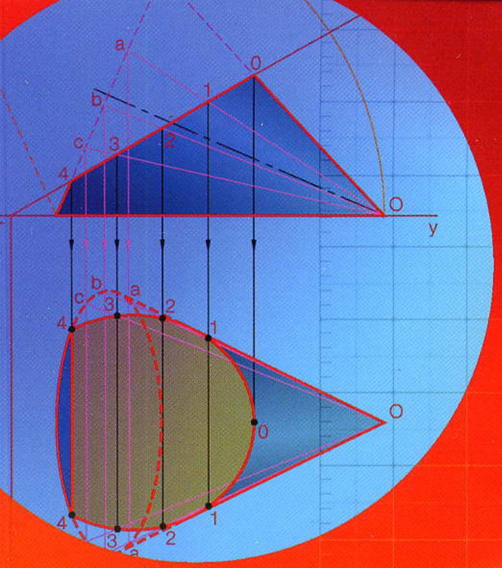
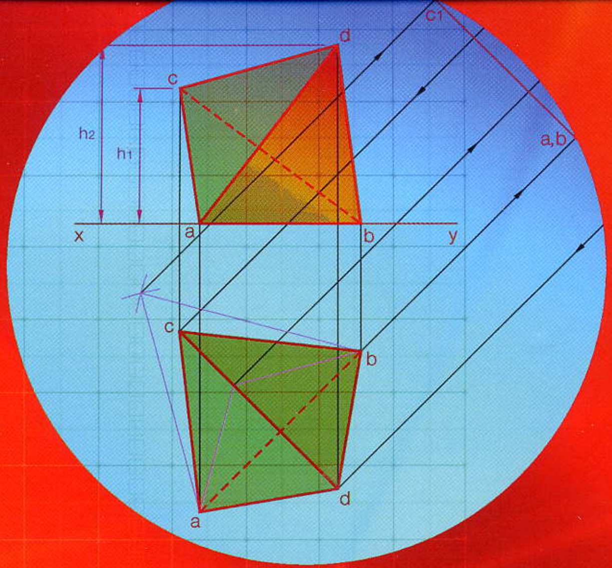
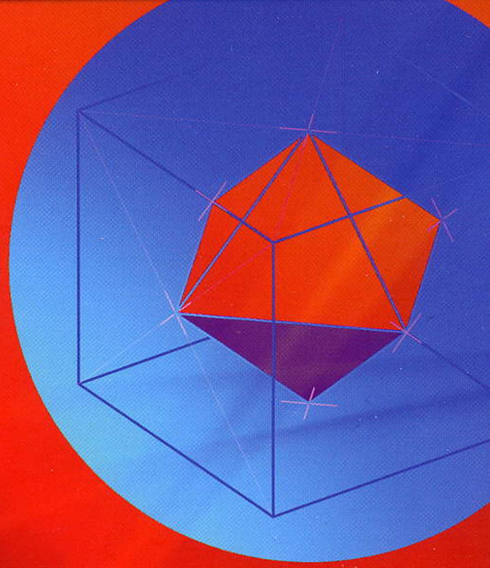
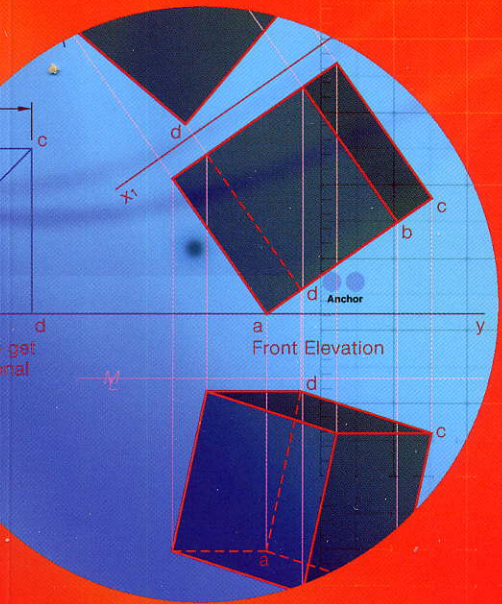


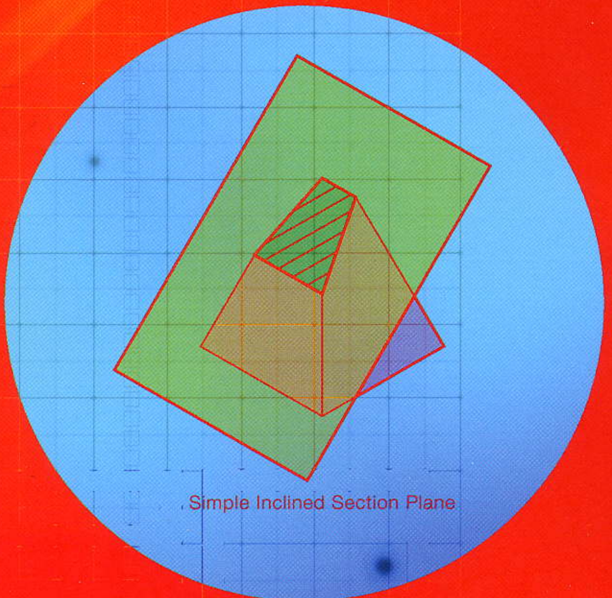


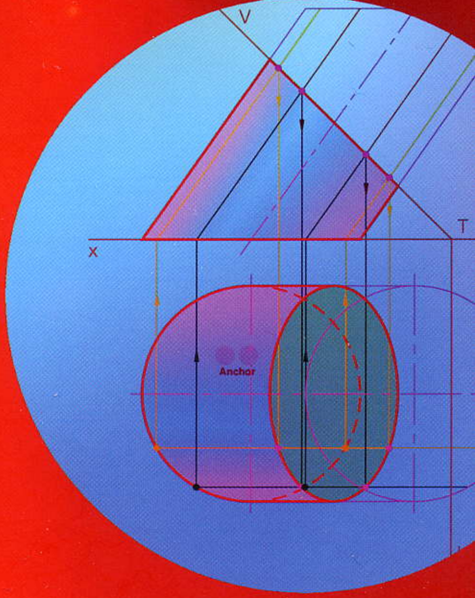


GRAPHICS IN DESIGN & COMMUNICATION 1

PLANE AND DESCRIPTIVE GEOMETRY







DAVID ANDERSON

10 Intersection of Solids

SYLLABUS OUTLINE

Areas to be studied:

- Intersection of surfaces of prisms, pyramids[1] and spheres, their frustra and composite solids and development of same.
- *Intersection of right and oblique solids and their surface development.*

Learning outcomes

Students should be able to:

Higher and Ordinary levels

- Find the intersection of given lines and planes with given planes and curved surfaces.
- Establish the surface intersections of prisms, pyramids, spheres, their frustra and composite solids, where the intersecting solids have their axes parallel to at least one of the principal planes of reference[2].

Higher level only

- *Complete the intersection details of regular and oblique solids wherein their axes are parallel to one of the principal planes of reference.*

1 Pyramid and prism are taken to include the cone and cylinder respectively.

2 Principal planes of reference refers to the horizontal and vertical planes.

Interpenetrations

Everyday life throws up numerous examples of solids joining into other solids. When these solids join we get a **line of interpenetration**. The line of interpenetration will be either straight or curved depending on the types of solids joining together.

Solids made up of flat or plane surfaces penetrated by a similar solid, will produce straight lines as join lines. Solids with curved surfaces, penetrated by other solids will produce curved lines of penetration. In this chapter we investigate various methods of finding the lines of intersection between solids.

Method One: Limits Method

For some of the less complex interpenetrations, involving solids with plane surfaces, this is the best method. The solids involved will produce an interpenetration line made up of straight lines. If we can find the start, bend points and finish of each penetration line, we can find the full line of interpenetration.

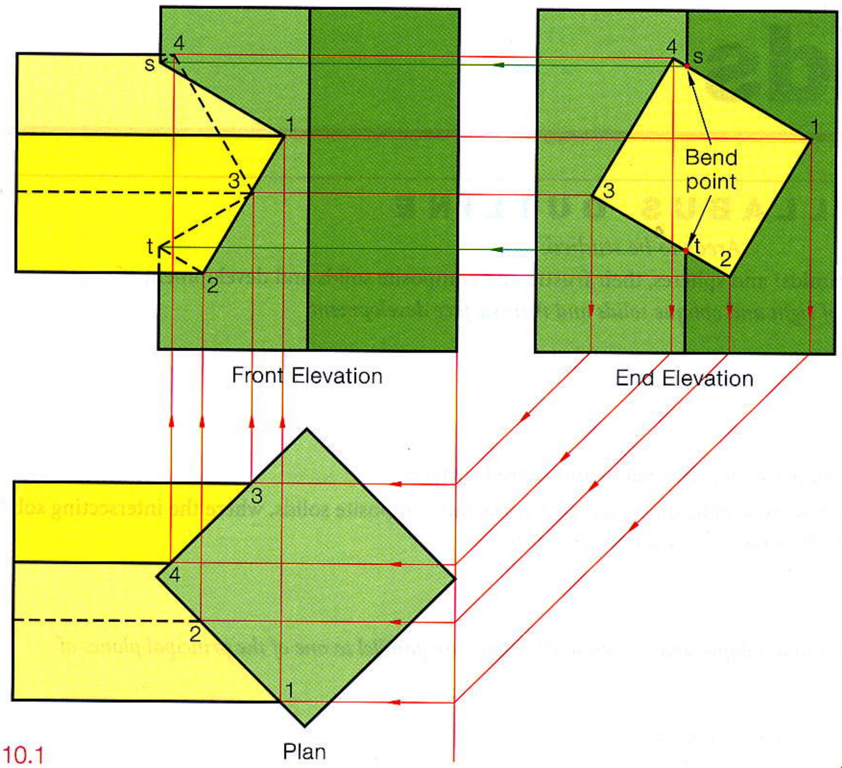


Fig. 10.1

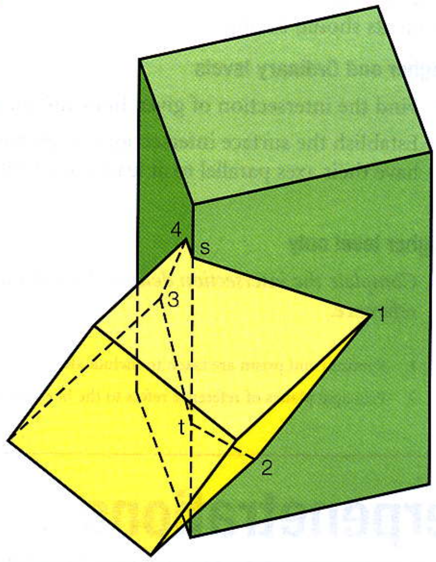


Fig. 10.2

The orthographic projection shown in Fig. 10.1 is a good example of the use of this method. Two square-based prisms are joined and the line of intersection between them is to be found. Information is found from the end elevation and the plan to complete the front elevation. The penetration points of edges 1, 2, 3 and 4 can be clearly seen in plan and are projected to elevation. Where a penetrating surface straddles two surfaces, the line of intersection will have a bend in it. An example of this is the surface containing 2 and 3. The bend point is found in the end view and is point t. The penetration line goes from '2' to 't' to '3'. The pictorial, Fig. 10.2, may help in the visualisation of this.

Given the plan and end elevation of two intersecting solids, a hexagonal prism and a triangular prism. Draw the given views and complete the front elevation of the solids, Fig. 10.3.

(1) Draw the given views. Both the plan and end view are complete and provide the necessary information for the front elevation.
(2) Identify the bend points r, s, t and u in the end elevation. Project these across to the front elevation.
(3) The end points for edges 1 and 2 are seen in plan and projected up to the front elevation.
(4) Edge 3 does not make contact with the triangular prism.
(5) It is important to use indexing in this type of question, Fig. 10.4.

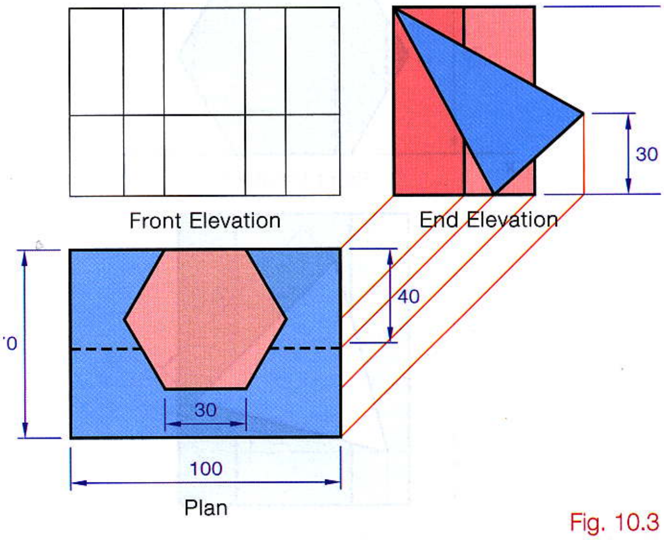


Fig. 10.3

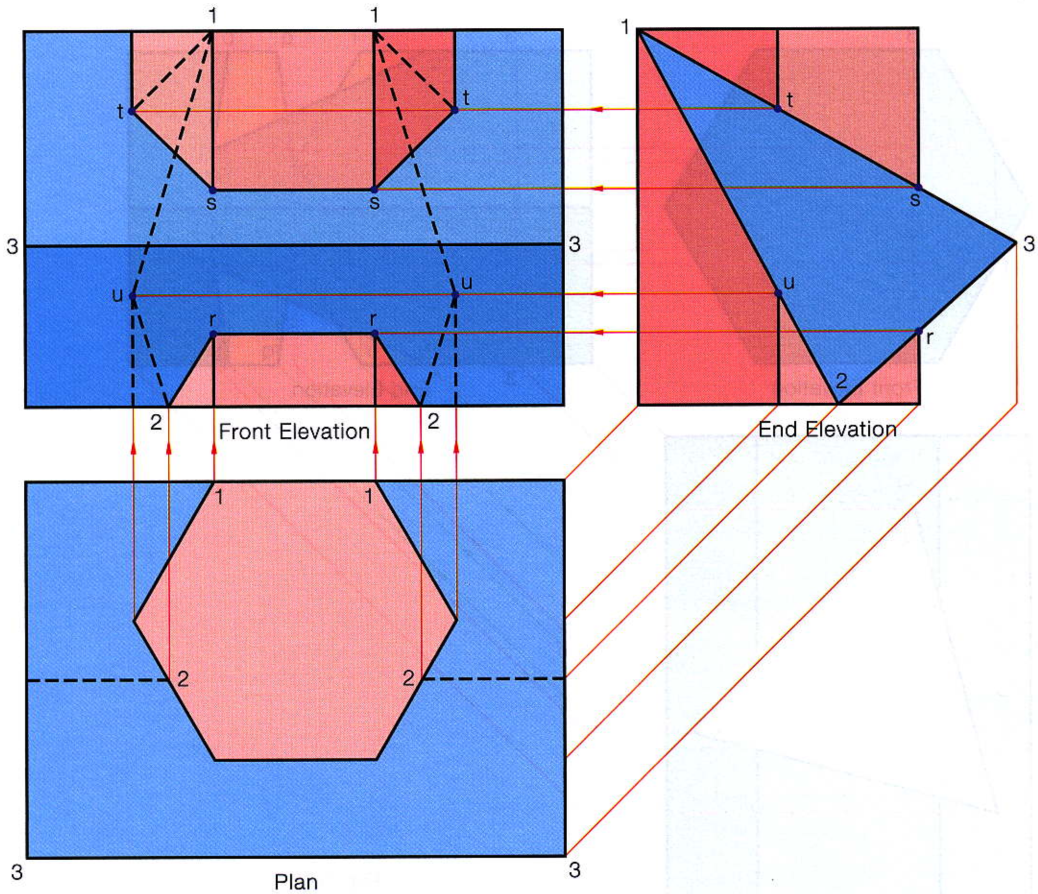


Fig. 10.4

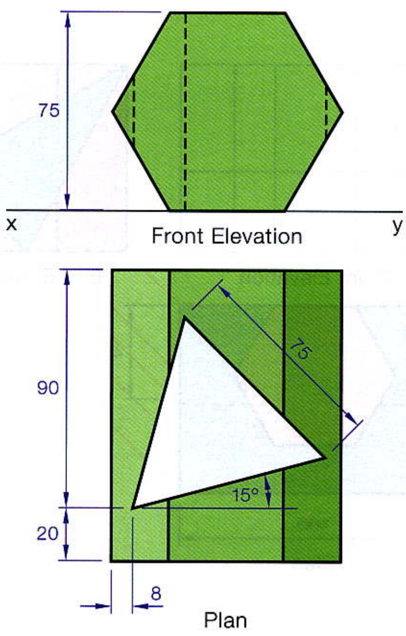


Fig. 10.5

Given the plan and elevation of a hexagonal prism with an equilateral triangular hole cut through it. Draw the given views and project an end view of the solid, Fig. 10.5.

(1) Draw the plan and elevation as given. Both of these views are complete.

(2) By projecting points from both of these views onto the end view we can build it up.

(3) Note the bend points p, q, r and s where one of the cutting planes crosses two or more planes.

(4) When all the points are found they are joined up in order. The order in which they are joined is best seen in the plan. 1 → p → q → 2 → r → 3 → s → 1, Fig. 10.6.

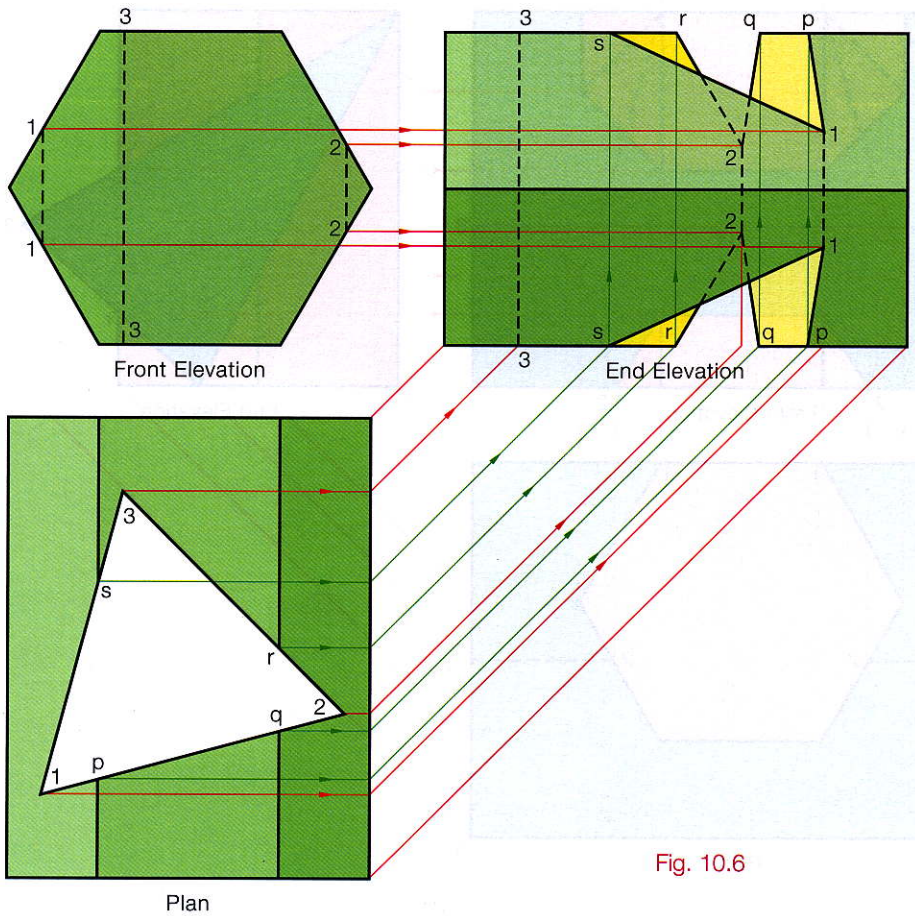


Fig. 10.6

Given the plan and end view of two intersecting prisms. Draw the given views and project the front elevation, Fig. 10.7.

(1) Draw the given views.

(2) The edge, 1 of the square prism intersects the triangular prism in two places. Similarly for edge 2 and edge 3 of the square prism. It can be seen from the end view that edge 4 does not intersect the triangular prism. All six of these points are found in plan and projected to elevation.

(3) The bend points p, q, r and s are seen in end view and projected across to the front elevation.

(4) The sequence of joining the points is found from the end elevation, s, 1, p, 2, q, 3, r, 3, 2, 1, s, Fig. 10.8.

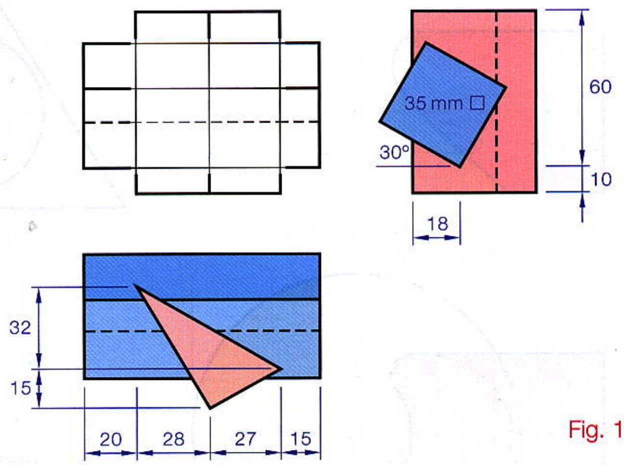


Fig. 10.7

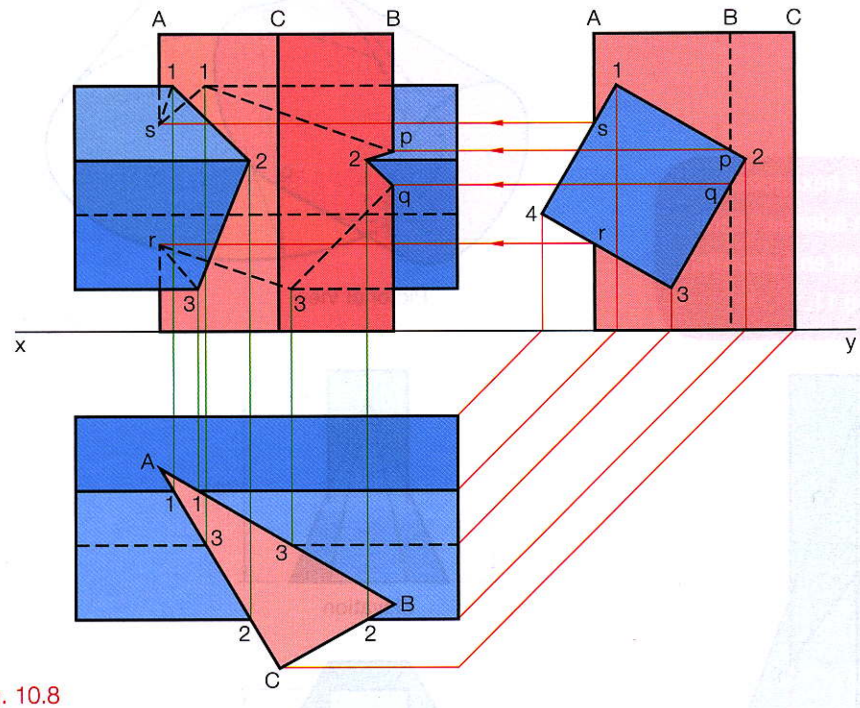


Fig. 10.8

Method Two: Radial Elements Method

This method can be very useful when cones or pyramids are being penetrated by other solids. The limits method used in the previous examples will not work for these types of solids.

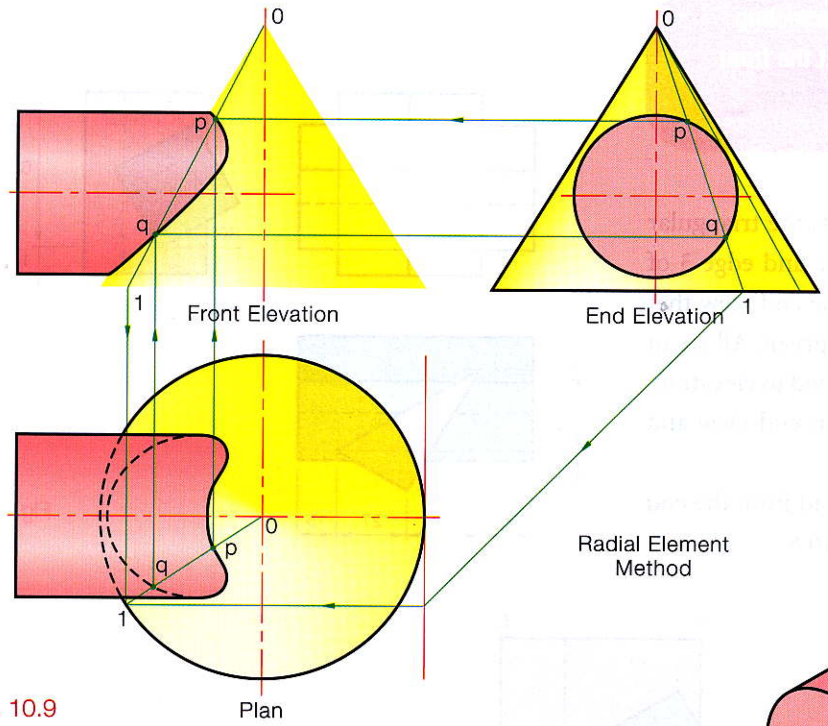


Fig. 10.9

In Fig. 10.9 we can see how a radial element 0, 1 is taken in end view from the apex of the cone and passing through the second solid. This element locates two points p and q. The element can easily be found in the other views and points p and q found on it. Points p and q are on both solids and are therefore on the line of intersection. Many more points on the line of intersection can be found by using more elements, Fig. 10.10.

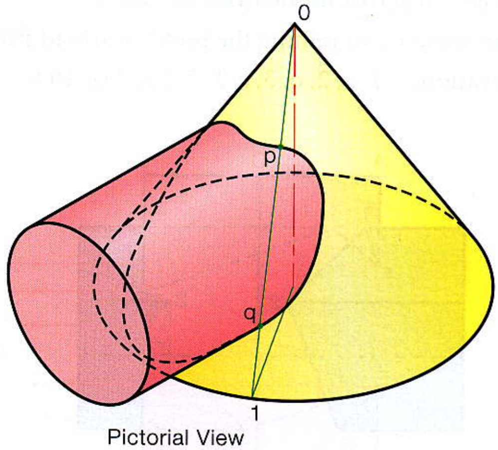


Fig. 10.10

Given the plan and partial elevation of a hexagonal-based pyramid and a square prism which intersect each other. Draw the complete plan, elevation and end elevation of the intersecting solids, Fig 10.11.

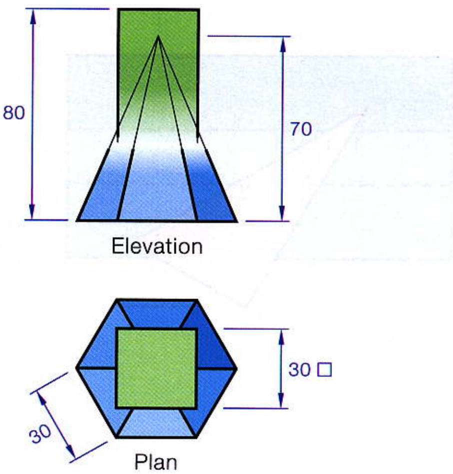


Fig. 10.11

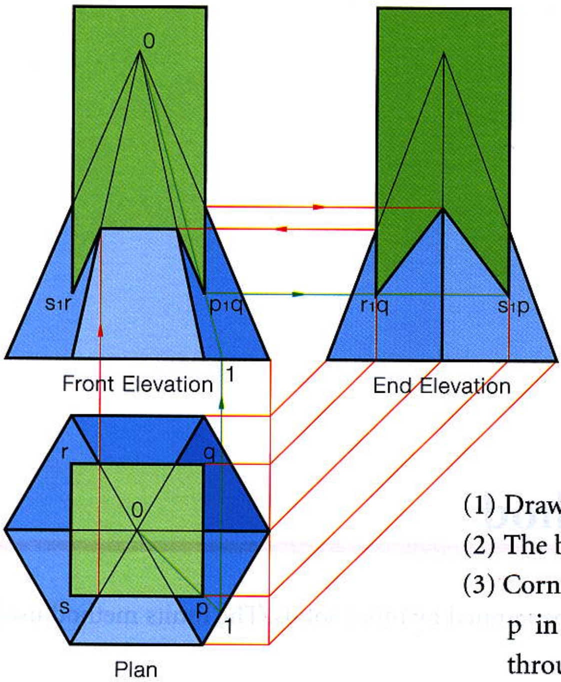


Fig. 10.12

(1) Draw the given plan and elevation.

(2) The bend points are easily found.

(3) Corners p, q, r and s fall on the sloping faces of the pyramid. To find point p in elevation we draw an element from the pyramid apex in plan, through point p to hit the pyramid base at point 1. This element 0, 1 can be found in elevation and p found on it.

(4) The front elevation and end elevation can now be completed. There is no necessity for any more elements as the answer is symmetrical, Fig. 10.12.

Fig. 10. 13 shows the plan of a hexagonal-based pyramid of height 80 mm. The pyramid has a square hole cut through it. Draw the given plan and project a front elevation and end elevation.

(1) The construction is as before.

(2) Draw the front elevation and end elevation simultaneously as some of the bend points are easier to find in one than the other.

(3) An element is drawn through point b in plan. This element is found in the front elevation and point b is found.

(4) Another element is drawn through point d in plan. This element is found in the end elevation and point d is found, Fig. 10.14.

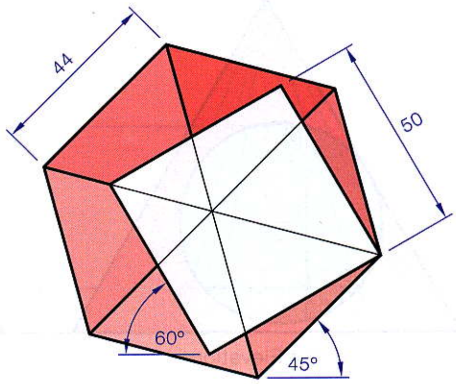


Fig. 10.13

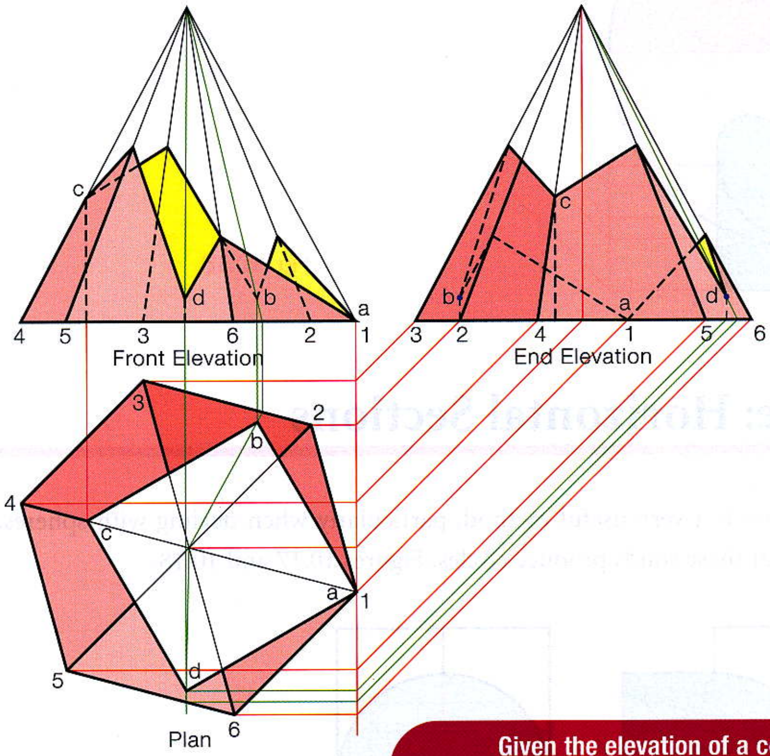


Fig. 10.14

Given the elevation of a cone and a cylinder intersecting each other. Draw the given view and project a plan and end view. The cylinder projects 10 mm beyond the base of the cone, Fig. 10.15.

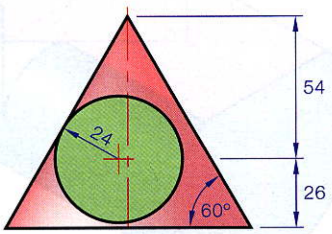


Fig. 10.15

When drawing in the elements on the cone in front elevation, it is advisable to space them equidistant each side of the centre line. This ensures there are less elements to project across to end view. Do not draw in too many generators as it can complicate the drawing. The method is clear from the drawing Fig. 10.16.

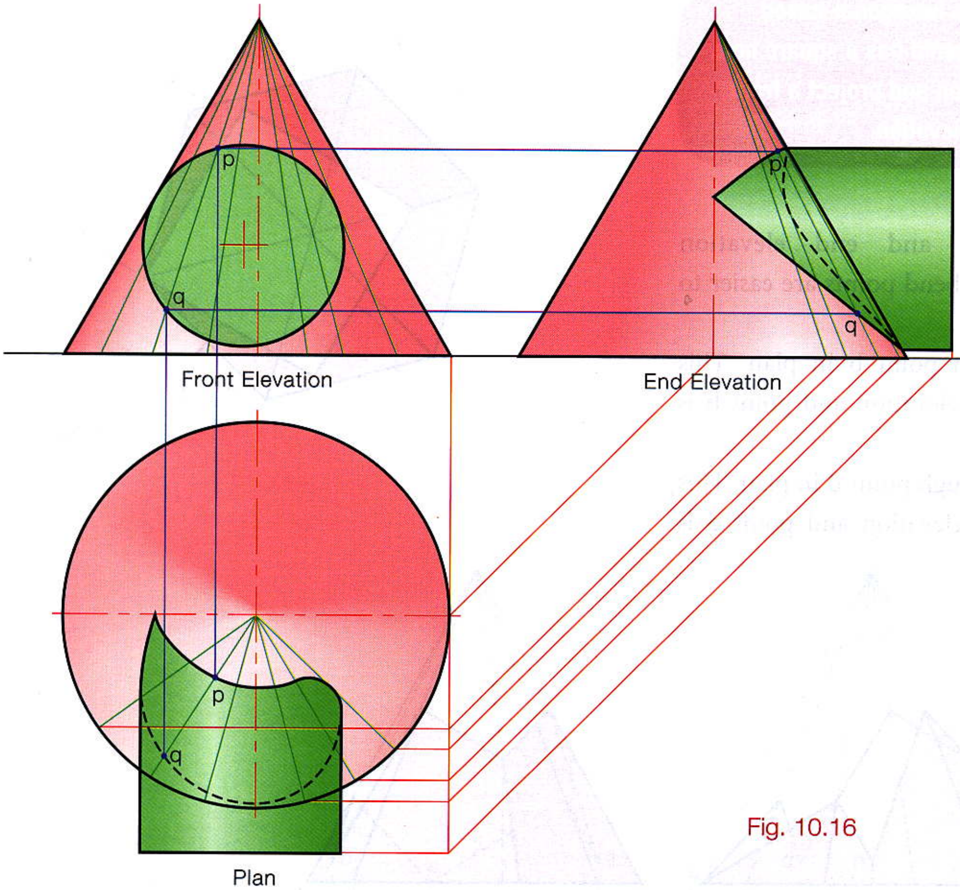


Fig. 10.16

Method Three: Horizontal Sections

The use of horizontal planes is a very useful method, particularly when dealing with spheres, cones and cylinders. The horizontal section of each of these solids produce circles, Figures 10.17 and 10.18.

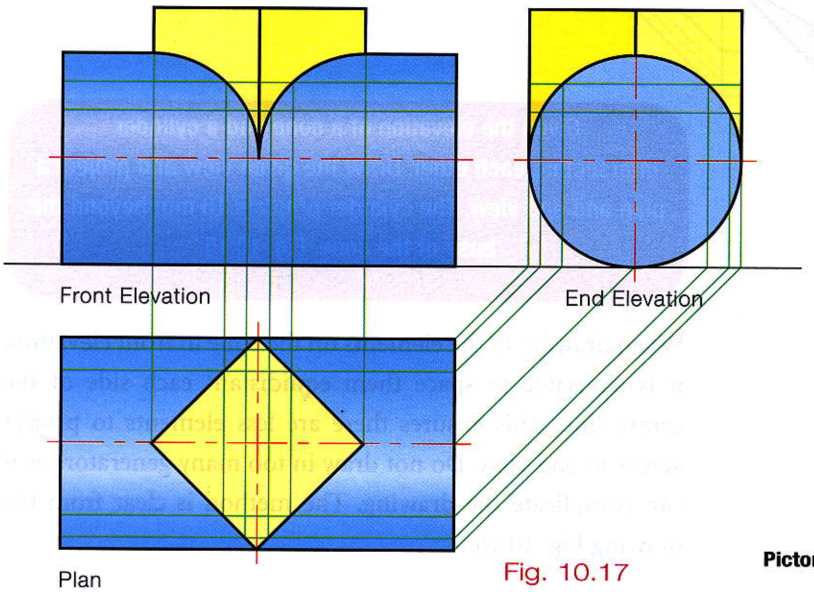
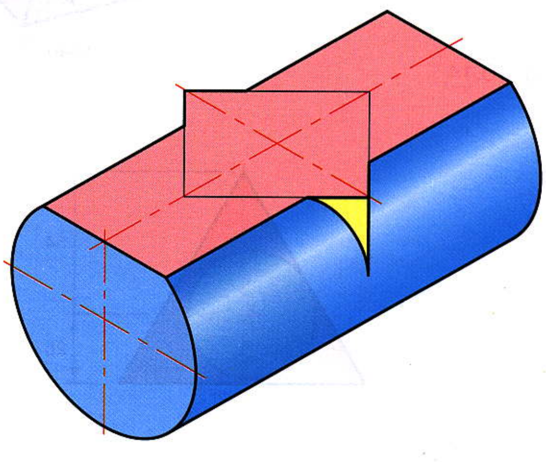


Fig. 10.17



Pictorial showing horizontal section

Fig. 10.18

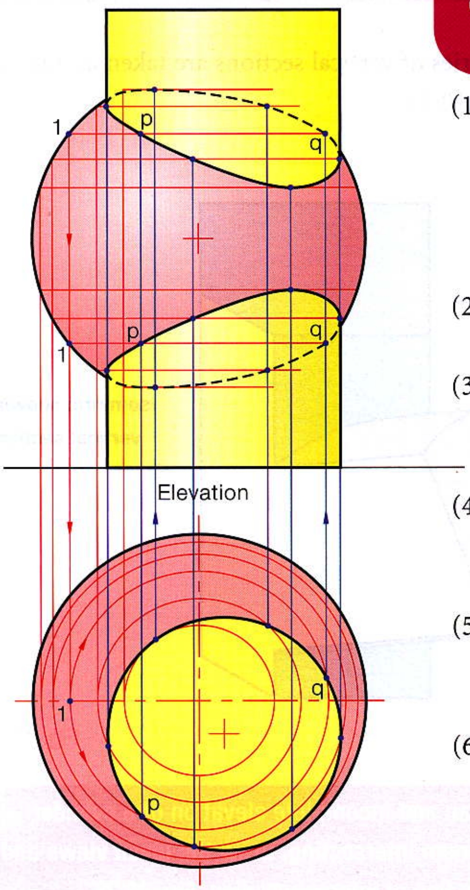


Fig. 10.20

To draw the plan and elevation of a cylinder piercing a sphere, showing clearly the line of intersection, Fig. 10.19.

(1) A horizontal section through these two solids will produce two intersecting circles. Draw the plan which is complete and draw the partial elevation.

(2) Take any horizontal section in elevation, e.g. at 1.

(3) The section of the sphere is a circle in plan which intersects the plan of the cylinder at points p and q.

(4) Project p and q onto the section line in elevation. Repeat for other sections.

(5) It is worth noting that if the section lines are taken too high or too low the circles produced will not intersect.

(6) The elevation is symmetrical about the horizontal axis, Fig. 10.20.

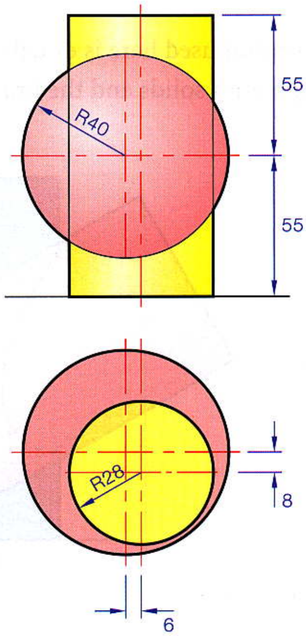


Fig. 10.19

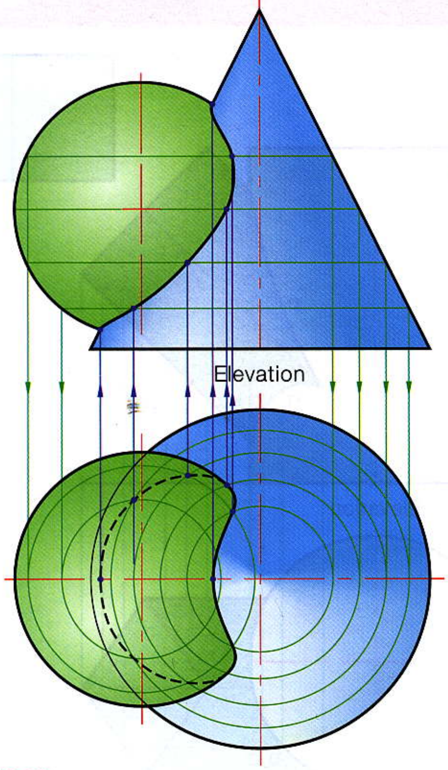


Fig. 10.22

Given the plan and elevation of a sphere and a cone which intersect each other. Draw the given views and find the line of interpenetration, Fig. 10.21.

(1) Draw the plan and elevation as given.

(2) It is advisable to space the horizontal sections at equal intervals each side of the sphere centre line.

(3) The intersections of the cone sections in plan with their corresponding sphere sections gives the points for the curve on the plan.

(4) Project these points of intersection to their corresponding horizontal sections to obtain the curve points on the front elevation, Fig. 10.22.

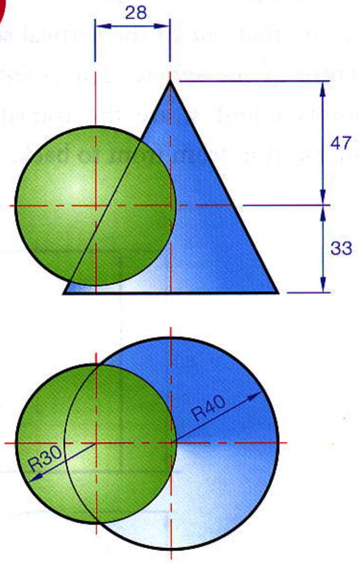


Fig. 10.21

Method 4: Vertical Sections

The method used here is exactly the same as that for horizontal sections. A series of vertical sections are taken at intervals through both solids and the line of intersection is built up, Figures 10.23 and 10.24.

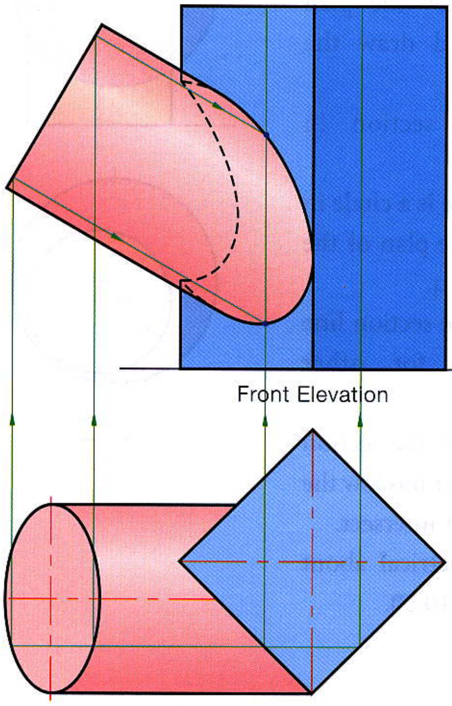


Fig. 10.23

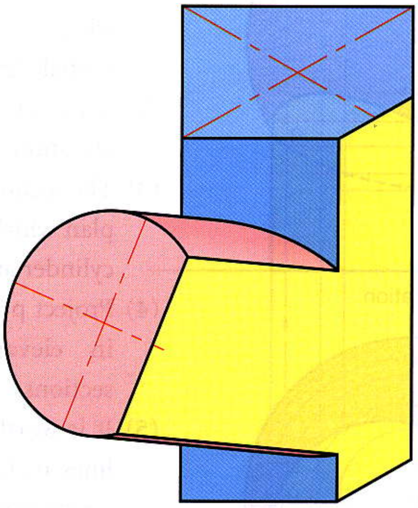


Fig. 10.24

Given the plan and incomplete elevation of a cylinder and a square prism intersecting. Draw the given views and find the line of interpenetration, Fig. 10.25.

(1) Set up the plan and elevation.

(2) Vertical sections will produce intersecting straight lines as shown in Fig. 10.26.

Ensure that one of the vertical sections taken is through the centre of the sphere. The points found using this section, points r and s, are the transition points for the line of intersection from front to back.

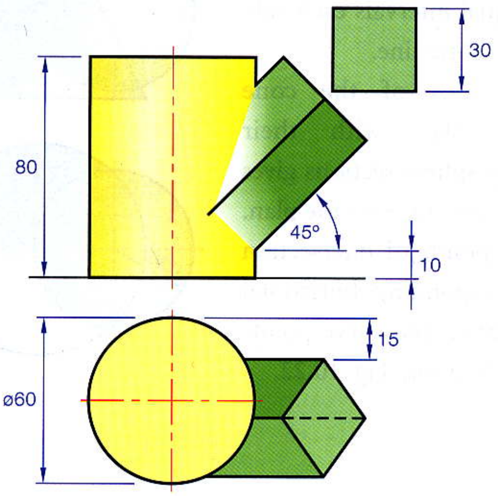


Fig. 10.25

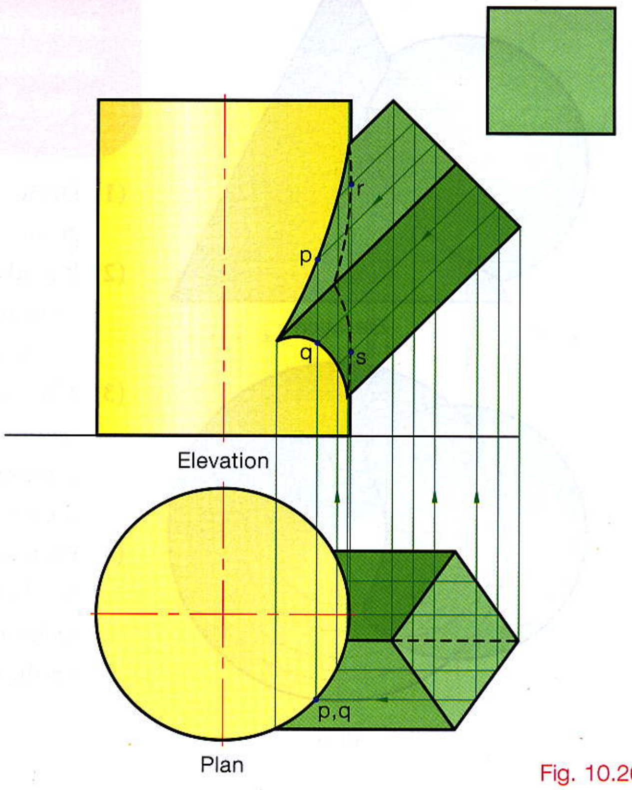


Fig. 10.26

Given the plan and incomplete elevation of a sphere intersecting a triangular prism. Draw the given plan and complete the elevation, Fig. 10.27.

(1) This problem could be solved by using horizontal or vertical cutting planes.
(2) It is advisable to take the cutting planes equidistant each side of the sphere centre line as this will reduce the number of sectional circles needed in elevation.
(3) One of these sectional circles forms part of the interpenetration line to the back of the two solids, Fig. 10.28.

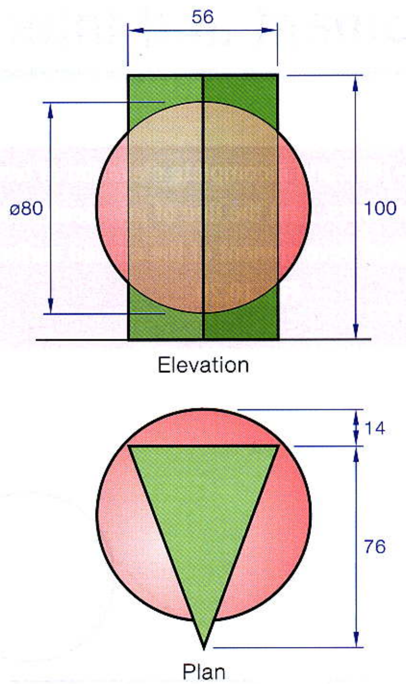


Fig. 10.27

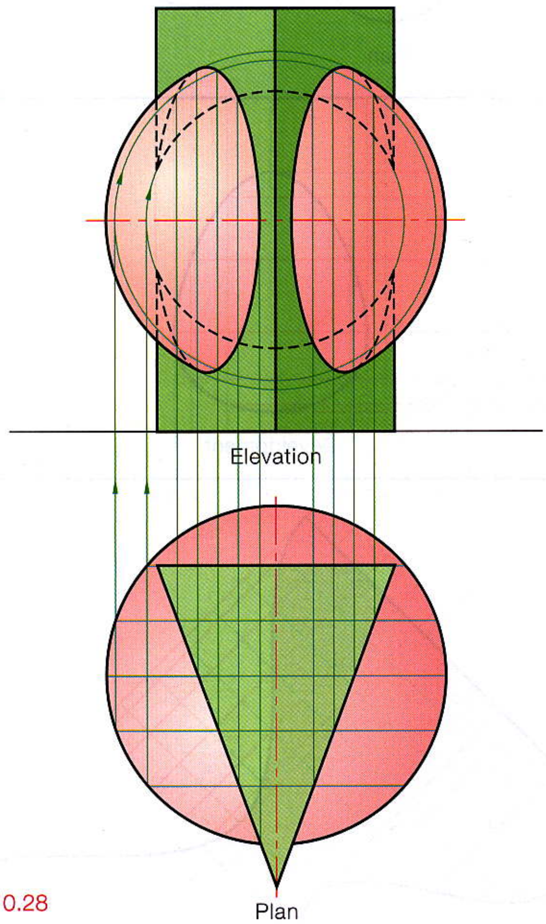


Fig. 10.28

Development and Interpenetration

Given the plan and incomplete elevation of two intersecting cylinders. Find the line of interpenetration and draw a surface development of the curved surfaces, Fig. 10.29.

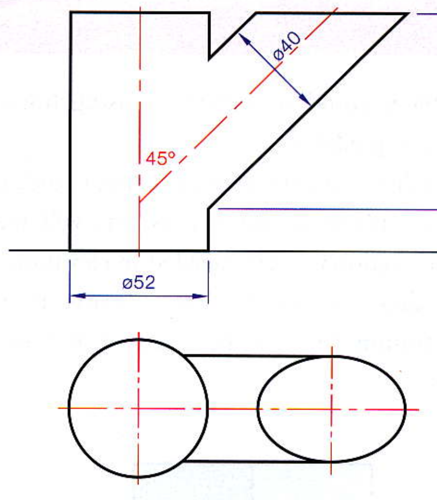


Fig. 10.29

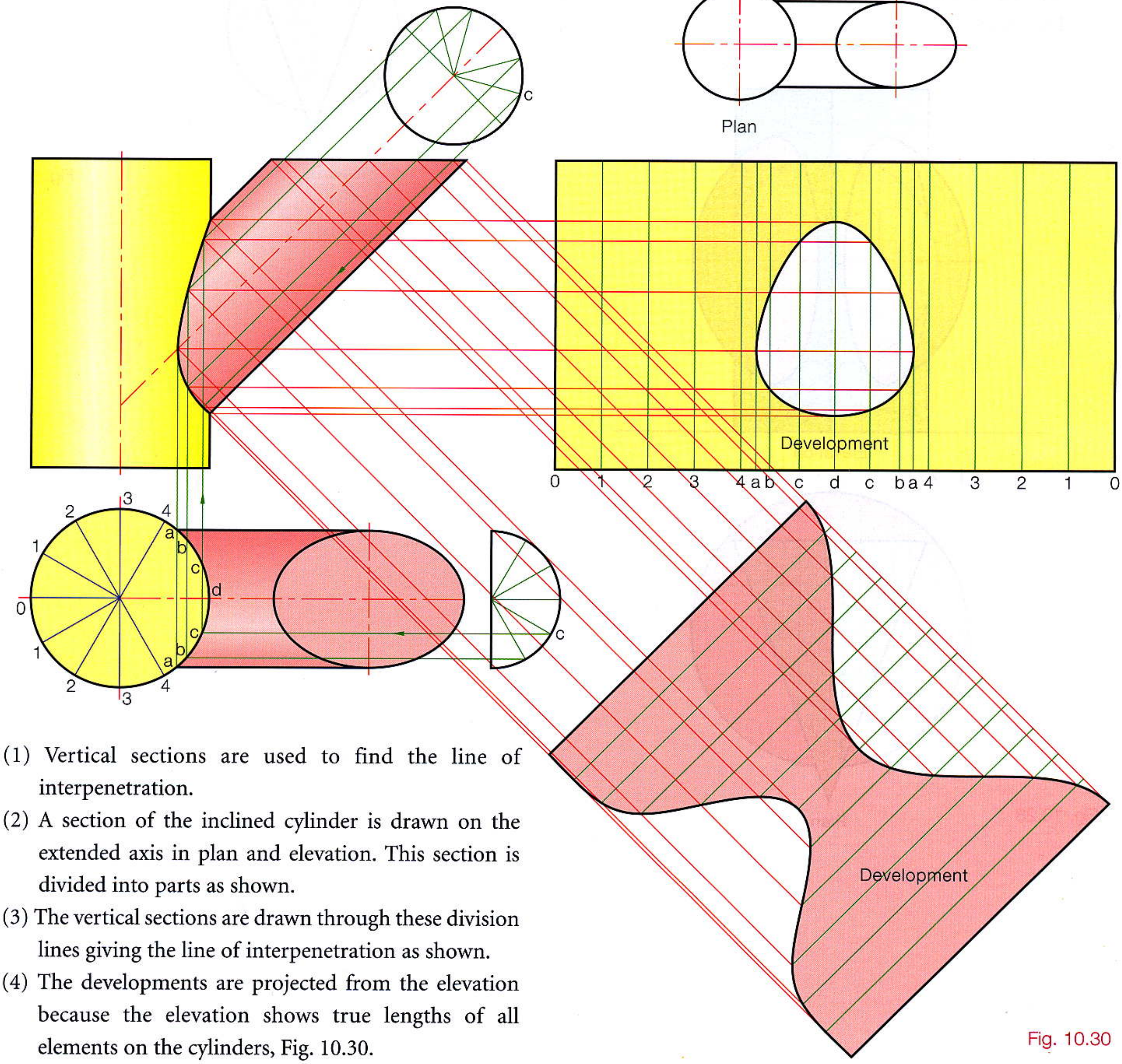


Fig. 10.30

(1) Vertical sections are used to find the line of interpenetration.

(2) A section of the inclined cylinder is drawn on the extended axis in plan and elevation. This section is divided into parts as shown.

(3) The vertical sections are drawn through these division lines giving the line of interpenetration as shown.

(4) The developments are projected from the elevation because the elevation shows true lengths of all elements on the cylinders, Fig. 10.30.

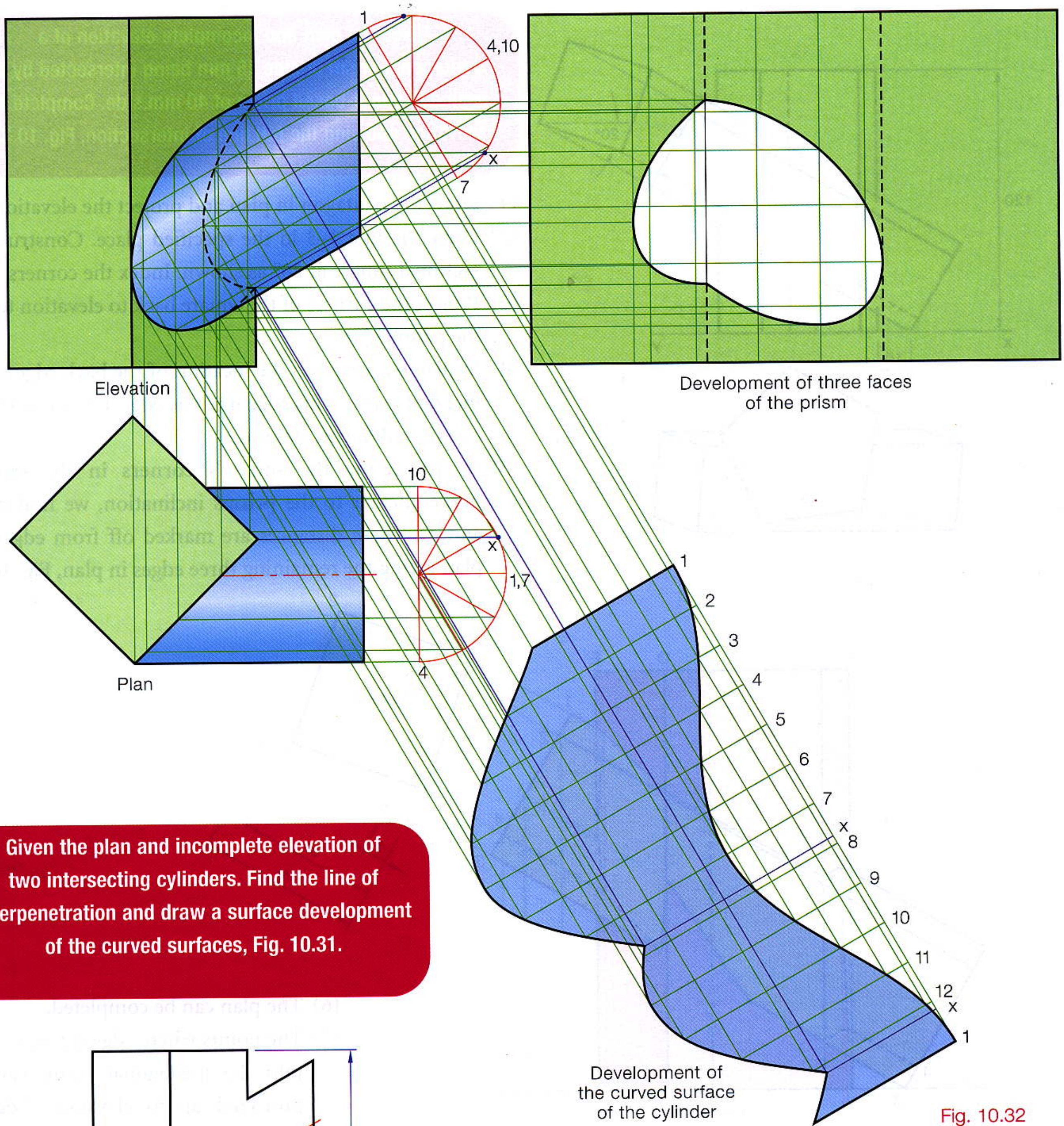


Fig. 10.32

Given the plan and incomplete elevation of two intersecting cylinders. Find the line of interpenetration and draw a surface development of the curved surfaces, Fig. 10.31.

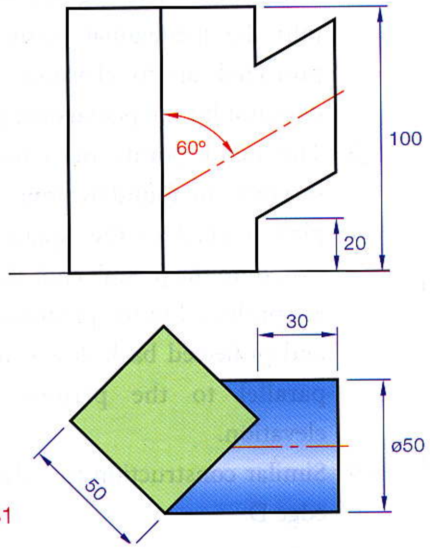


Fig. 10.31

(1) Draw a partial section of the cylinder on the extended axis in plan and elevation. A semicircle is sufficient. Divide each into six equal divisions as shown.

(2) The interpenetration is found by projection of elements as shown.

(3) Point X must be located in plan, where the cylinder hits the corner of the prism. Once found on the section in plan it is transferred with the compass to the section in elevation. Thus we can find the bend points in elevation.

(4) Point X is also needed in the development of the cylinder.

(5) Complete as shown, Fig. 10.32.

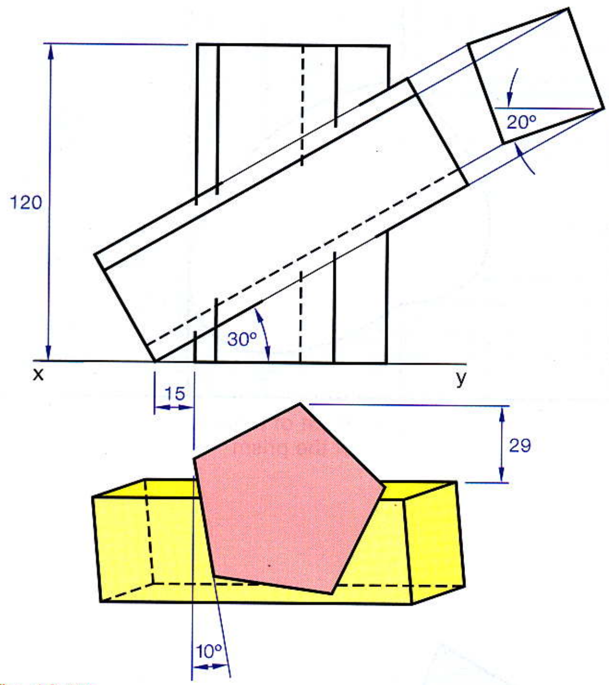


Fig. 10.33

Given the plan and incomplete elevation of a pentagonal prism of side 45 mm being intersected by an inclined square-based prism of 40 mm side. Complete the elevation showing the full line of intersection, Fig. 10.33.

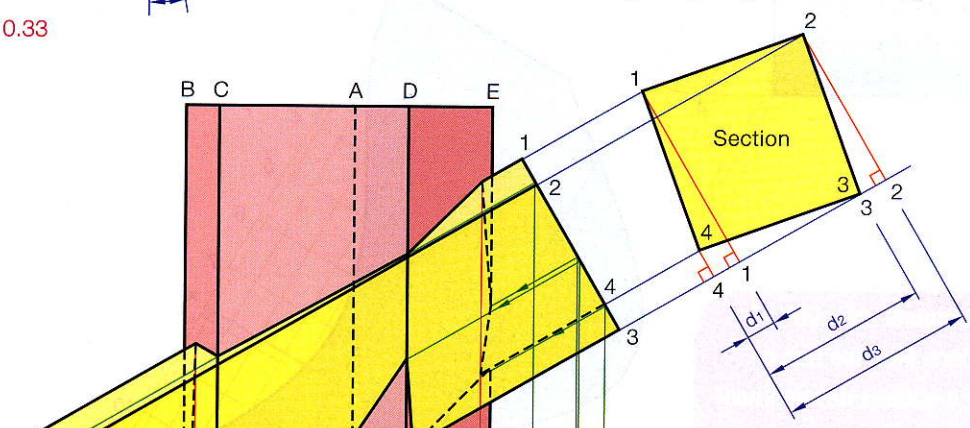
(1) Draw the pentagon in plan and project the elevation.

(2) Draw the line 3–3 in the specified place. Construct the sectional square at 20° as shown. Index the corners.

(3) Project the corners of the square back to elevation to give the prism edges.

(4) In plan we are given the location of the back edge of the inclined prism. This edge must be edge 4 as this is shown as a dotted line in elevation.

(5) By projecting the square's corners in the section, perpendicular to the prisms inclination, we find d_1, d_2 and d_3. These distances are marked off from edge 4 in plan giving the remaining three edges in plan, Fig. 10.34.



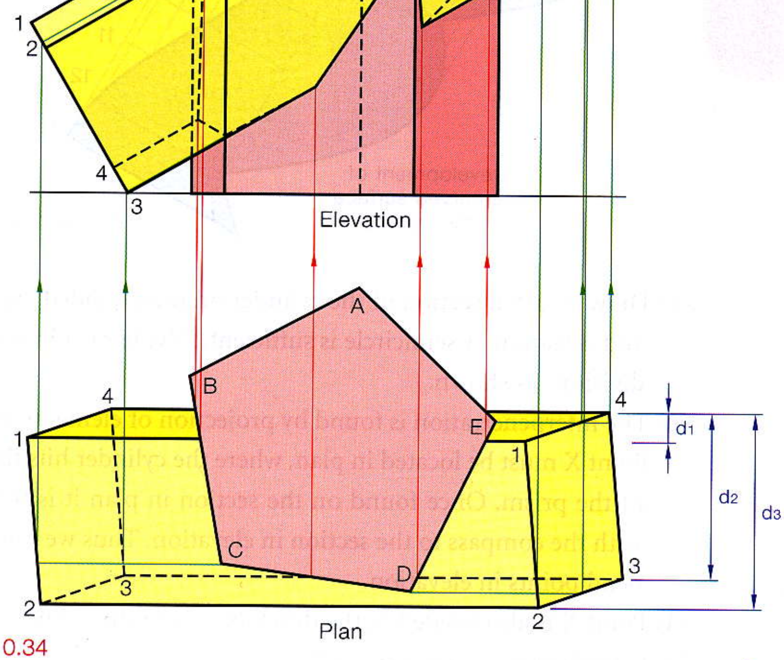


Fig. 10.34

(6) The plan can be completed.

(7) The points where edge 4,1 and 3 run into the pentagonal prism can be projected up to elevation. Edge 2 does not hit the pentagonal prism.

(8) The bend points on edge E, for instance, are found by projecting E in plan parallel to the square prism's axis, onto the prism's end. The points are projected to the prism's elevation and projected back down to edge E parallel to the prism's axis in elevation.

(9) Similar construction for edge C and edge D.

Use of Auxiliary Plans

Fig. 10.35 shows the projections of a square-based prism of 50 mm side. This solid is being intersected by an equilateral triangular-based prism of 50 mm side. Draw the projections of the solids showing all lines of interpenetration

(1) Draw the square prism in plan and elevation.

(2) Details for the lowest line of the inclined triangular prism are given. Draw this line in elevation. Extend this line to the right and construct the equilateral section as given.

(3) Index the corners and project back to elevation.

(4) Project the corners of the section triangle perpendicularly to the prism axis thus finding d_1 and d_2.

(5) Details of one of the triangular prism edges is given in plan. This edge must be edge 1. Step distances d_1 and d_2 in plan to find the other two edges.

(6) An auxiliary plan is drawn, viewing down along the triangular prism's axis. This auxiliary shows which faces are intersected and finds bend points p, q, r and s.

(7) From the auxiliary, corner 2 penetrates surface ABCD and AABB.

AABB is seen as an edge view in plan and the penetration point is seen in plan and projected to elevation.

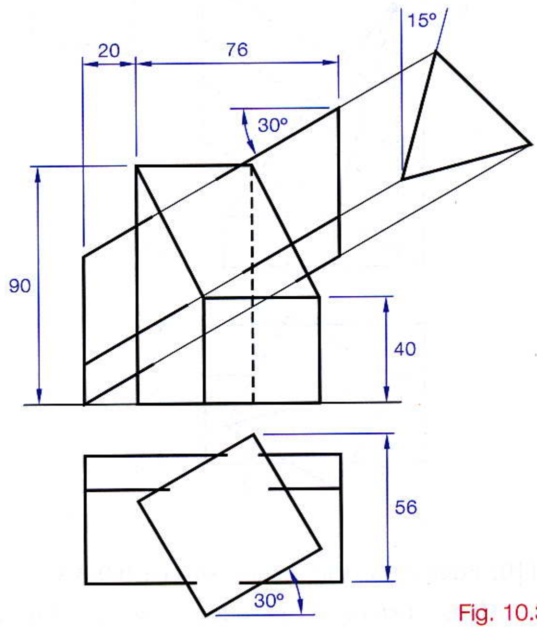


Fig. 10.35

(8) To find point 2 on surface ABCD we draw a line from one of the corners through 2, e.g. from B through 2 to hit edge AD. Find this line in elevation. Point 2 is on this new line in elevation and on line 2,2 so therefore is pinpointed at these lines' intersection point. Similar construction for point 1 on surface ABCD, Fig. 10.36.

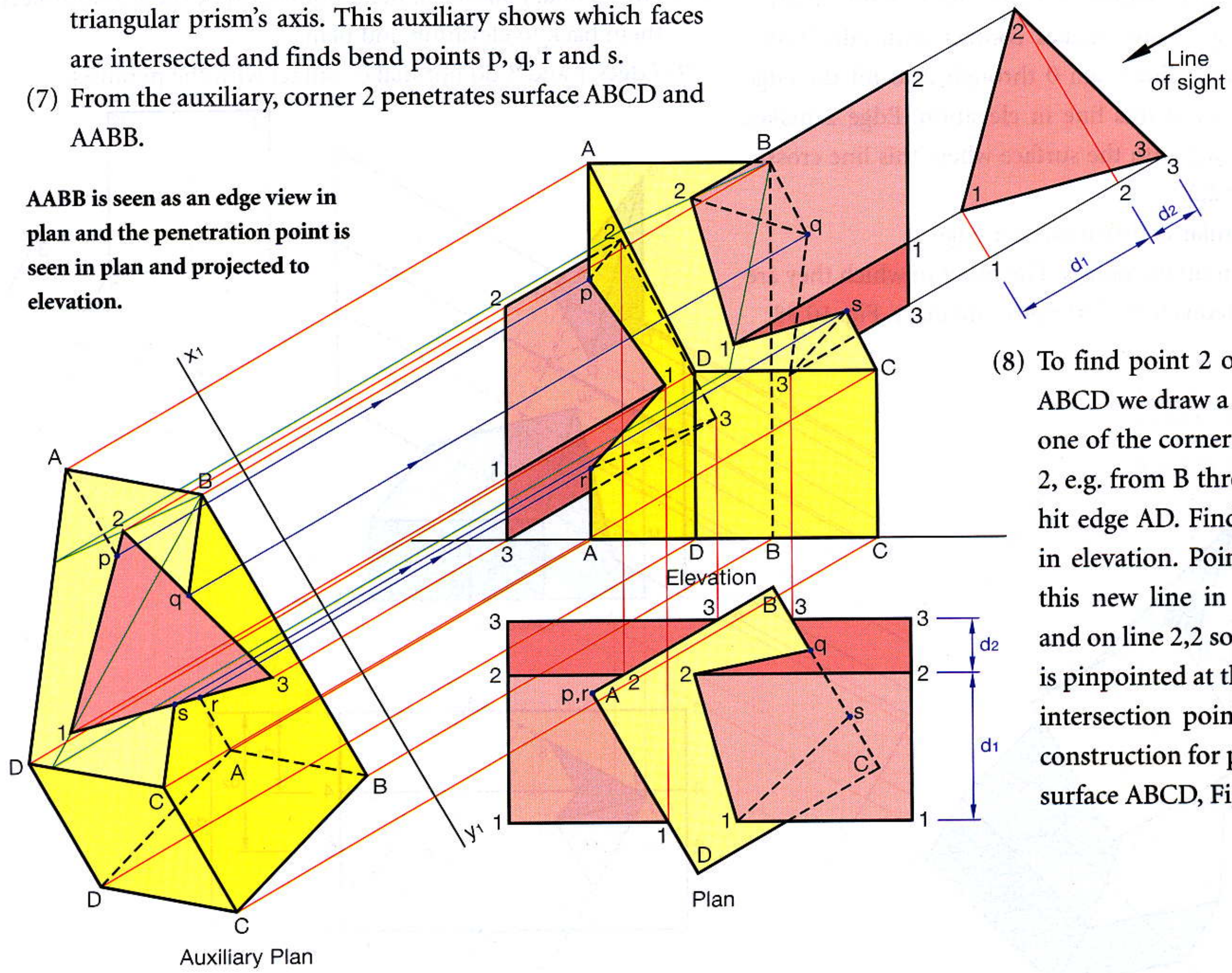


Fig. 10.36

HIGHER LEVEL

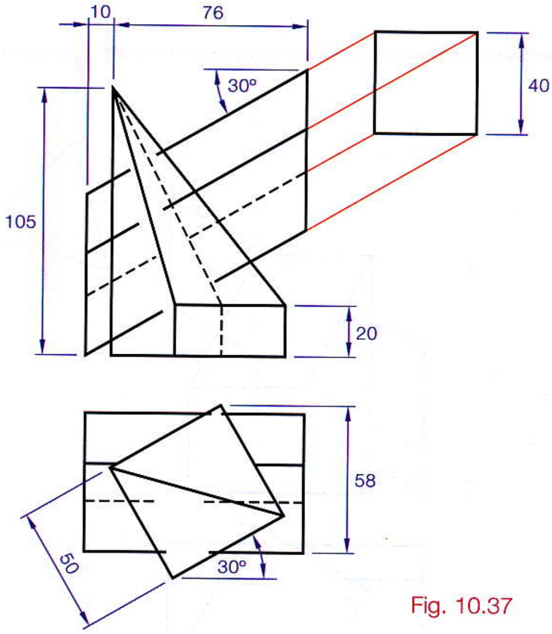


Fig. 10.37

Fig. 10.37 shows the incomplete projections of an oblique pyramid with a 50 mm square base. This solid is penetrated by a square-based prism of 40 mm side. Draw the projections of the solid

(1) Draw the oblique pyramid in plan and elevation.

(2) Draw the lowest line of the penetrating square prism as given. Extend this line and draw the square section as shown.

(3) Index the corners and project them back to the elevation.

(4) Find distances d_1, d_2 and d_3 from the sectional view by projecting the corners of the section perpendicular to the prism axis.

(5) One of the prism edges is given in plan. This must be edge 3, because edge 1 is appearing as a dotted line in elevation and is therefore at the back.

(6) Using distances d_1, d_2 and d_3, locate the other edges of the prism in plan.

(7) Draw the auxiliary plan.

(8) The auxiliary shows all bend points. Index these and project them back to elevation and plan.

(9) Edges 1 and 3 do not make contact with the pyramid.

(10) Edge 2 hits the vertical surface 0,0,AA. This surface is an edge view in plan and therefore shows the penetration point clearly.

(11) Edge 2 also makes contact with edge OBC. Draw a line from 0 through 2 to hit the edge BC. Find this line in elevation. Edge 2 makes contact with the surface where this line crosses line 2,2.

(12) Similar construction for edge 4.

(13) Join up the points. The order in which they are joined is found from the auxiliary, Fig. 10.38.

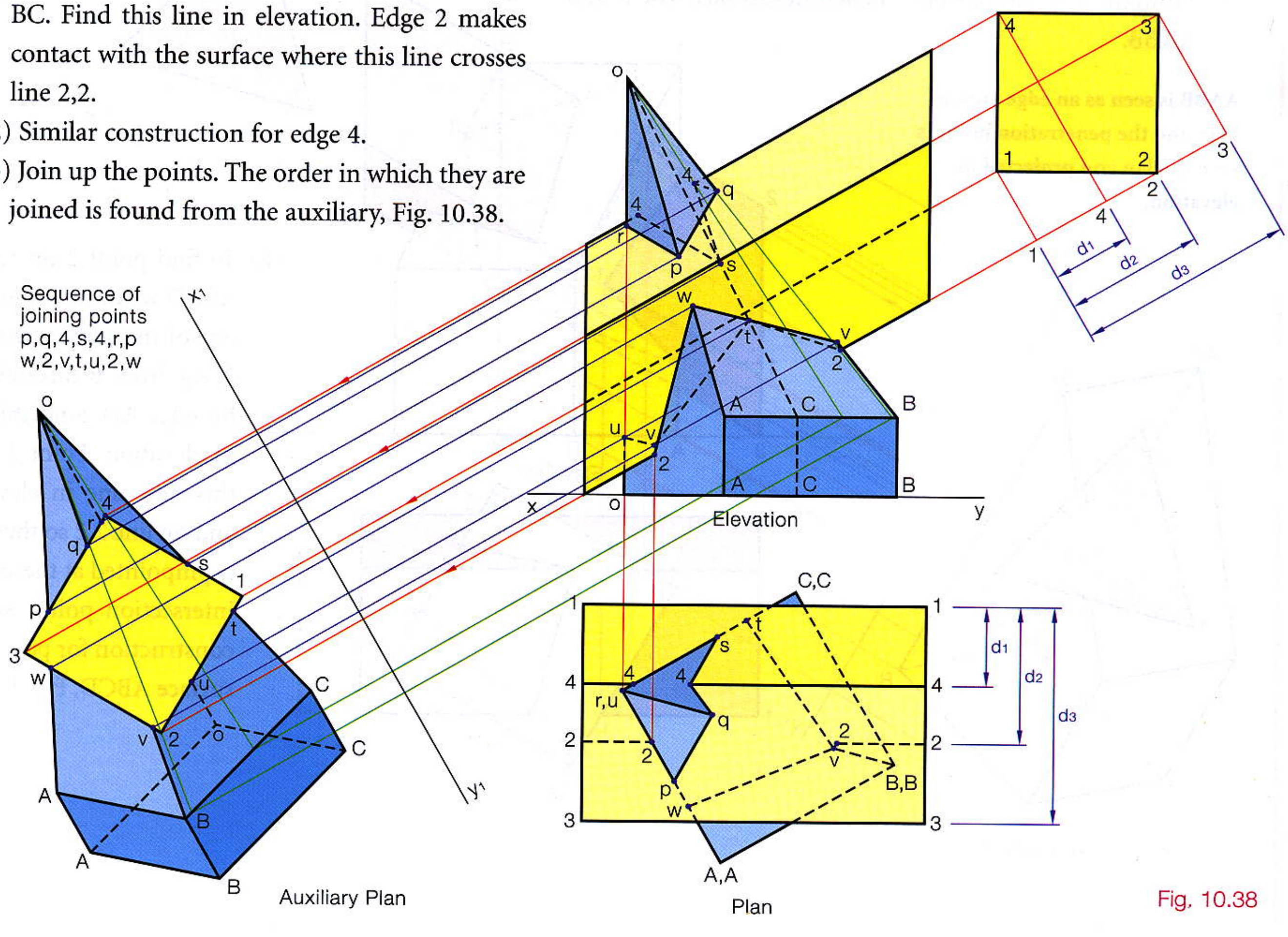


Fig. 10.38

To show a complete surface development of all the surfaces of the intersecting solids shown in Fig. 10.38, see Fig. 10.39.

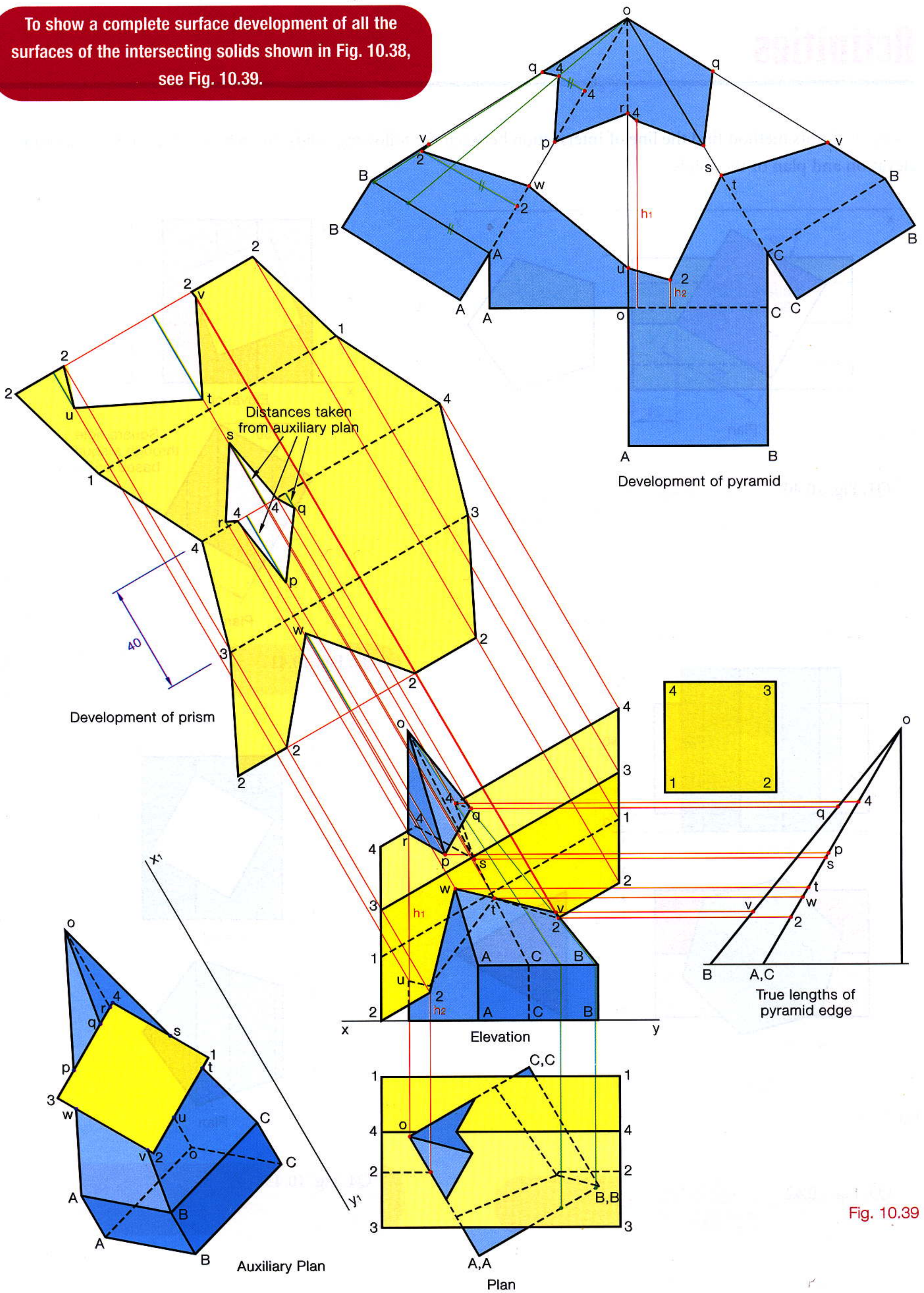


Fig. 10.39

Activities

Using the limits method find the line of intersection between the following solids. In each case draw a front elevation, end elevation and plan of the solids.

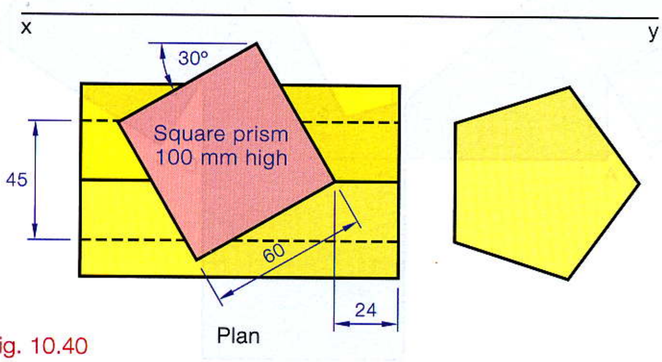


Fig. 10.40

Q1. Fig. 10.40

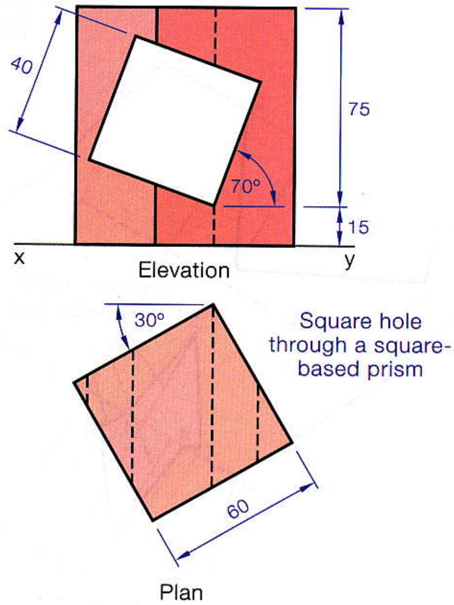


Fig. 10.41

Q2. Fig. 10.41

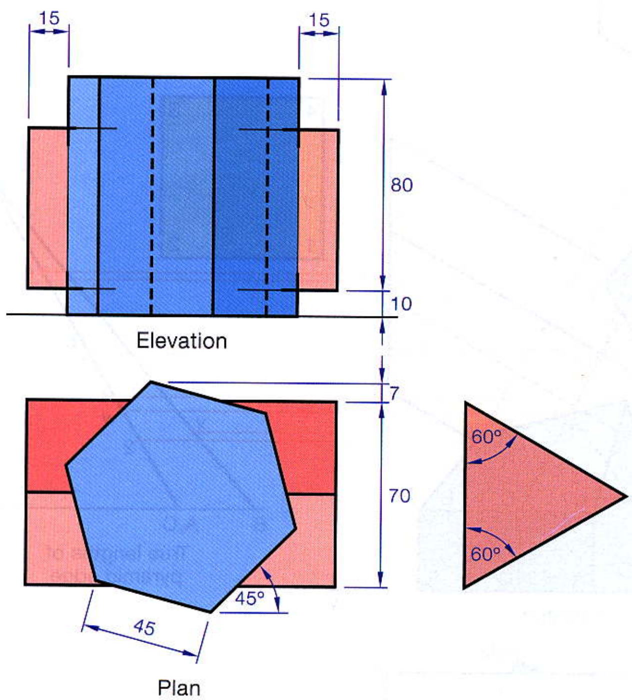


Fig. 10.42

Q3. Fig. 10.42

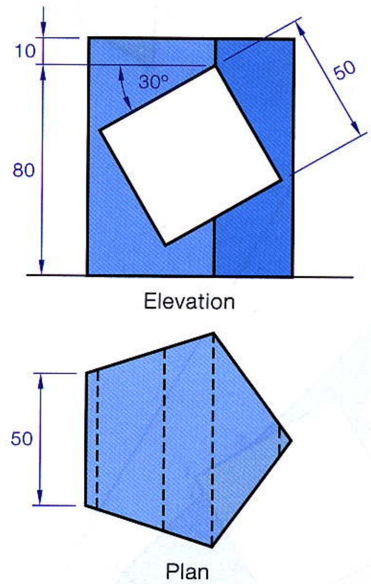


Fig. 10.43

Q4. Fig. 10.43

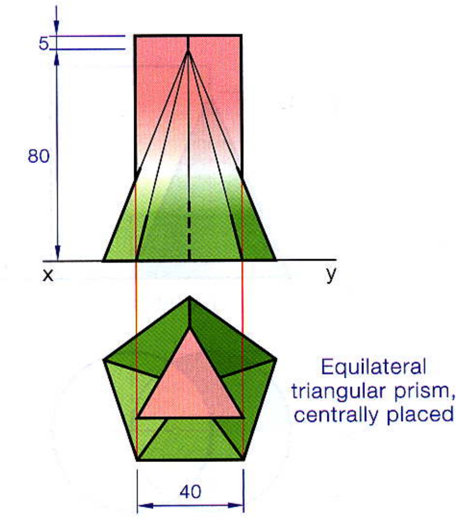


Fig. 10.44

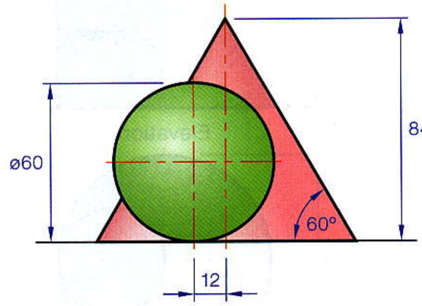


Fig. 10.45

Q5. Fig. 10.44

Q6. Fig. 10.45

Solve the following questions using horizontal sections. In each case draw a front elevation, end elevation and plan showing the full line of intersection.

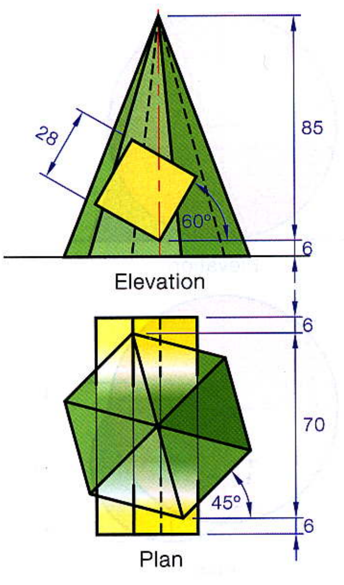


Fig. 10.46

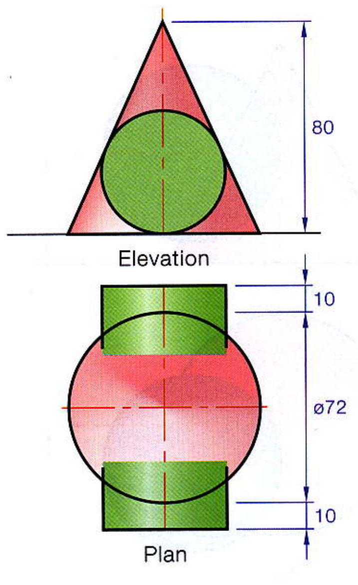


Fig. 10.47

Q7. Fig. 10.46

Q8. Fig. 10.47

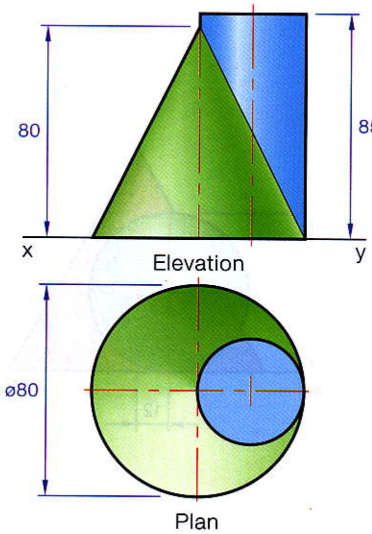


Fig. 10.48

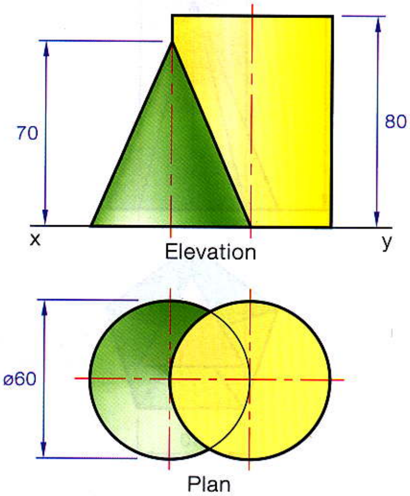


Fig. 10.49

Q9. Fig. 10.48

Q10. Fig. 10.49

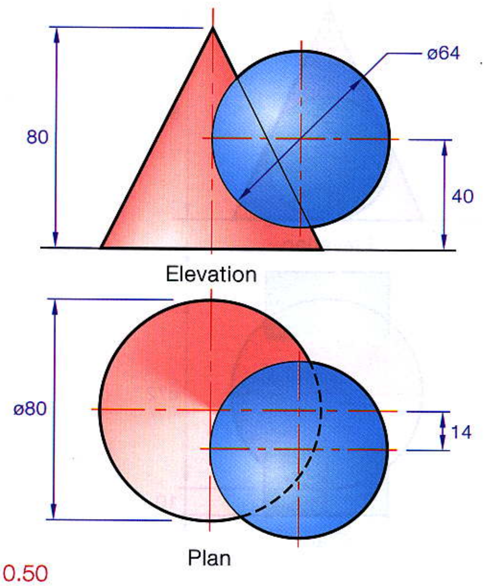


Fig. 10.50

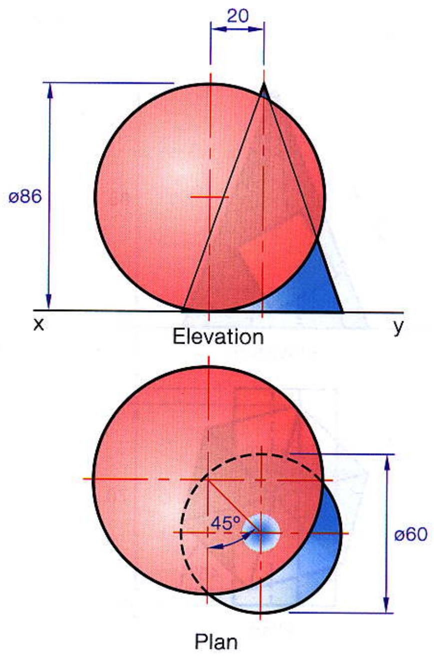


Fig. 10.51

Q11. Fig. 10.50

Q12. Fig. 10.51

Solve the following questions using vertical sections. In each case draw a front elevation, end elevation and plan showing all lines of intersection.

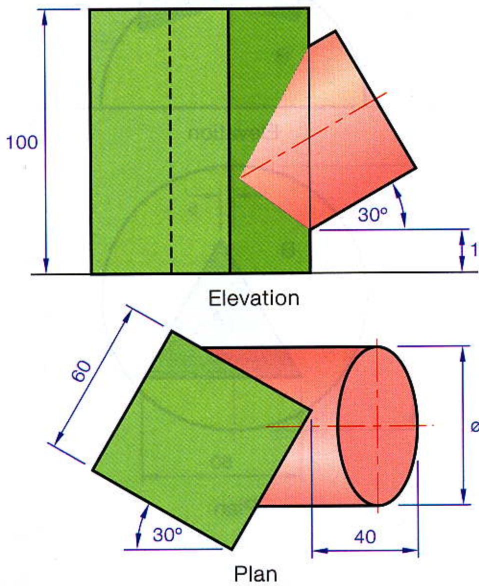


Fig. 10.52

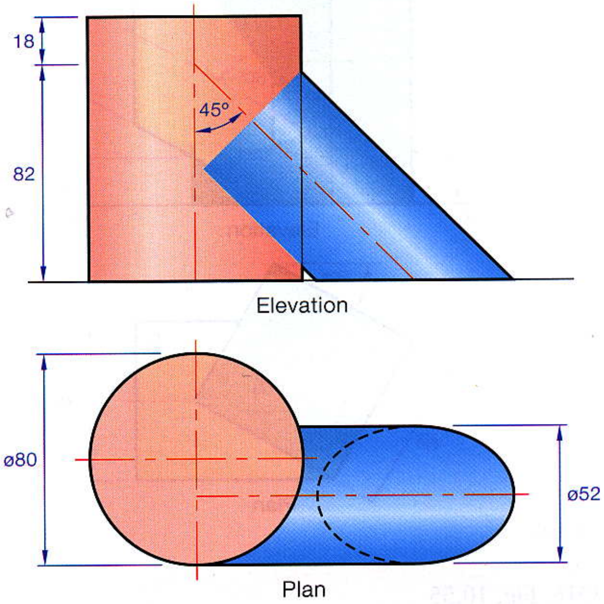


Fig. 10.53

Q13. Fig. 10.52

Q14. Fig. 10.53

For each of the following questions determine the line of intersection between the solids A and B. Develop the surfaces of the solid A.

Q15. Fig. 10.54

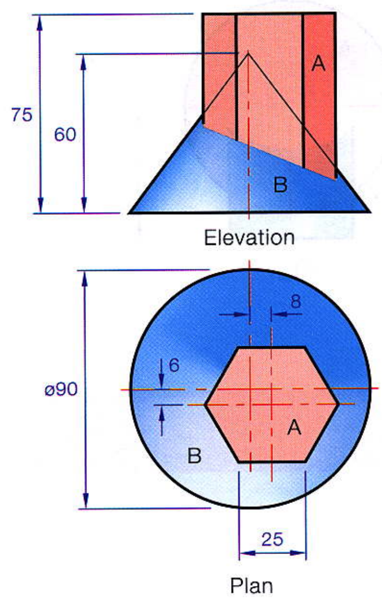


Fig. 10.54

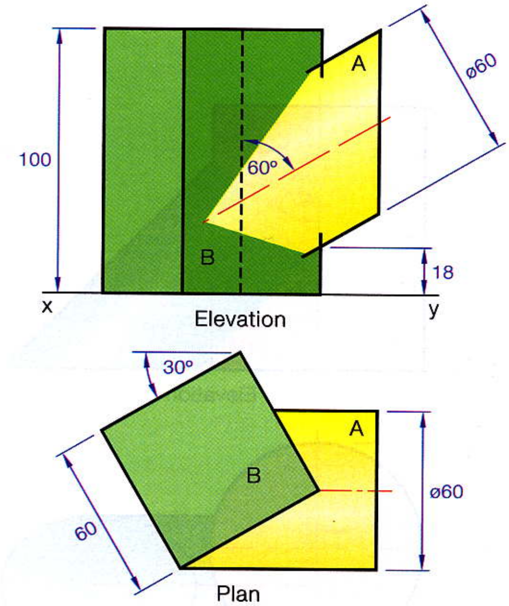


Fig. 10.56

Q16. Fig. 10.55

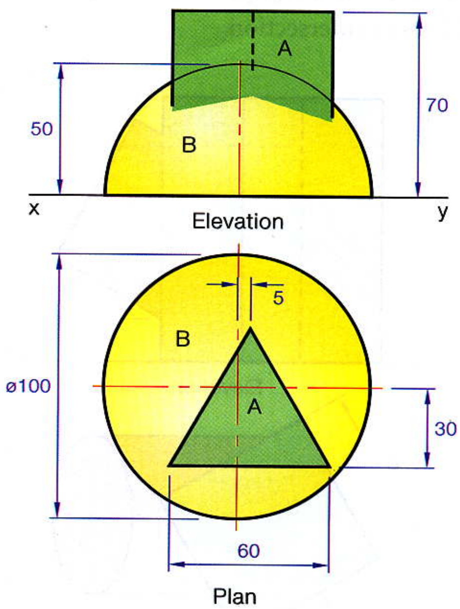


Fig. 10.55

Q17. Fig. 10.56

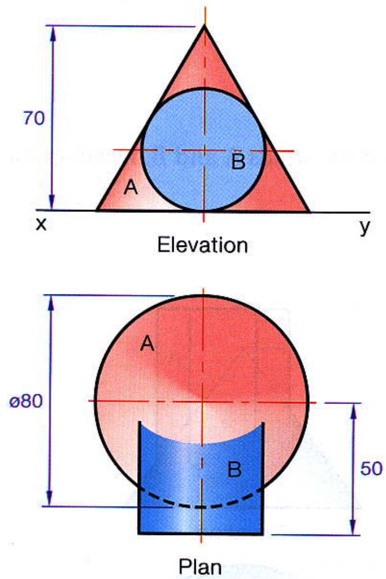


Fig. 10.57

Q18. Fig. 10.57

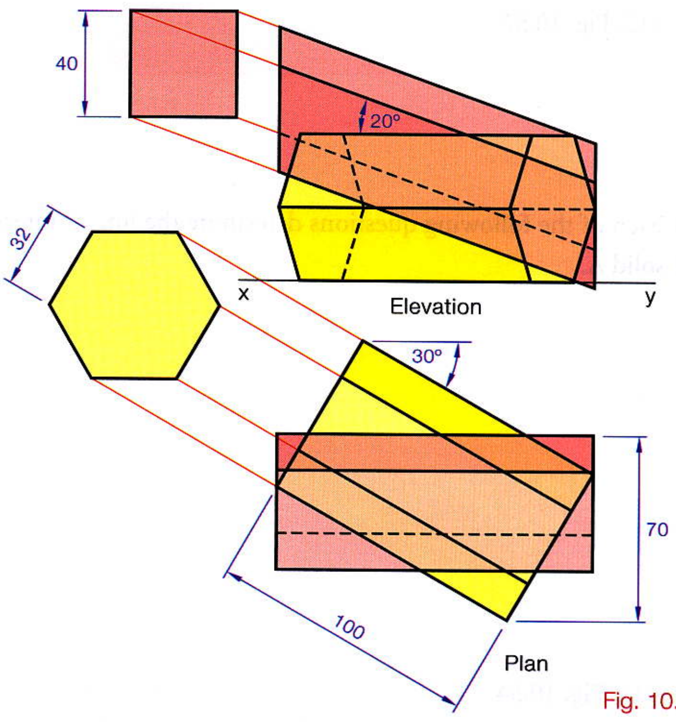


Fig. 10.58

Q19. Shown in Fig. 10.58 are the incomplete plan and elevation of a hexagonal-based prism being intersected by an inclined square-based prism. Draw the projections of the solids and find all lines of intersection.

Q20. Fig. 10.59 shows the incomplete plan and elevation of a truncated equilateral triangular prism of 100 mm side resting on the horizontal plane. This solid is penetrated by a 60 mm side equilateral triangular prism which is inclined at 30° to the HP. Draw the projections of the solids showing all interpenetration lines.

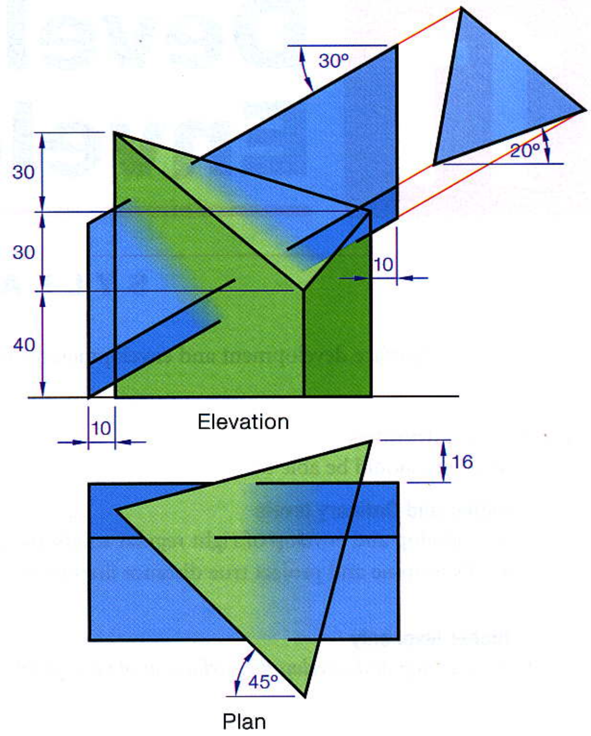
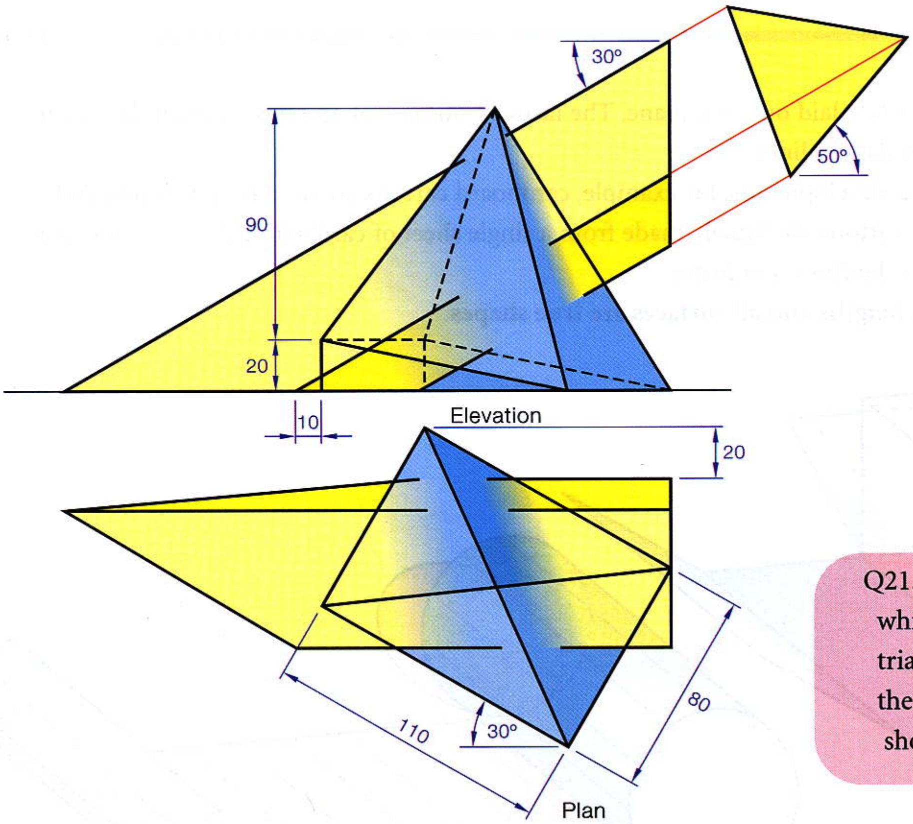


Fig. 10.59



Q21. Fig. 10.60 shows a shaped solid which is penetrated by an equilateral triangular prism of side 70 mm. Draw the given views and complete them to show all lines of interpenetration.

Fig. 10.60