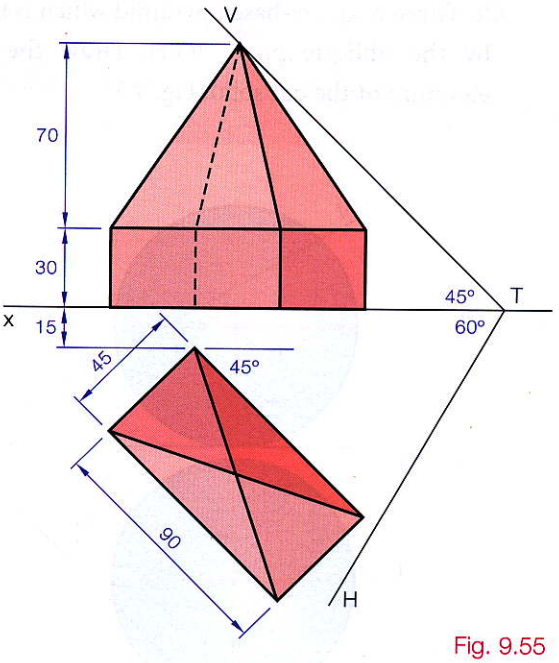


Fig. 9.55

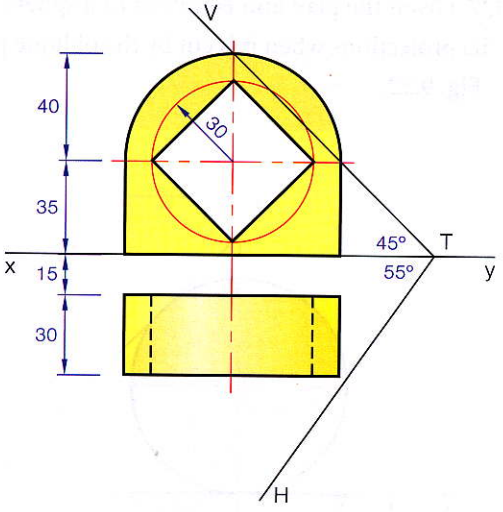


Fig. 9.56

Q11. Fig. 9.56 shows the elevation and plan of a solid which is to be cut by the plane VTH.

(i) Draw the plan and elevation of the solid when it is cut by the oblique plane VTH.

(ii) Draw the true shape of the cut surface of the solid.

Q12. The plan and elevation of a solid which is to be cut by the oblique plane VTH.

(i) Draw the plan and elevation of the cut solid.

(ii) Draw the true shape of the cut surface, Fig. 9.57.

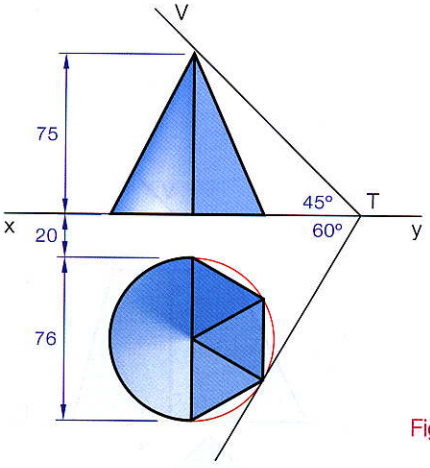


Fig. 9.57

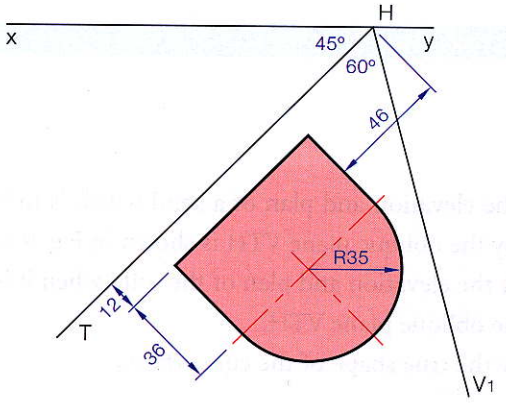


Fig. 9.58

Q13. Fig. 9.58 shows the rebatment of an oblique plane which contains a plane figure. Find the VT and the plan and elevation of the figure.

Q14. The object shown in Fig. 9.59 is to be cut by the oblique plane VTH.
(i) Draw the plan and elevation of the cut solid.
(ii) Draw the true shape of the cut solid.

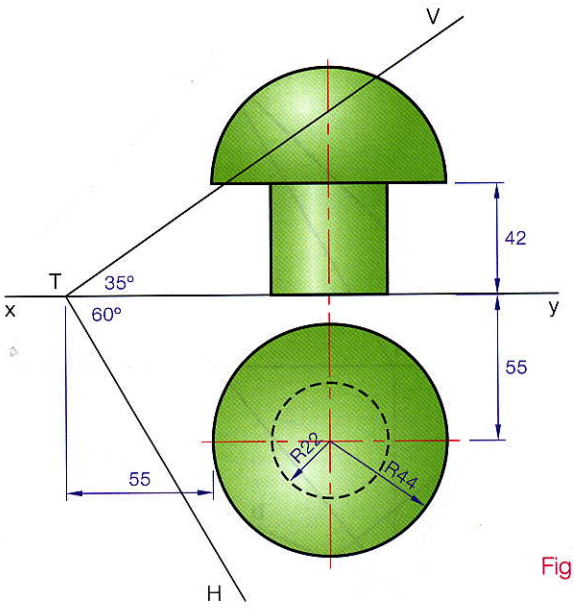


Fig. 9.59

Given two planes VTH and $V_1T_1H_1$. Find the line of intersection between the planes. Determine the dihedral angle between the two planes. Fig. 9.60

(1) Find the line of intersection in plan and elevation.
(2) Find the true length of the line of intersection.
(3) At any point c on the true length draw a perpendicular to the length giving cab. The line cab represents the edge view of the triangle which fits between the planes and measure the dihedral angle.
(4) Rebat the triangle onto the horizontal plane by rotating vertex c as shown. Edge ab is perpendicular to the plan of the line of intersection.

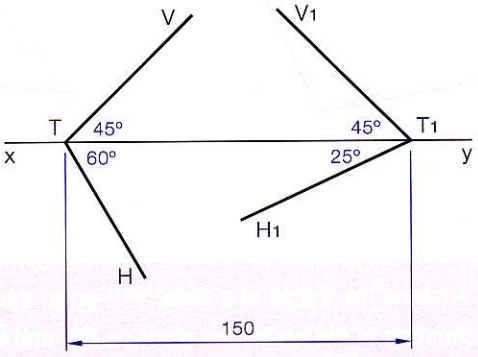


Fig. 9.60

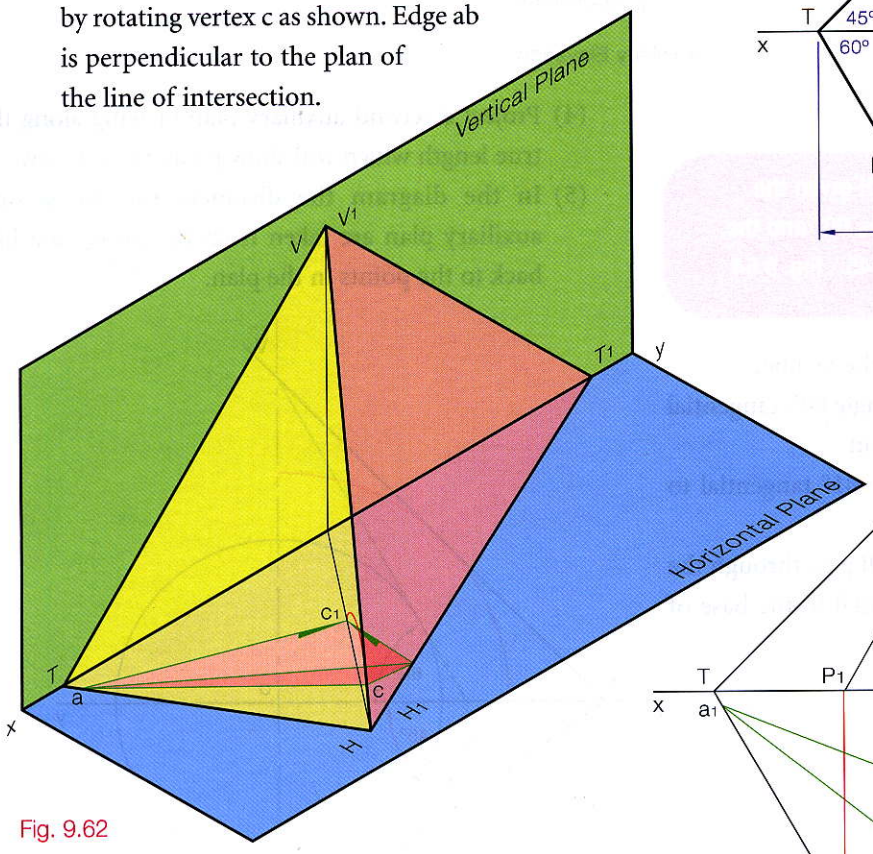


Fig. 9.62

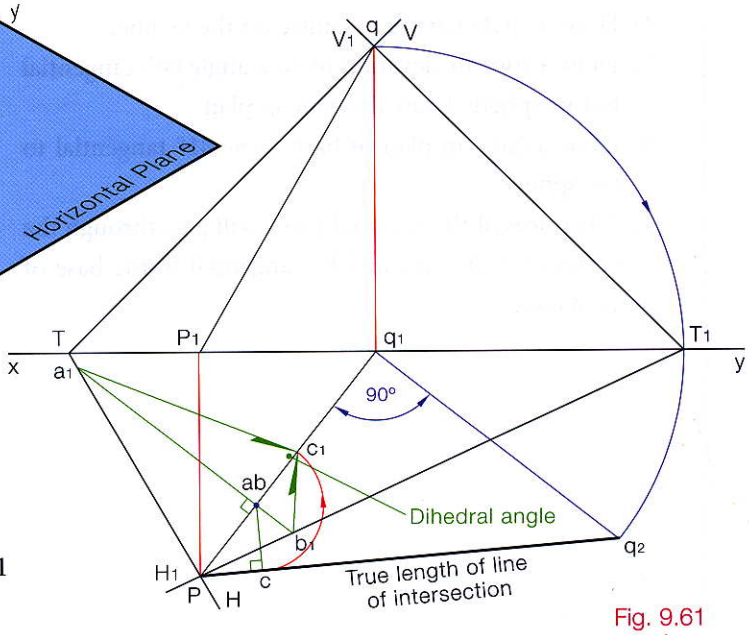


Fig. 9.61

(5) Angle acb is the required dihedral angle, Figures 9.61 and 9.62.

HIGHER LEVEL

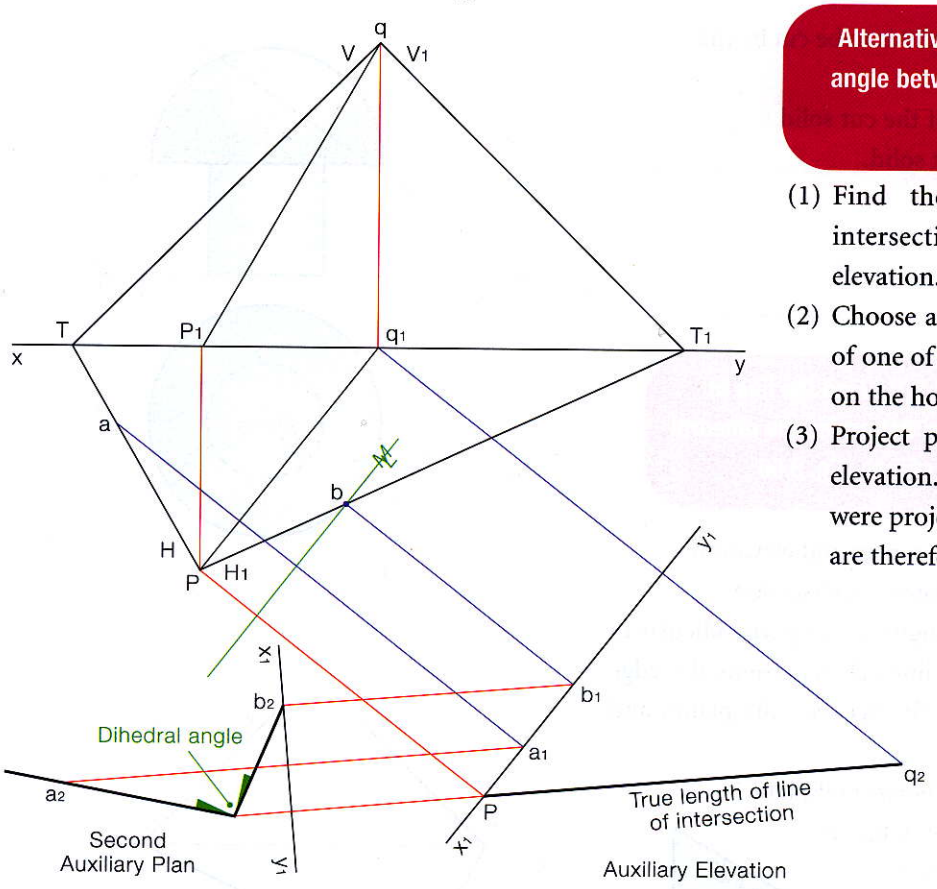


Fig. 9.63

Alternative method of finding the dihedral angle between two oblique planes VTH and $V_1T_1H_1$. Fig 9.63

(1) Find the true length of the line of intersection, pq, by using an auxiliary elevation.

(2) Choose any point 'a' on the horizontal trace of one of the planes and choose any point 'b' on the horizontal trace of the other plane.

(3) Project points a and b onto the auxiliary elevation. Both points rest on the x_1y_1 as they were projected from the horizontal trace and are therefore on the horizontal plane.

(4) Project a second auxiliary plan viewing along the true length which will show pq as a point view.

(5) In the diagram the distances for the second auxiliary plan are taken from the measuring line back to the points in the plan.

To draw the traces of a plane VTH given the inclination to the horizontal plane as 60° and the inclination to the vertical plane as 45°. Fig. 9.64

(1) Draw a circle having its centre on the xy line.

(2) Draw a cone in elevation of base angle 60°, tangential to this sphere. Draw the cone in plan.

(3) Draw a cone in plan of base angle 45° tangential to the sphere.

(4) The traces of the required plane will pass through the vertex of each cone and be tangential to the base of each cone.

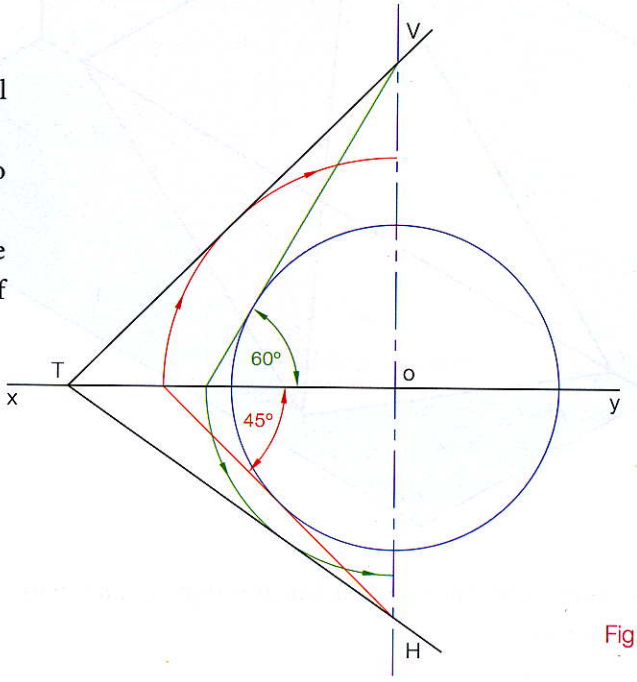


Fig. 9.64

Given the plan and elevation of a point P. Find the traces of a plane that contains point P and is inclined at 60° to the HP and 45° to the VP. Fig. 9.65

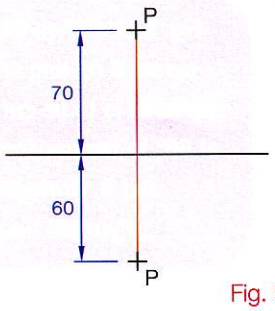


Fig. 9.65

(1) Using P as the apex of the cone in elevation draw a cone with base angle of 60°.

(2) Draw the plan of this cone. The horizontal trace must be tangential to this cone in plan.

(3) Draw the hemisphere that fits under this cone and is tangential to it. Find the plan of the hemisphere.

(4) The angle required to the vertical plane is 45°. Draw a line at 45° to the xy line in plan that is tangential to the plan of the hemisphere. This line intersects the centre line of the cone and hemisphere at point s. Point s is a point on the HT line

(5) Draw the HT from s, tangential to the plan of the cone.

(6) The VT line can be found by projecting the cone apex parallel to the HT to xy. Then vertically and horizontally across in elevation to find a point on the vertical trace.

(7) Alternatively the VT line can be found by drawing it tangentially to the 45° angle rotated about point o, Fig. 9.66.

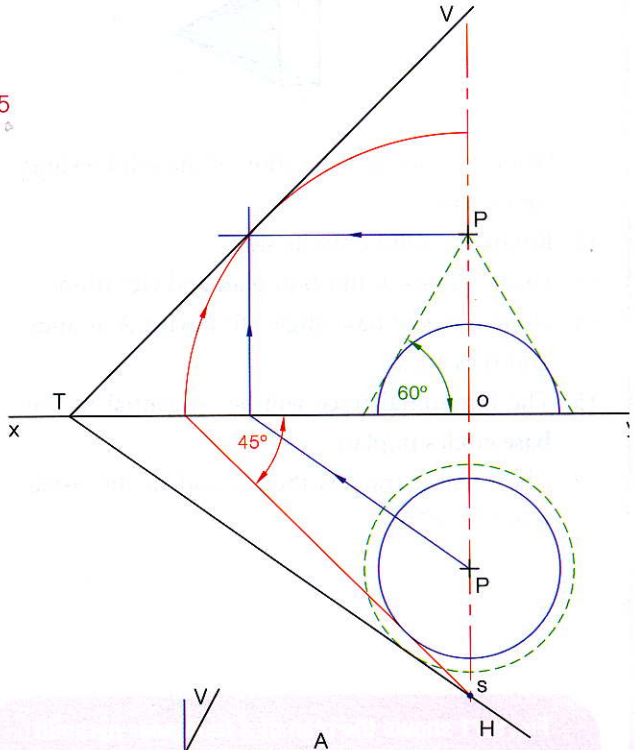


Fig. 9.66

Given the projections of a line AB. Find the traces of the oblique plane that contains line AB and is inclined at 70° to the horizontal plane. Fig. 9.67

(1) Draw the plan and elevation of the line as given.

(2) Draw the plan and elevation of a cone of base angle 70°, resting on the horizontal plane and having its vertex at A.

(3) Draw a similar cone having B as vertex.

(4) The HT line will be tangential to the cones in plan.

(5) The VT line is found by projecting the vertex of either cone parallel to the HT as far as the xy line. Then projecting vertically and across horizontally in elevation to find a point on the VT which can then be drawn in, Fig. 9.68.

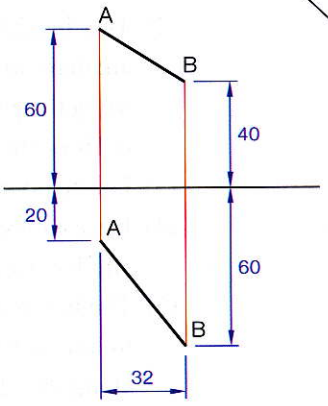


Fig. 9.67

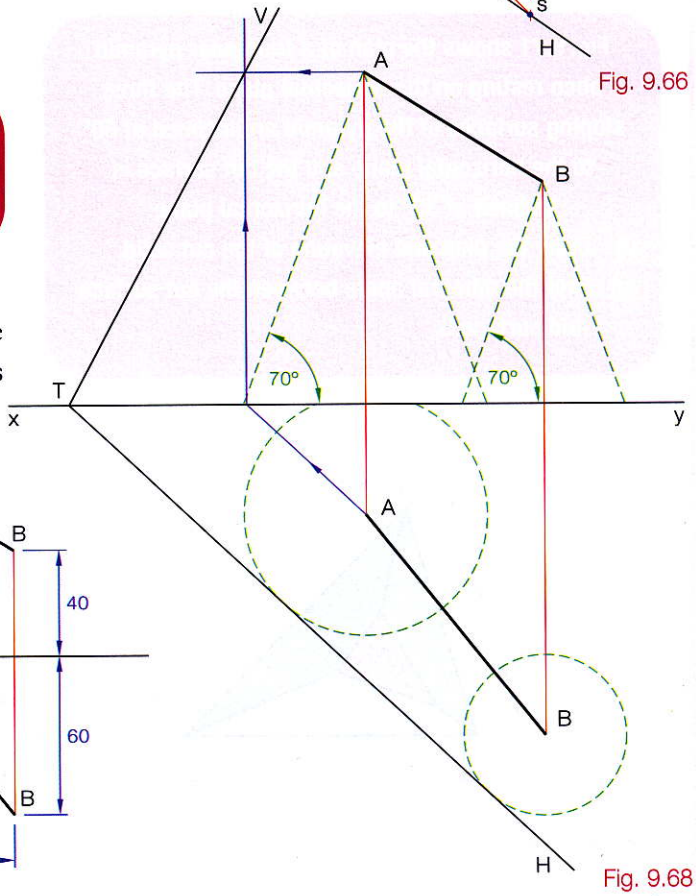


Fig. 9.68

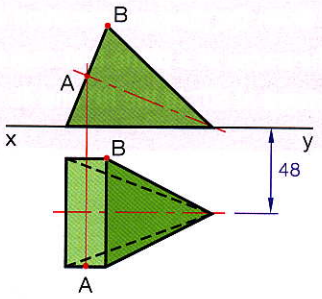


Fig. 9.69

(1) Draw the plan and elevation of the solid resting on its base.
(2) Rotate the solid onto its side.
(3) Locate points A and B in plan and elevation.
(4) Draw cones of base angle 60° having A as apex and B as apex.
(5) The horizontal trace will be tangential to the base circles in plan.
(6) The vertical trace is then found in the usual way, Fig. 9.70.

Fig. 9.69 shows the projections of a right square-based pyramid resting on the horizontal plane as shown. The pyramid is to be cut by an oblique plane which is inclined at 60° to the horizontal plane and passes through points A and B. Draw the projections of the cut pyramid. Pyramid base = 60 mm, Altitude = 75 mm.

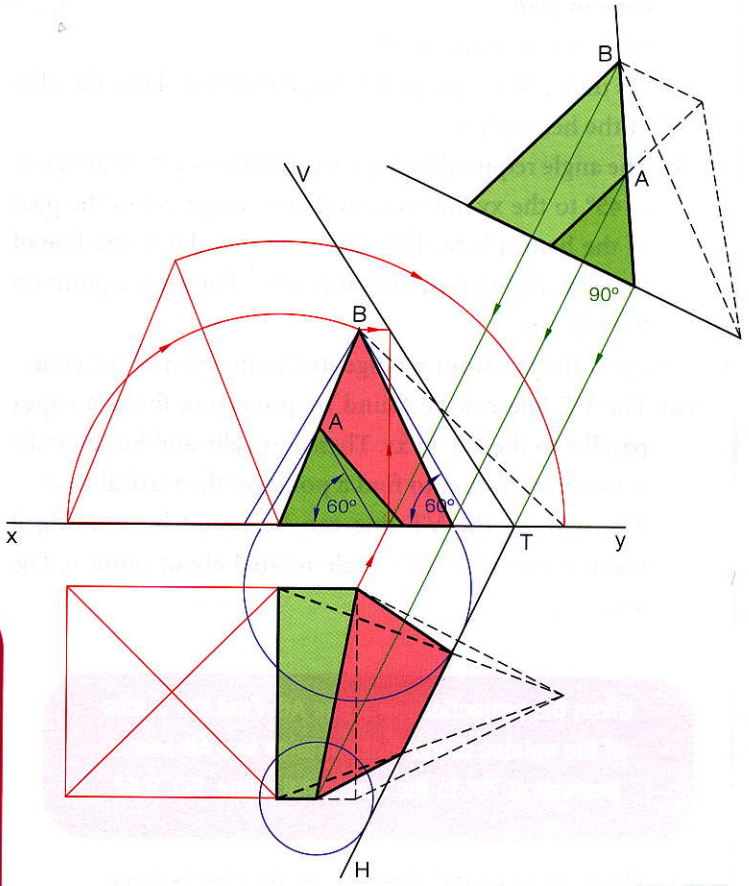


Fig. 9.70

Fig. 9.71 shows the plan of a triangular pyramid abco resting on the horizontal plane. The three sloping surfaces of the pyramid are inclined at 60° to the horizontal plane and the cut surface is inclined at 30° to the horizontal plane.
(i) Draw the plan and elevation of the cut solid.
(ii) Find the dihedral angle between the cut surface and surface S.

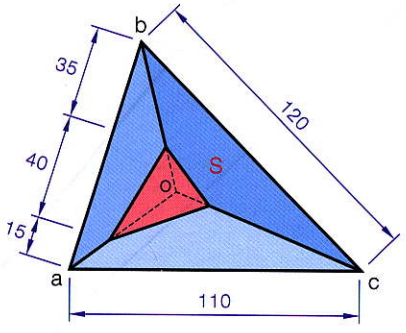


Fig. 9.71

(1) Draw the plan outline. Since the three sloping surfaces of the pyramid have equal inclinations to the HP the lines ao, bo and co will bisect the angles.
(2) The height of the pyramid is found by drawing an auxiliary to show an edge view of one of the surfaces and using the given inclination and the projection of the apex o. Draw the elevation.
(3) Locate points 1 and 2 in plan and elevation, Fig. 9.72.
(4) Place cones, of base angle 30°, underneath points 1 and 2 and having them at their apex.
(5) The horizontal trace of the cut surface will be tangential to the cone base circles. Project an auxiliary viewing along the HT line. This will show the cut point 3 on the line co. Complete the plan and elevation.