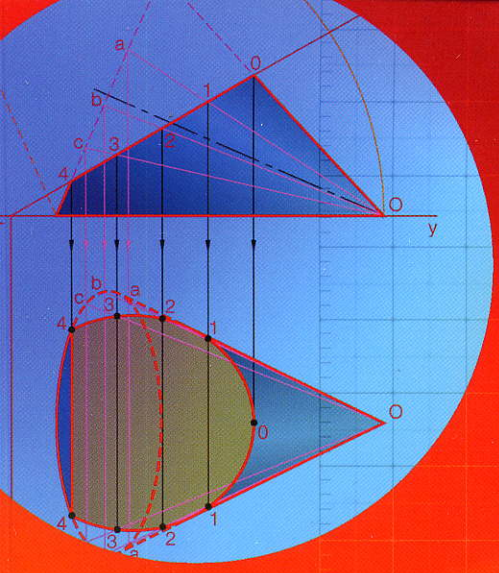
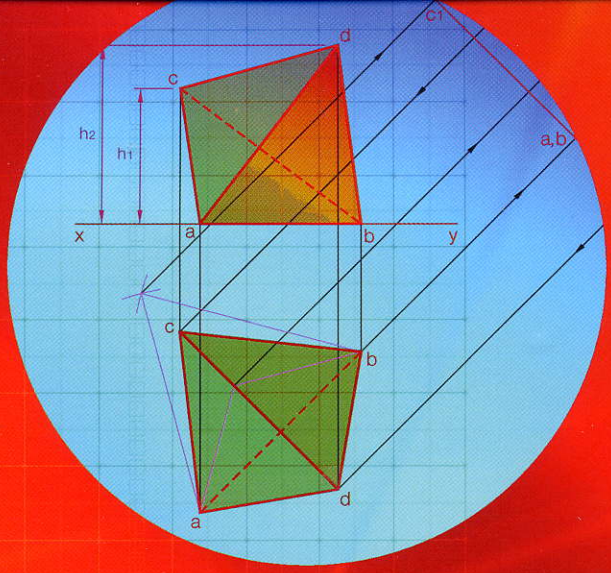
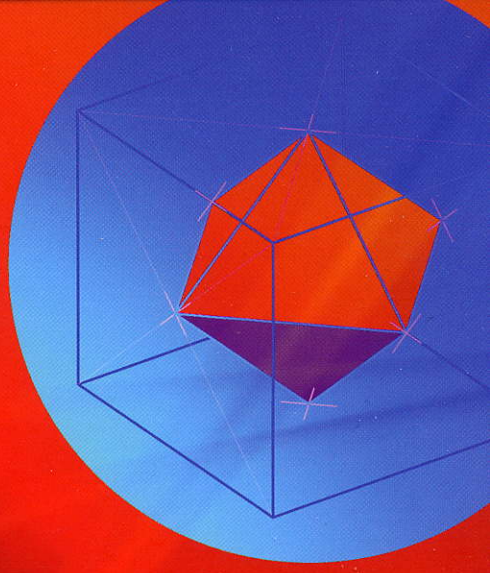
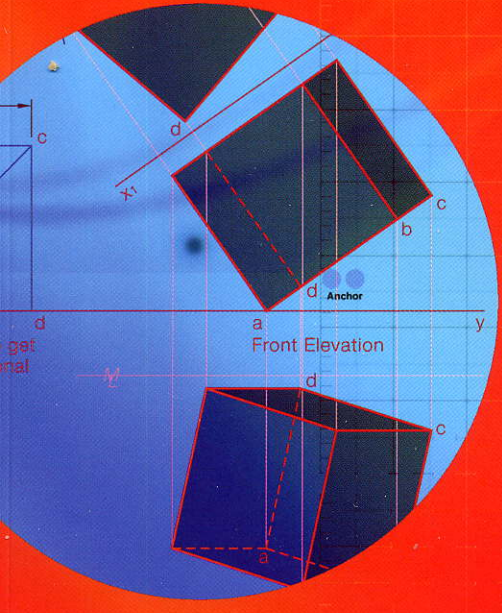


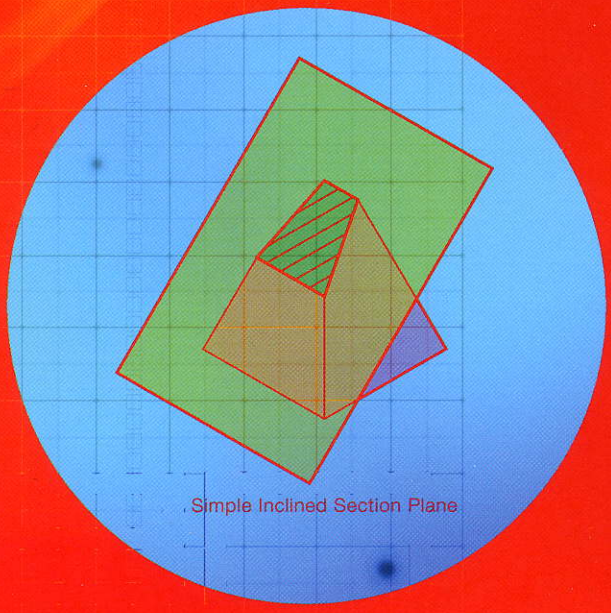


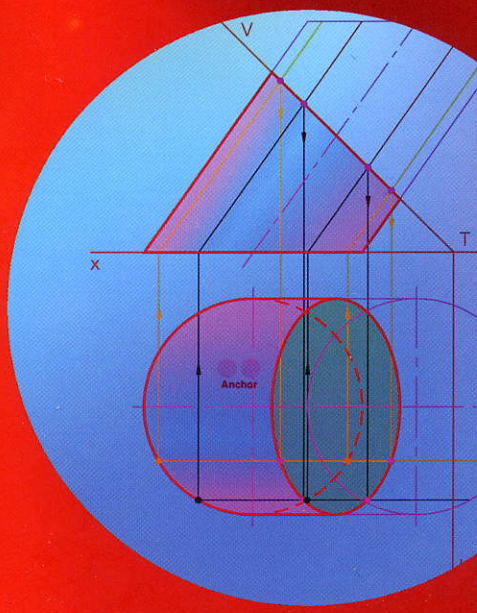


GRAPHICS IN DESIGN & COMMUNICATION 1

PLANE AND DESCRIPTIVE GEOMETRY







DAVID ANDERSON

11 Developments and Envelopments

SYLLABUS OUTLINE

Areas to be studied:

• Surface development and envelopment of right solids. • *Surface development and envelopment of oblique solids.*

Learning outcomes

Students should be able to:

Higher and Ordinary levels

- Develop and envelop of right regular solids, their composites and frustra.
- Determine and project true distance lines between specified points on the surfaces of solids.

Higher level only

- *Develop and envelop the surfaces of oblique prisms and pyramids.*

Developments

The development of a surface is that surface laid out on a plane. The faces or surfaces of an object are unfolded onto a single plane. Fold lines are indicated by dashed lines.

A large number of industries rely on developments, for example, cardboard cartons are used for packaging and marketing manufactured goods. These cartons are usually made from a single sheet of cardboard. Sheet metalworkers continuously use developments as does the clothes industry.

In a development all lines are true lengths and all surfaces are true shapes.

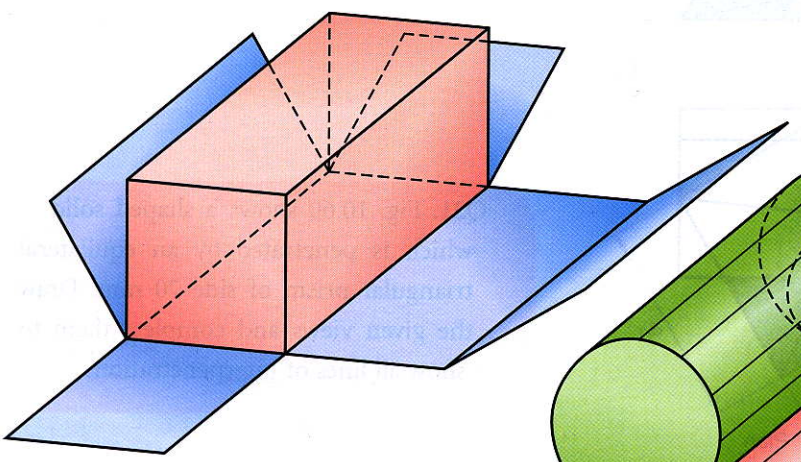


Fig. 11.1a

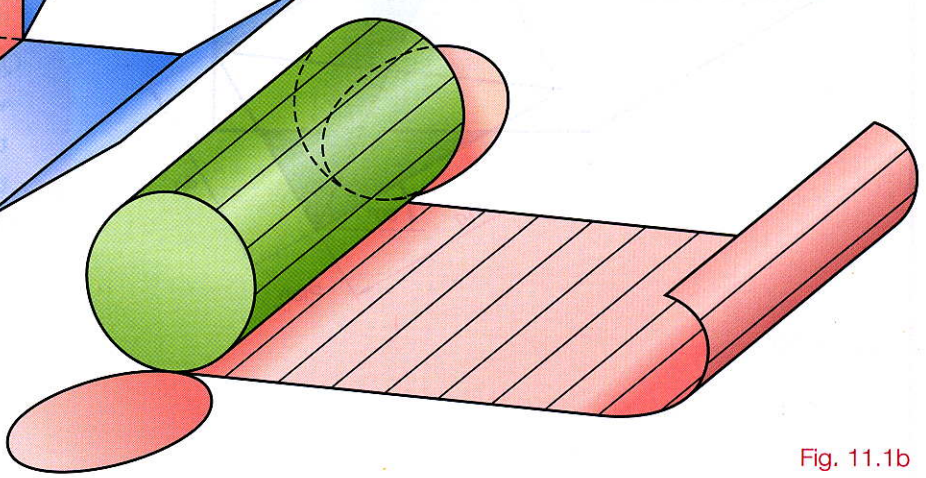


Fig. 11.1b

Envelopments

As explained earlier, the word 'development' describes the process of opening out the surfaces of an object. 'Envelopment' can be seen as the opposite, the closing over of a development to create the object.

Given the incomplete development of a solid. Draw the front elevation, end elevation and plan of the solid and complete the development.
Fig. 11.36a

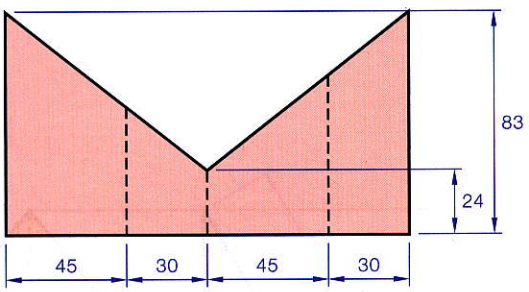


Fig. 11.36a

(1) The base must be rectangular as alternate sides are equal in length. The solid must also be a prism as the development is made up of parallel height lines.
(2) Decide which face of the development will form the front of the object. Draw the front elevation in line with the development.
(3) Project the plan which will equal the base in size.
(4) Draw the end elevation and complete the development, Fig. 11.36b.

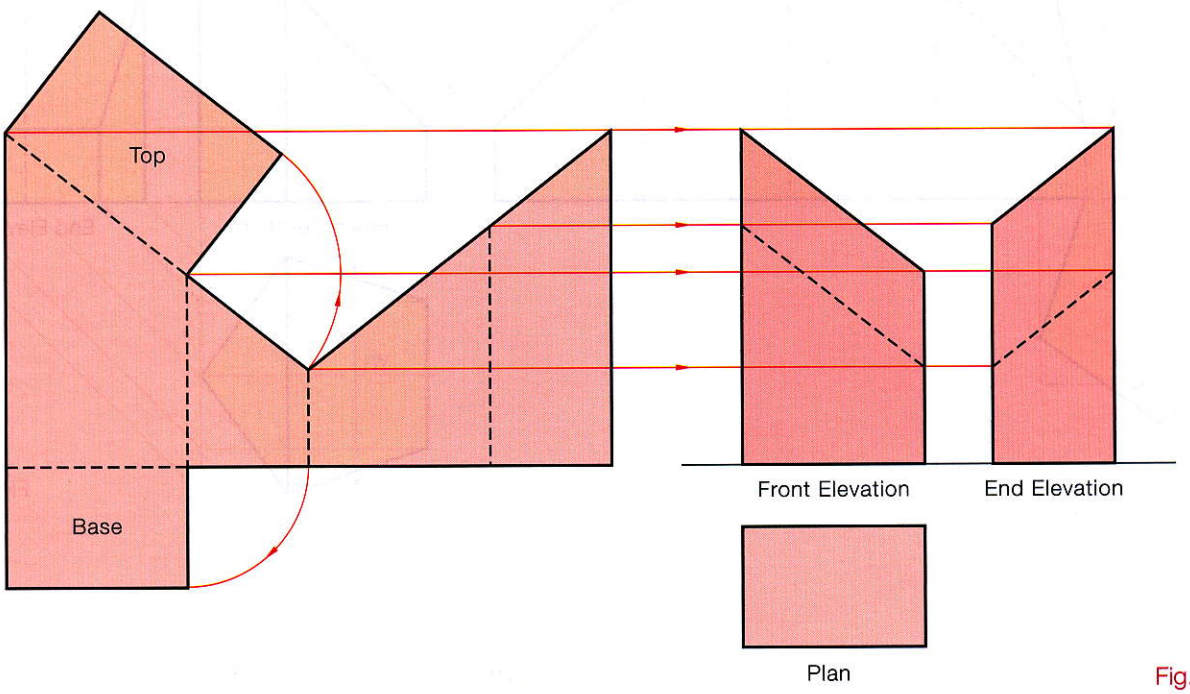


Fig. 11.36b

Given the partial development of a solid. Draw the front elevation, end elevation and plan of the solid. Complete the development.
Fig. 11.37a

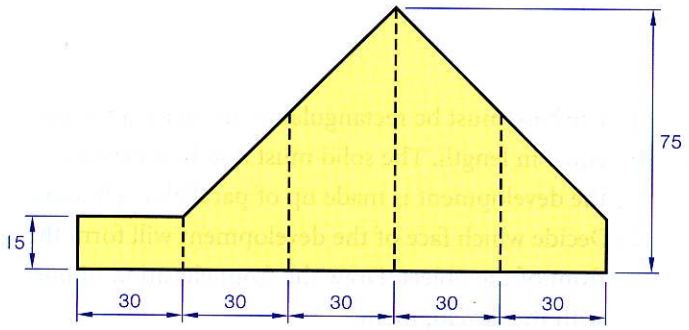


Fig. 11.37a

The solid must be a pentagonal prism because it has five equal sides and parallel height lines. The prism must also be truncated because of the variation in heights.

(1) Draw the development.
(2) Draw the base of the development and the plan.
(3) Project the elevation from the plan and from the development.
(4) Project the end view.
(5) Find the true shape of the cut surfaces by taking widths from the plan and lengths from the elevation, Fig. 11.37b.

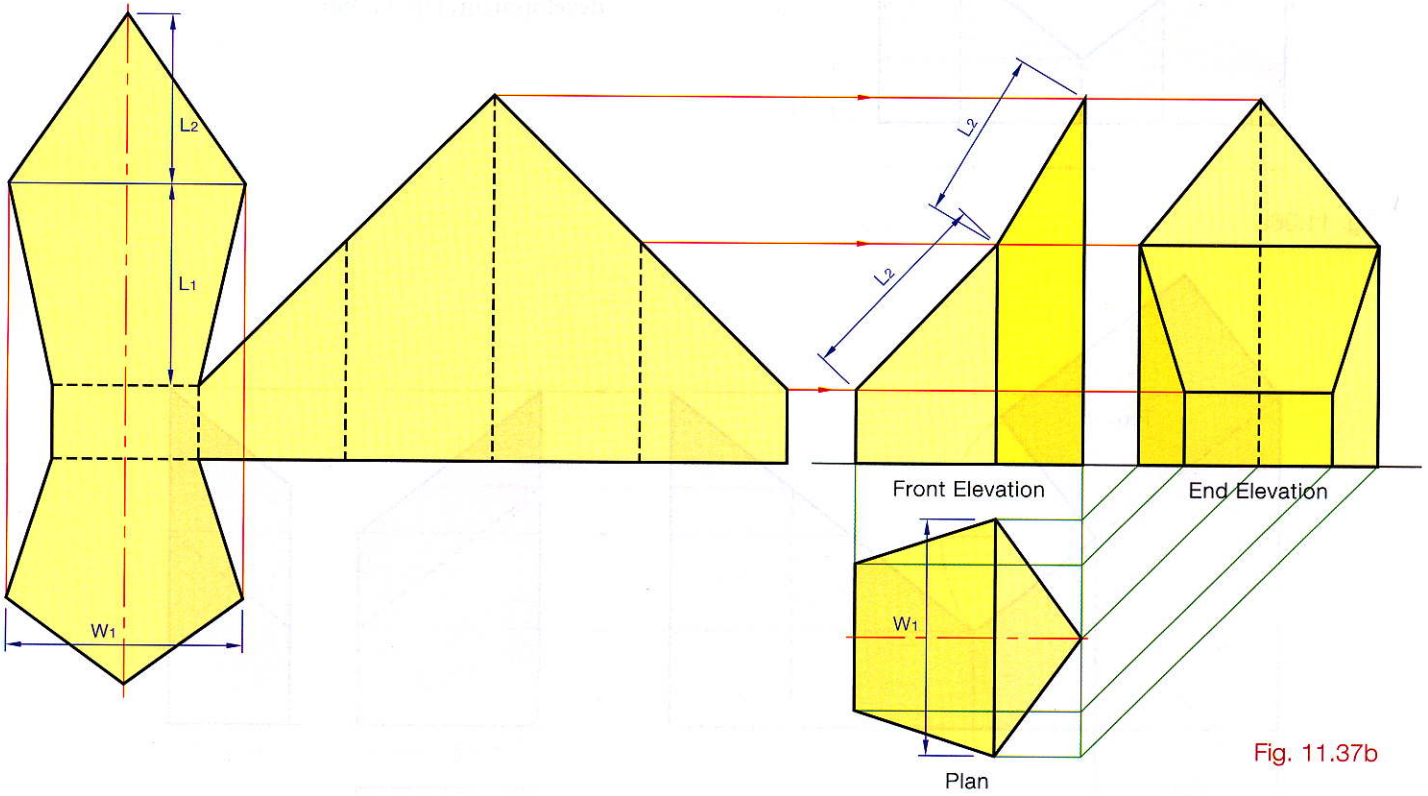


Fig. 11.37b

The development of a cylinder which is open at both ends is shown. Draw a front elevation and plan of the object. Fig. 11.38a

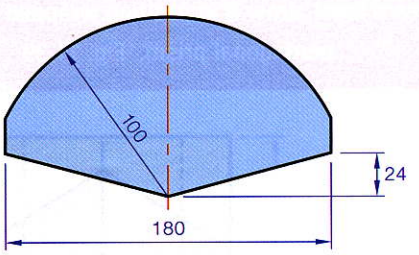


Fig. 11.38a

(1) The length of the development must equal the circumference of the cylinder

$$2\pi R = 180 \text{ mm} \Rightarrow R = 28.6 \text{ mm}$$

(2) Draw the plan of the cylinder as a circle of radius 28.6 mm.

(3) Divide this circle into twelve.

(4) Divide the development into twelve equal parts.

(5) Complete by projection, Fig. 11.38b.

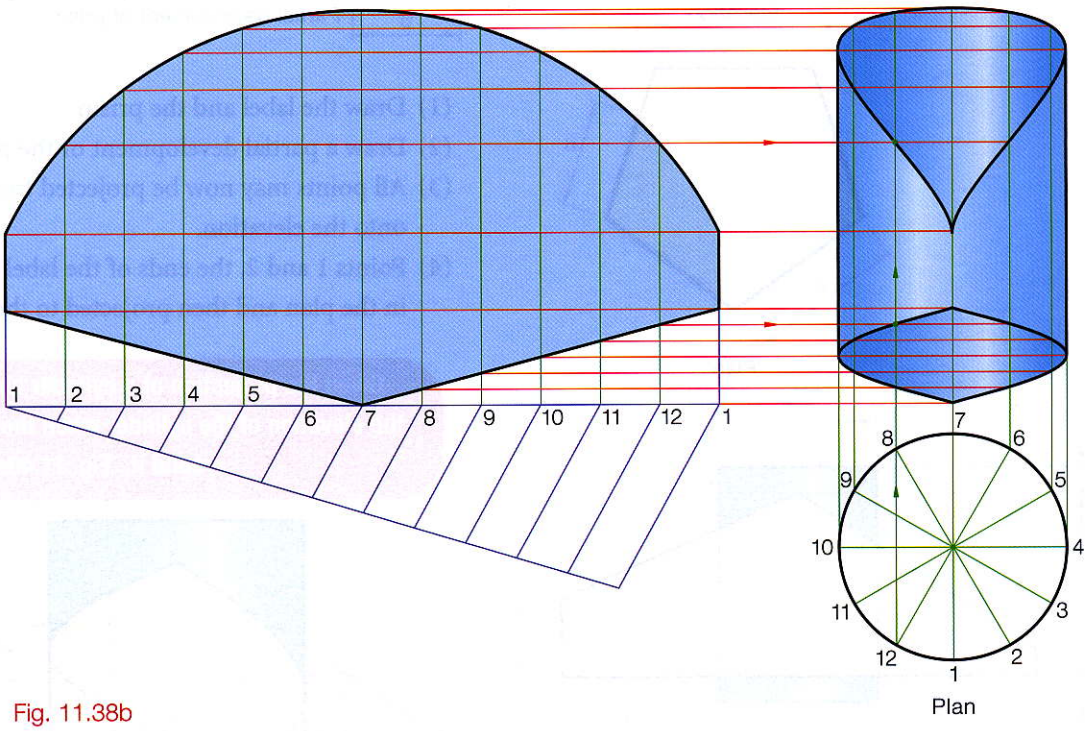
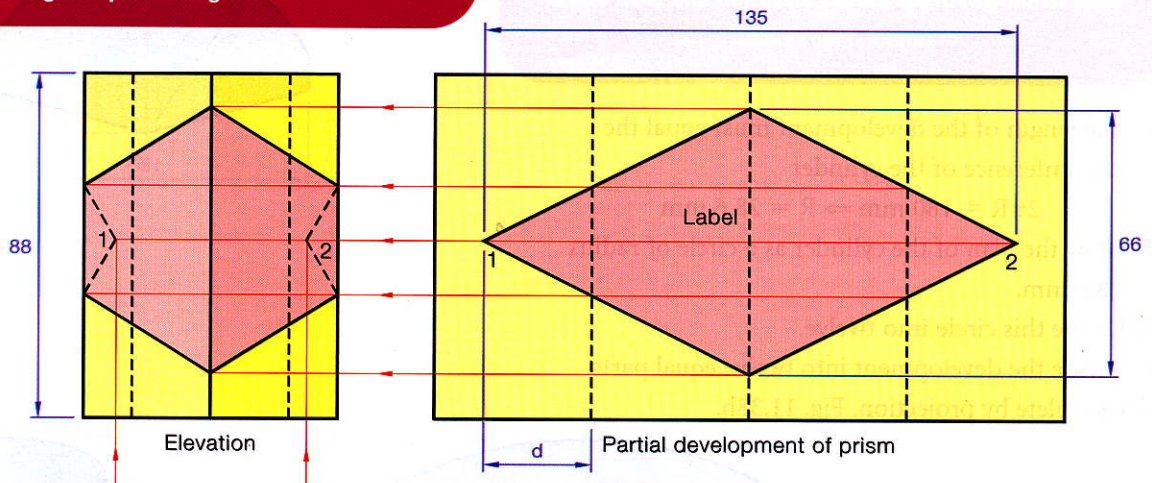


Fig. 11.38b

Partial Envelopments

A partial development is when a label or sticker is wrapped around a solid. The label can be much more complicated in appearance when developed out than it would suggest when wrapped around the jar or bottle.

Given a label, draw it in position wrapped around a pentagonal prism. Fig. 11.39



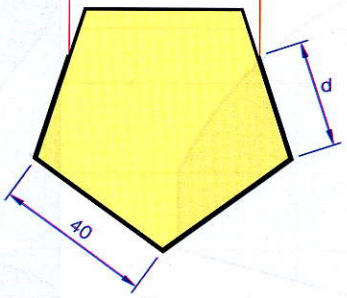


Fig. 11.39

(1) Draw the label and the prism.
(2) Draw a partial development of the prism over the label.
(3) All points may now be projected from the development onto the elevation.
(4) Points 1 and 2, the ends of the label must first be found in the plan and then projected to the elevation.

Given the projection of a cylinder and a label. Draw the elevation of the cylinder when the label is wrapped around it. Fig. 11.40a

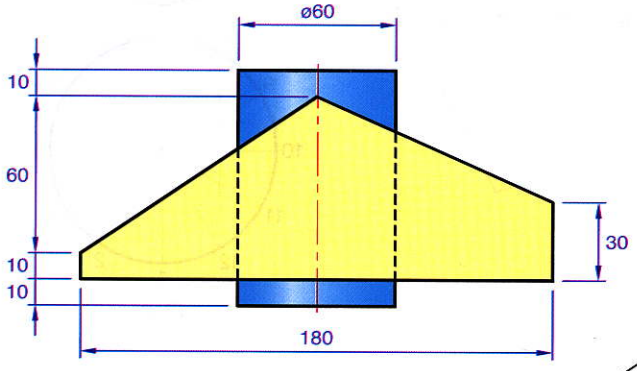


Fig. 11.40a

(1) Draw the given view and draw the plan.
(2) Draw the stretched-out label in plan.
(3) Divide the plan into twelve equal parts.
(4) From the centre line of the label step-off the divisions to the left and right and index.
(5) Draw the generators in elevation.
(6) Project across the heights from the label to the generators in elevation.
(7) The ends of the label must be found in plan first and then in elevation, Fig. 11.40b.

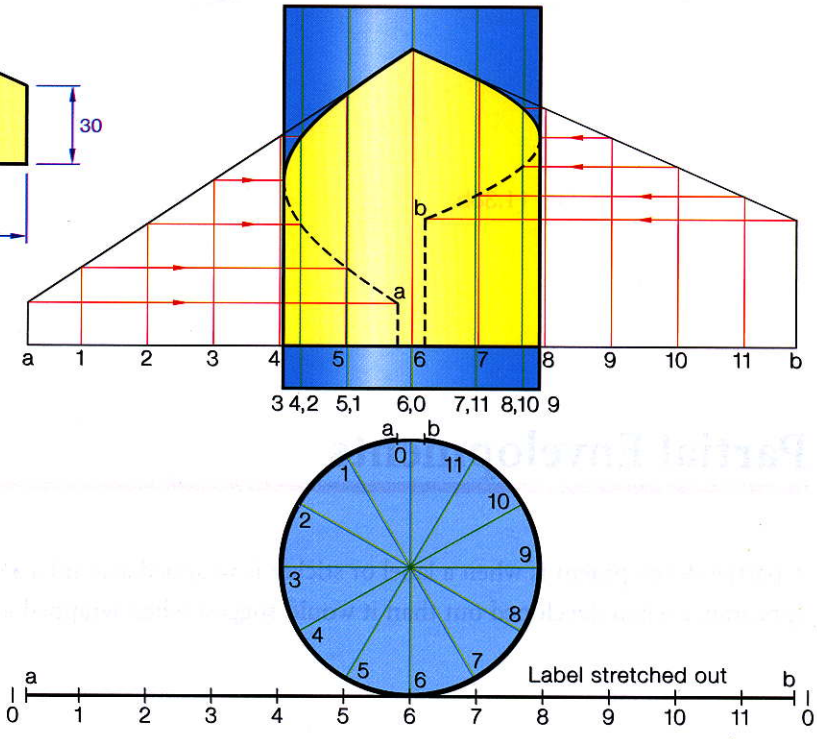


Fig. 11.40b

Given the elevation of a cylinder and the development of an elliptical label. Draw the elevation of the cylinder when the label is wrapped around it. Fig. 11.41a

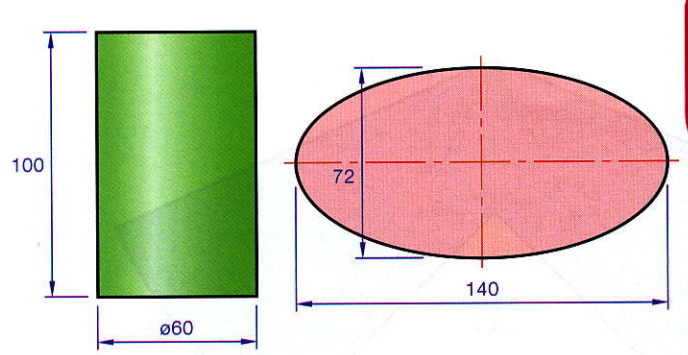


Fig. 11.41a

(1) The construction is the same as in the previous example. Drawing the developed label to the side produces a less complex drawing and is a neater presentation.

(2) The ends of the label, points a and b, will fall between 5 and 6 on the cylinder. They are found in plan first and then projected to elevation,

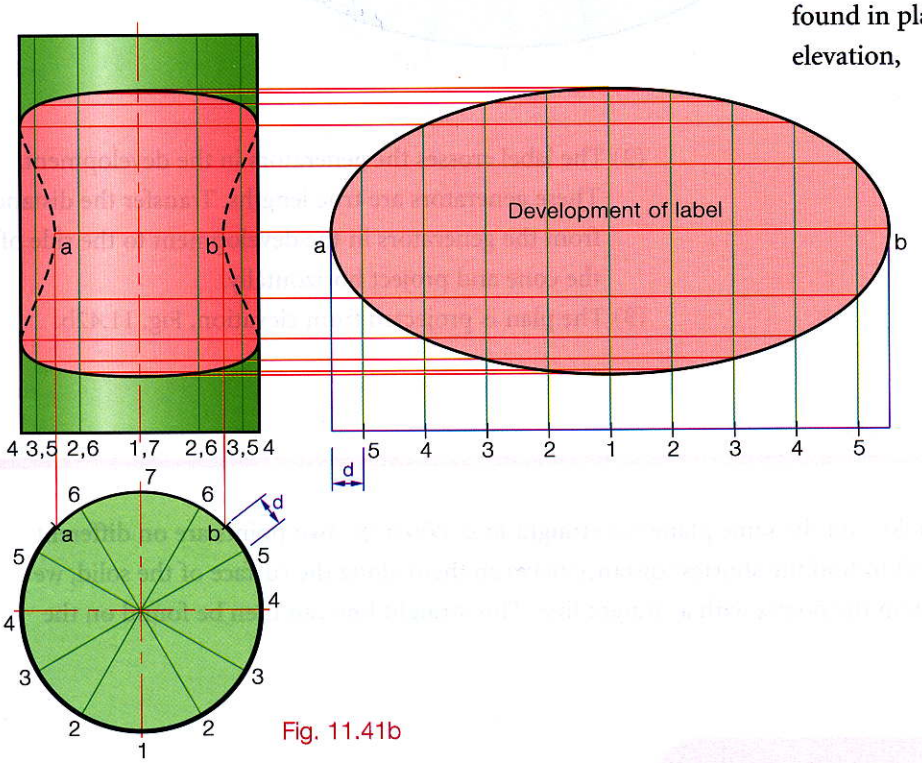


Fig. 11.41b

Given the projection of a cone and the development of a triangular label. Draw the elevation and plan of the cone when the label is wrapped around it. Fig. 11.42a

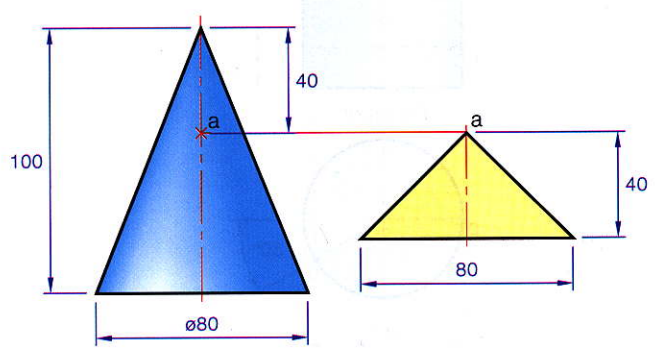
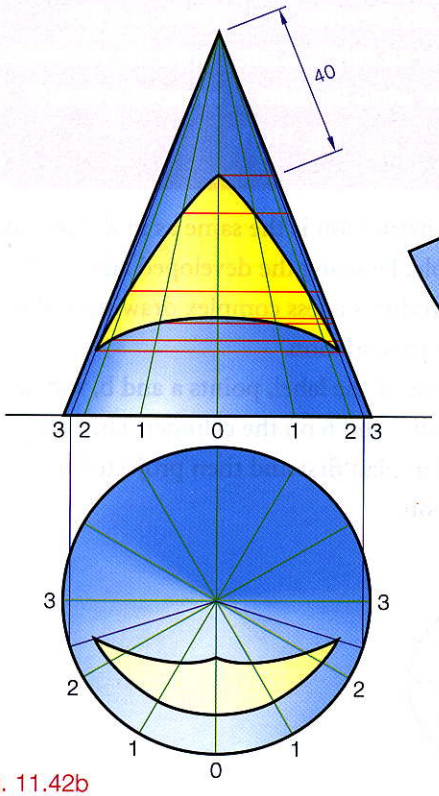


Fig. 11.42a

(1) Develop the surface of the cone and place the label in position.



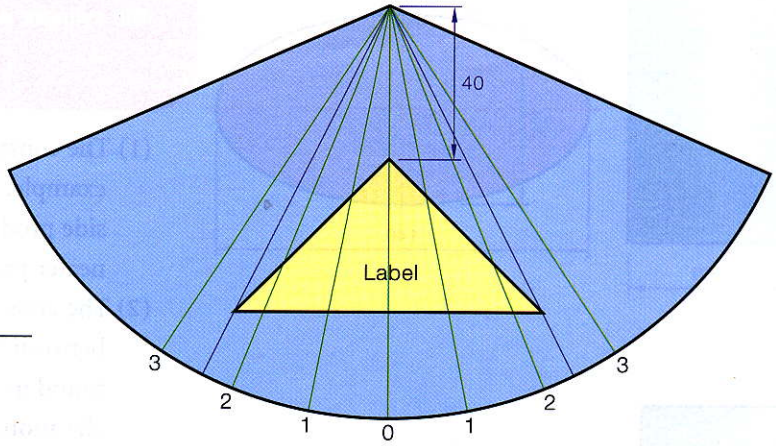


Fig. 11.42b

(2) The label crosses the generators in the development. These generators are true lengths. Transfer the distances from the generators in the development to the side of the cone and project horizontally.

(3) The plan is projected from elevation, Fig. 11.42b.

Shortest Distance

The shortest distance between two points on the same plane is a straight line. When the two points are on different surfaces of the same solid and we wish to find the shortest distance between them along the surface of the solid, we develop the surface of the solid and join the points with a straight line. This straight line can then be found on the projections of the solid.

Given the plan and elevation of a cylinder and two points p and q on its surface. Draw the projection of the shortest distance between these two points along the surface of the cylinder.
Fig. 11.43a

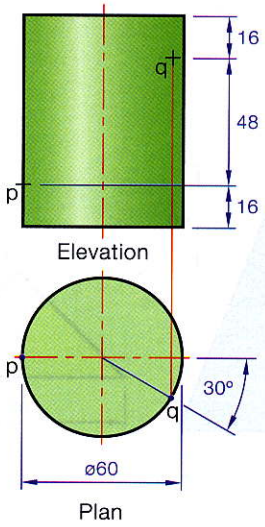


Fig. 11.43a

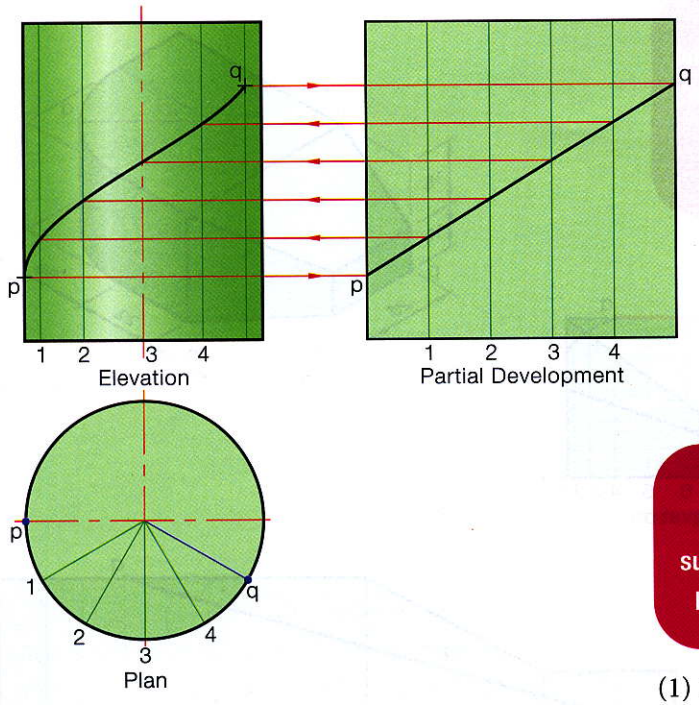


Fig. 11.43b

(1) Develop the surface of the cylinder between p and q.
(2) Project p and q onto the development and join with a straight line.
(3) Project this line back to elevation. The line joining p and q in elevation forms part of a helical curve, Fig. 11.43b.

The front elevation and plan of a solid are shown in Fig. 11.44a. Also shown are two points p and q on the surface of the solid. Draw the path of the shortest distance between these two points along the surface of the solid.

(1) Draw the given plan and elevation and locate points p and q on its surface.
(2) Develop the surface of the solid and locate p and q on the development.
(3) Join p and q with a straight line on the development.
(4) Distances Oa, Ob, Oc and Od are true lengths on the development and are stepped down on generator Op in elevation because it too is a true length.
(5) The elevation and plan are finished by projection, Fig. 11.44b.

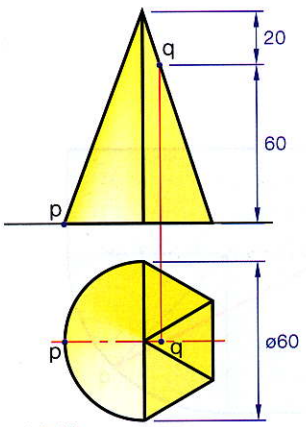
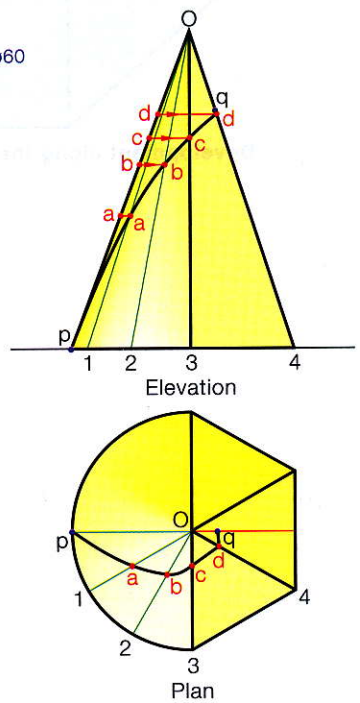


Fig. 11.44a



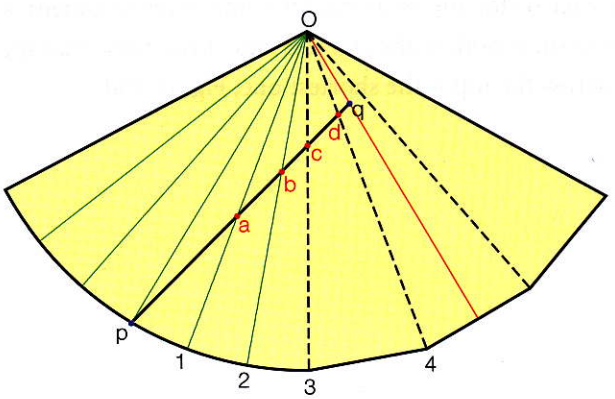


Fig. 11.44b

Given the isometric projection of a solid having two points, p and q, on its surface. Draw the front elevation, end elevation and plan of the solid showing the shortest path between points p and q.
Fig. 11.45a

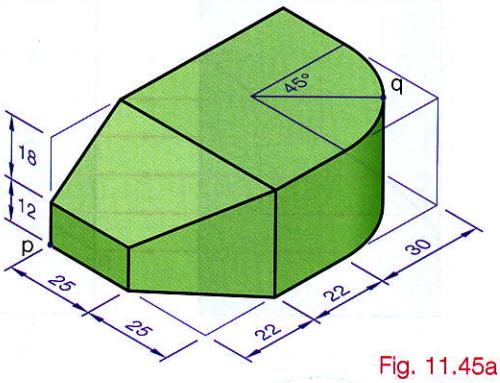
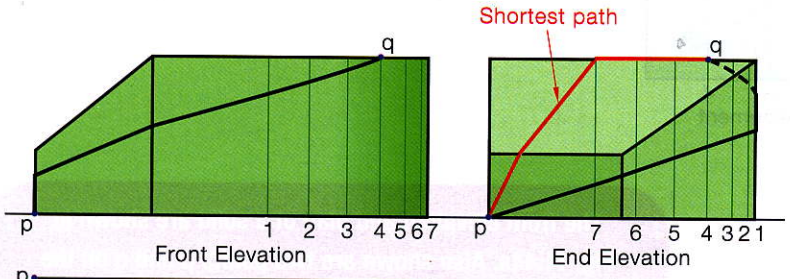


Fig. 11.45a



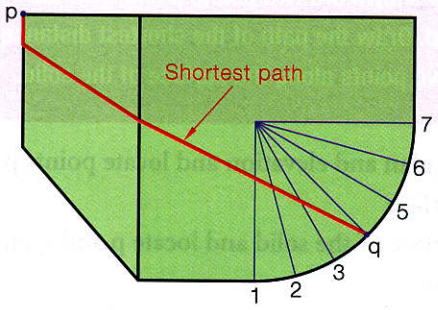
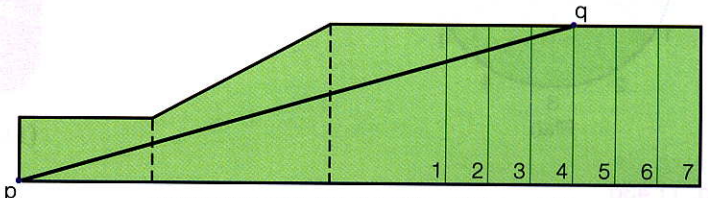


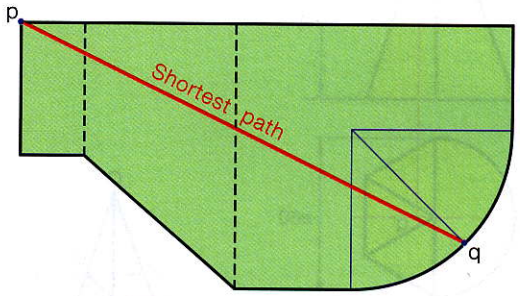
Fig. 11.45b



Development around side of the solid

Fig. 11.45c

In this example it is unclear which route would be the shorter, across the top or around the side. A development is drawn to show both paths and it is clear from these that the route across the top is the shortest one, Fig. 11.45d.



Development along the top of the solid

Fig. 11.45d

Activities

ENVELOPMENTS

Q1. TO Q3.

Figures 11.46 to 11.48 show the incomplete developments of prisms. Draw the front elevation, end elevation and plan of each solid and complete the development

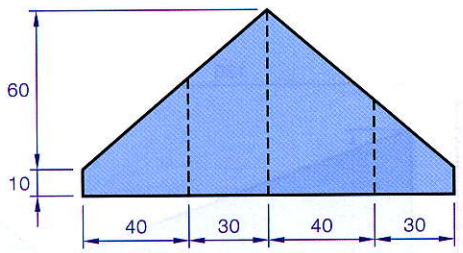


Fig. 11.46

Q1. Fig. 11.46

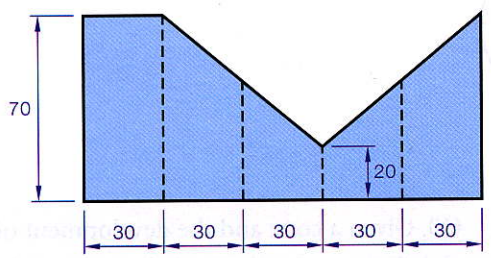


Fig. 11.48

Q3. Fig. 11.48

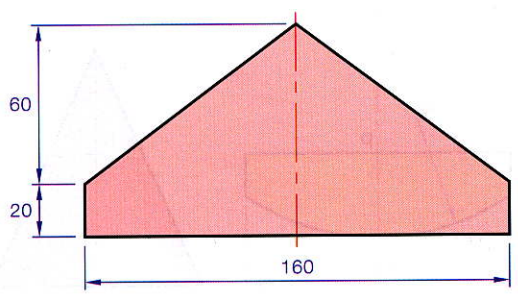


Fig. 11.50

Q5. Fig. 11.50

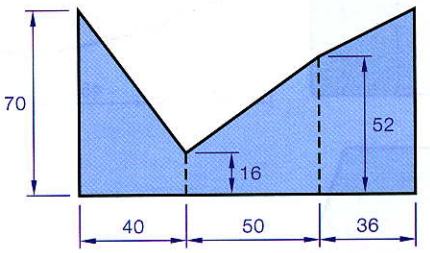


Fig. 11.47

Q2. Fig. 11.47

Q4. TO Q6.

Figures 11.49 to 11.51 show the development of cylinders which are open at both ends. Draw a plan and elevation of the cylinder.

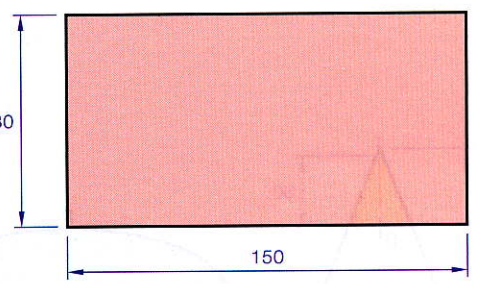


Fig. 11.49

Q4. Fig. 11.49

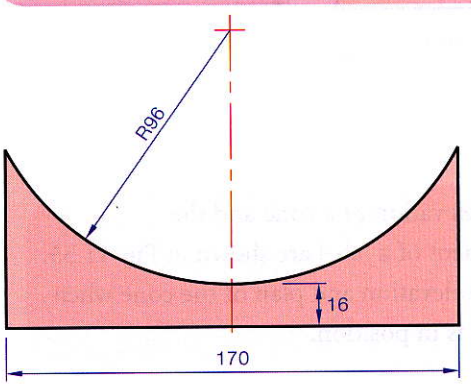


Fig. 11.51

Q6. Fig. 11.51

PARTIAL ENVELOPMENTS

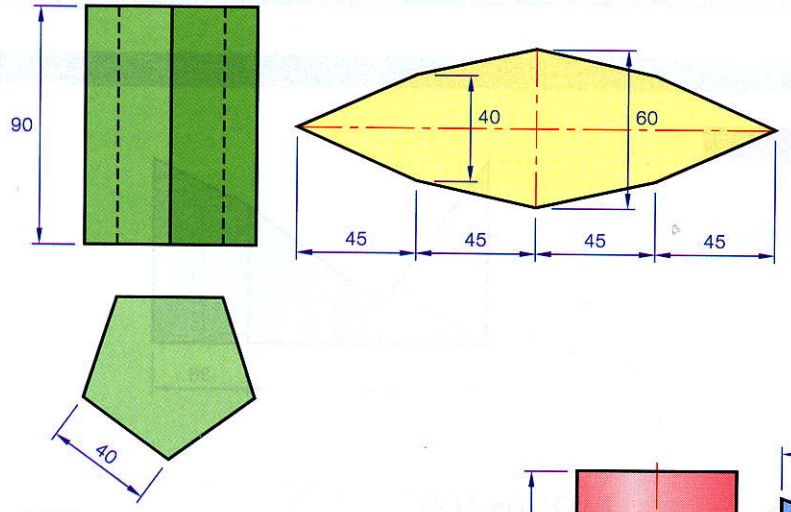
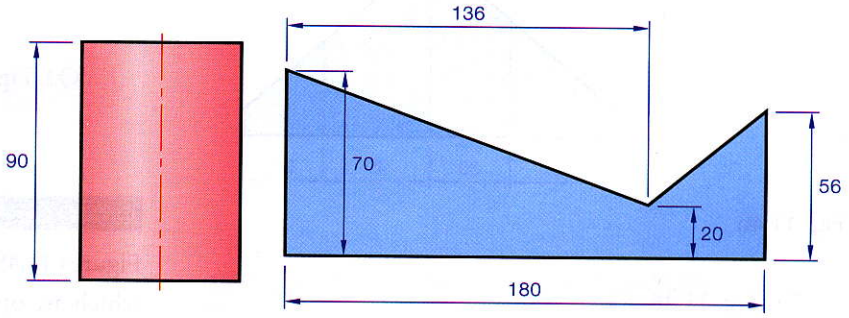


Fig. 11.52

Q7. Given the plan and elevation of a pentagonal prism and the development of a label. Draw the plan and elevation of the prism when the label is wrapped around it, Fig. 11.52.



Q8. Given the plan and elevation of a cylinder and the development of a label. Draw the elevation of the cylinder when the label is wrapped around it, Fig. 11.53.

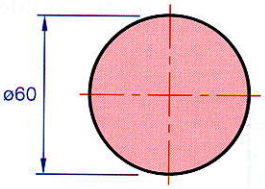


Fig. 11.53

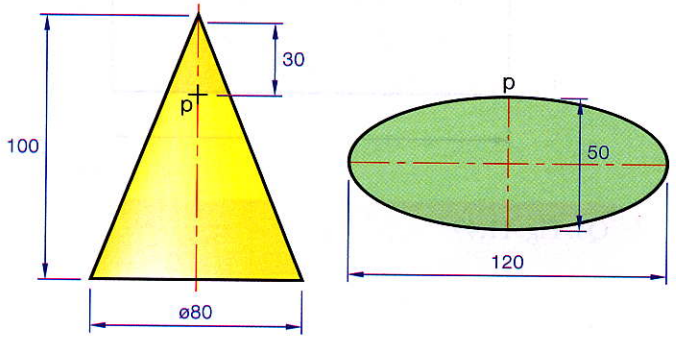


Fig. 11.54

Q9. Given a cone and the development of a label. Draw the plan and elevation of the cone when the label is wrapped around it, Fig. 11.54.

Q10. The elevation of a cone and the development of a label are shown in Fig. 11.55. Draw the elevation and plan of the cone when the label is in position.

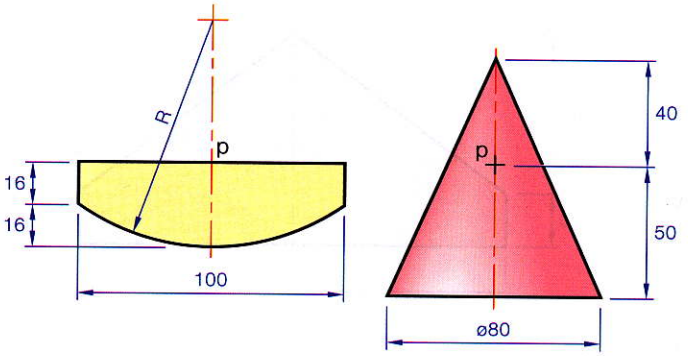


Fig. 11.55

Q11. AND Q12.

In each case, given the pictorial view of a solid having two points on its surface, p and q. Draw a front elevation, end elevation and plan of the solid showing the shortest route, on the surface of the solid, between p and q.

Q11. Fig. 11.56

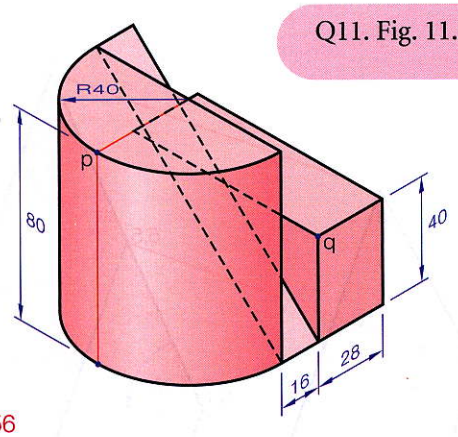


Fig. 11.56

Q12. Fig. 11.57

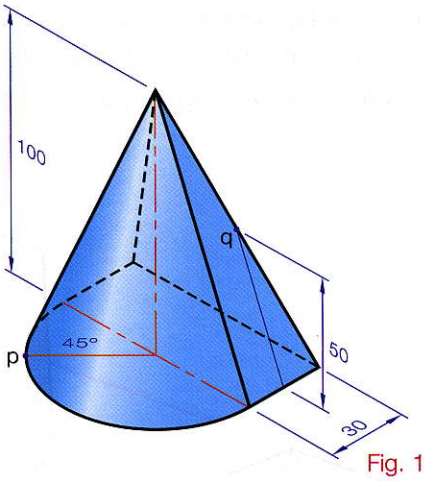


Fig. 11.57

Development of Oblique Solids

To draw the development of an oblique pentagonal prism. Fig. 11.58

(1) Draw the plan and elevation.
(2) The edge lines are true length lines in elevation. Project the ends of these lines perpendicularly.
(3) Choose a starting point for edge 1.

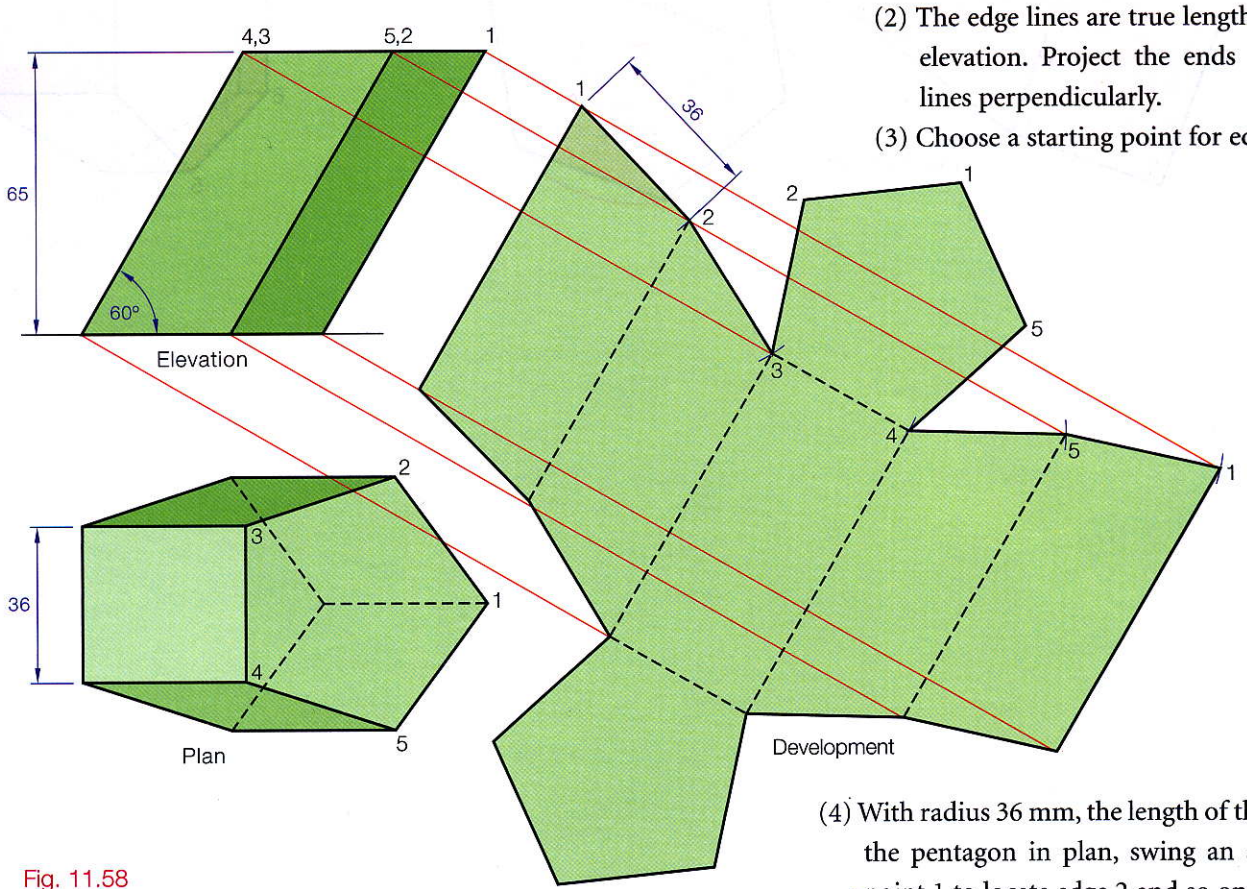


Fig. 11.58

(4) With radius 36 mm, the length of the side of the pentagon in plan, swing an arc from point 1 to locate edge 2 and so on.

HIGHER LEVEL

To draw the development of the surfaces of a truncated oblique hexagonal prism. Fig. 11.59

The development method is the same as above. The surface on the HP gives true lengths which are used as before to find the edges 1,1 and 2,2 etc.

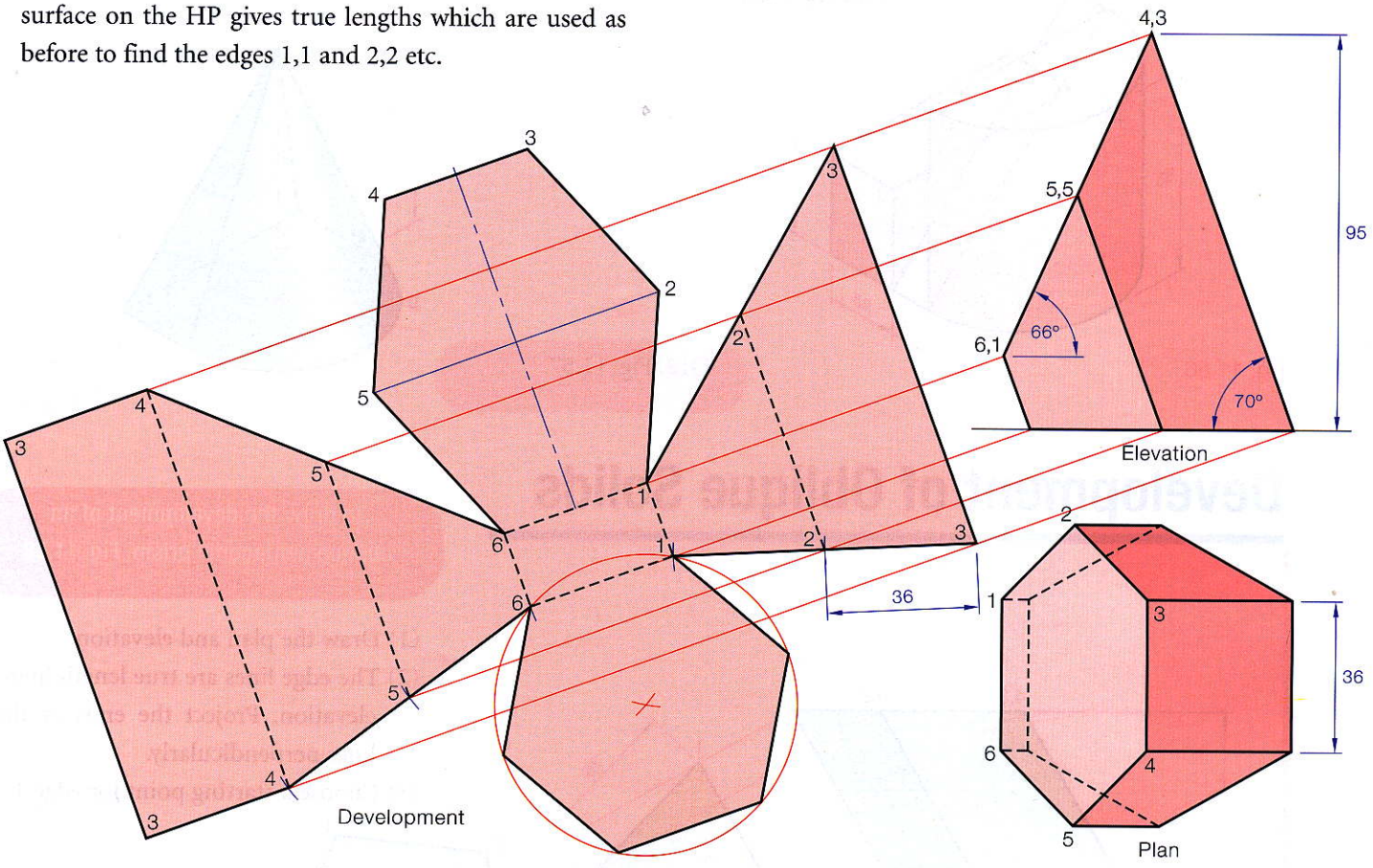


Fig. 11.59

To draw the surface development of an oblique cylinder. Fig. 11.60

(1) Draw the plan and elevation.
(2) Divide the base circle into 12 equal parts.
(3) Project these points to elevation and draw in the generators.
(4) Project out the ends of each generator at right angles.
(5) Choose a starting point 1.
(6) Take distance d from plan and swing an arc from point 1 to locate point 2.
(7) Continue in this manner to locate the other points.
(8) Construct lines parallel to the cylinder edge from each point and find the top edge of the development.

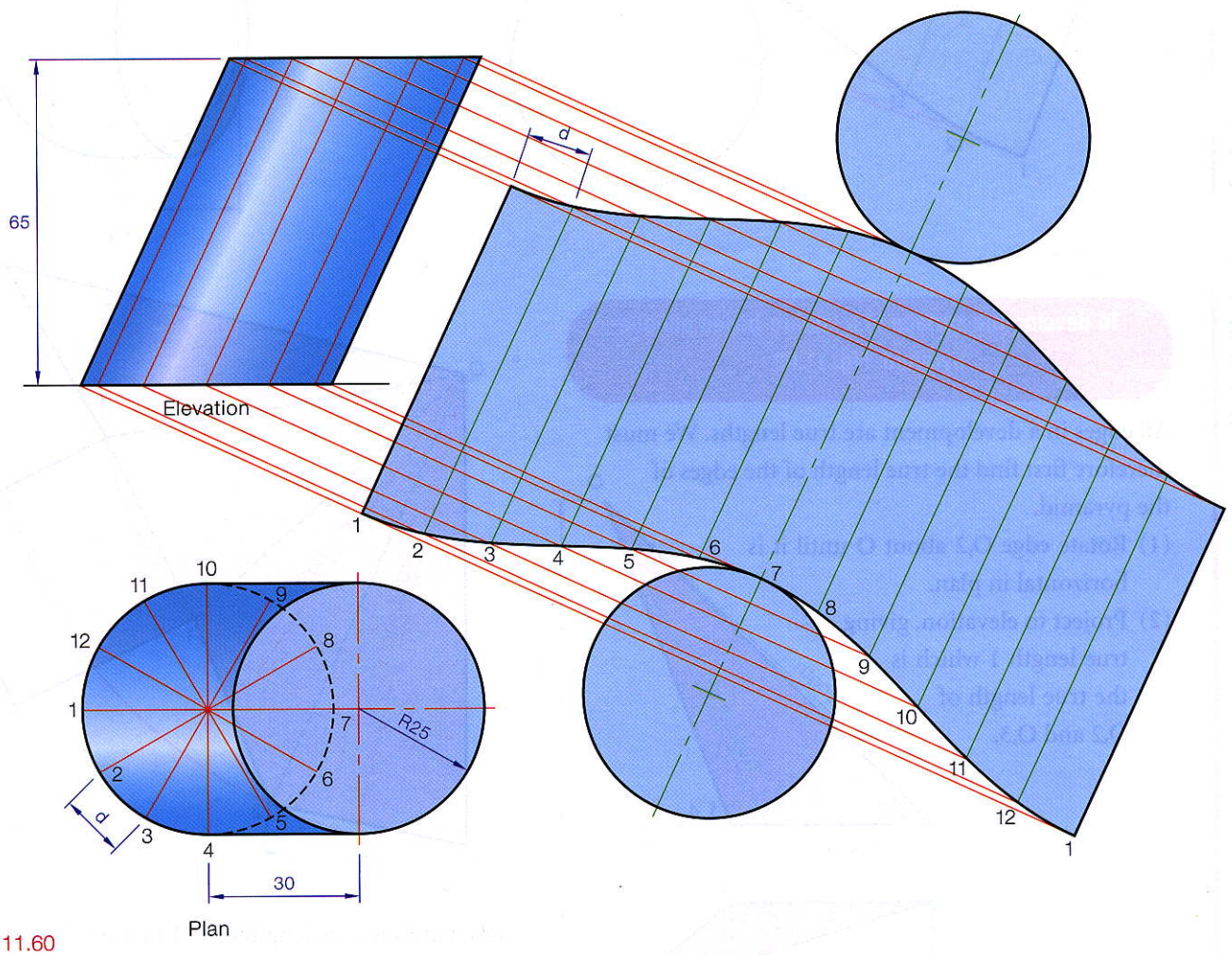


Fig. 11.60

To draw the complete surface development of an oblique truncated cylinder. Fig. 11.61

The construction is as Fig. 11.60.

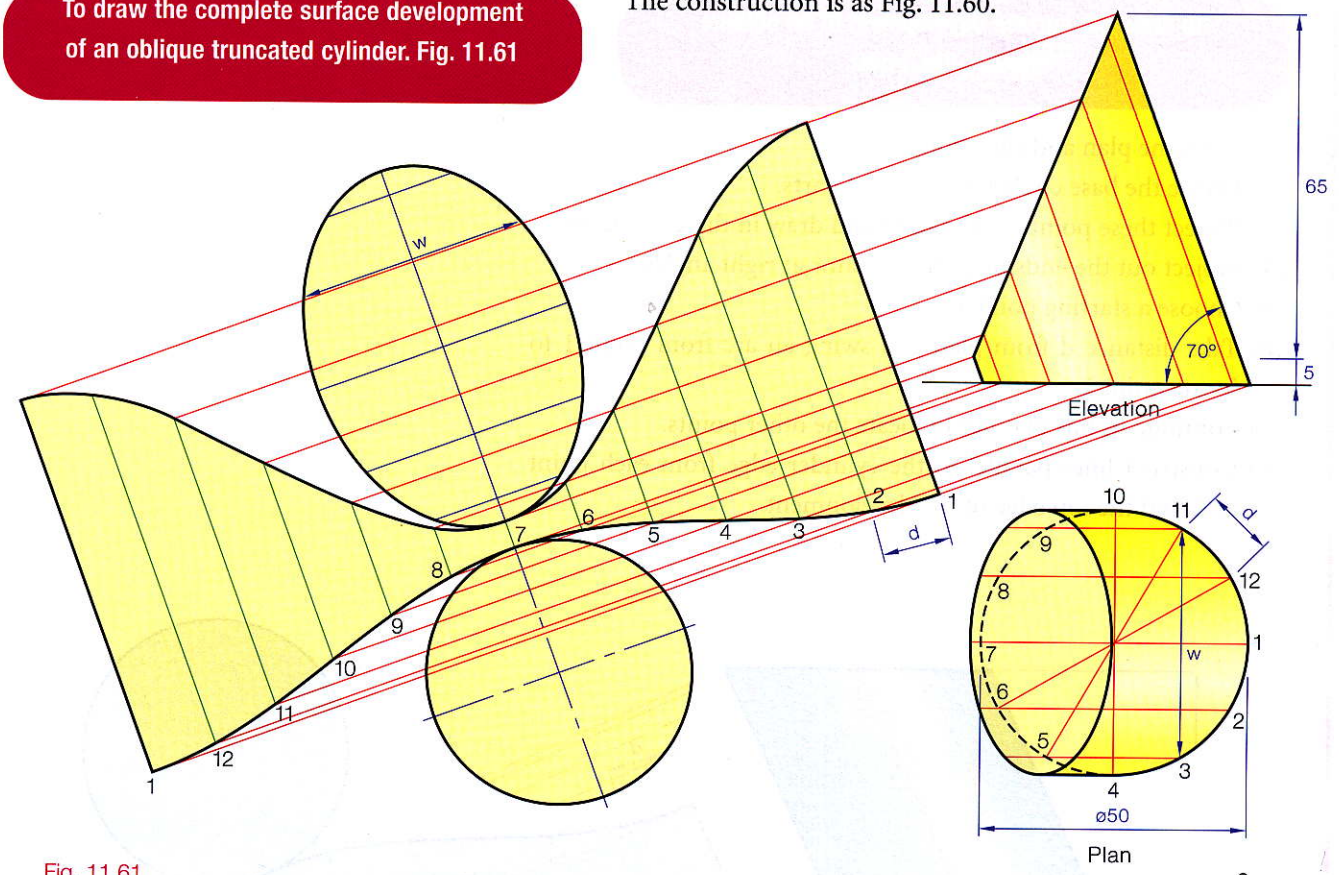


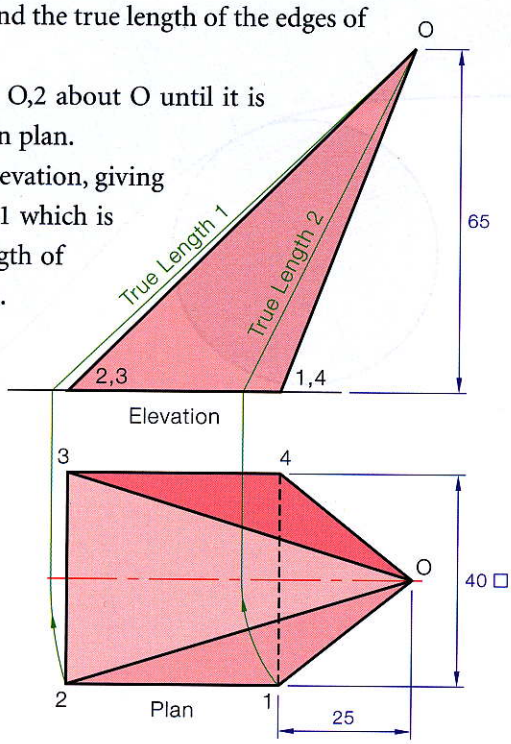
Fig. 11.61

To develop the surface of a square-based oblique pyramid. Fig. 11.62

All edges in a development are true lengths. We must therefore first find the true length of the edges of the pyramid.

(1) Rotate edge O,2 about O until it is horizontal in plan.

(2) Project to elevation, giving true length 1 which is the true length of O,2 and O,3.



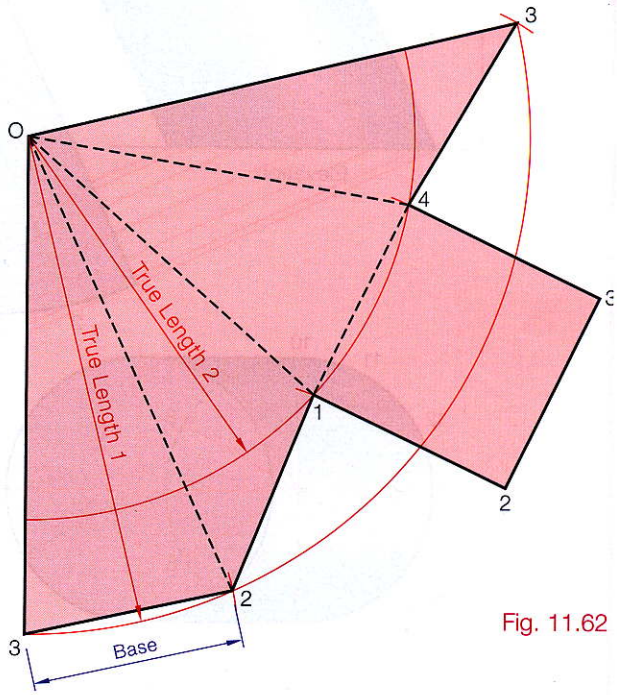


Fig. 11.62

(3) Find the true length of O,1 in a similar way.

(4) Choose a starting point O for the development.

(5) Swing two arcs using the true lengths.

(6) Step the length of the base around and add the base.

To develop the surface of a pentagonal-based oblique pyramid. Fig. 11.63

(1) Find the true length of all the edges. True length 1 shows the true length of edges O,3 and O,4. True length 2 shows the true length of O,2 and O,5. Edge O,1 already appears as a true length in elevation.

(2) Project the cut points in elevation over to these true lengths.

(3) Construct the development as before, using the true lengths found.

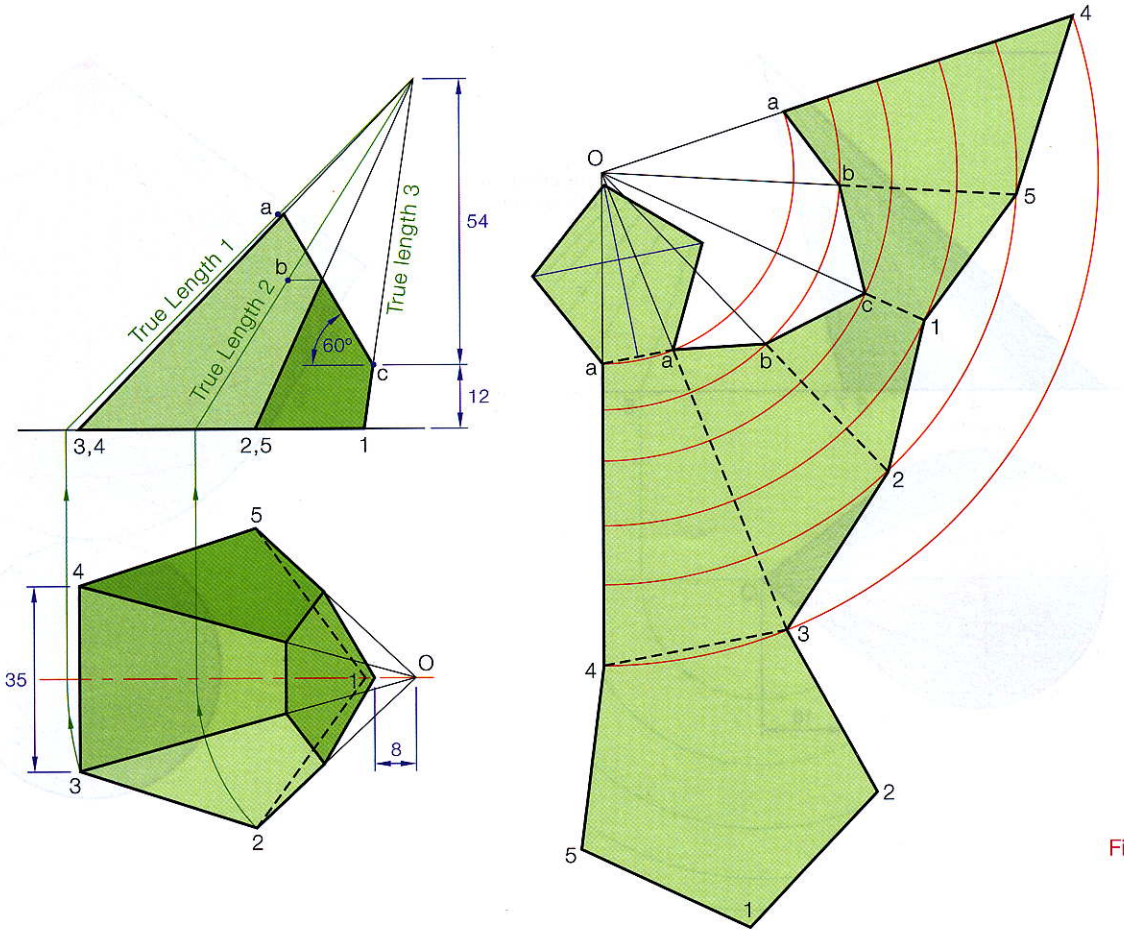
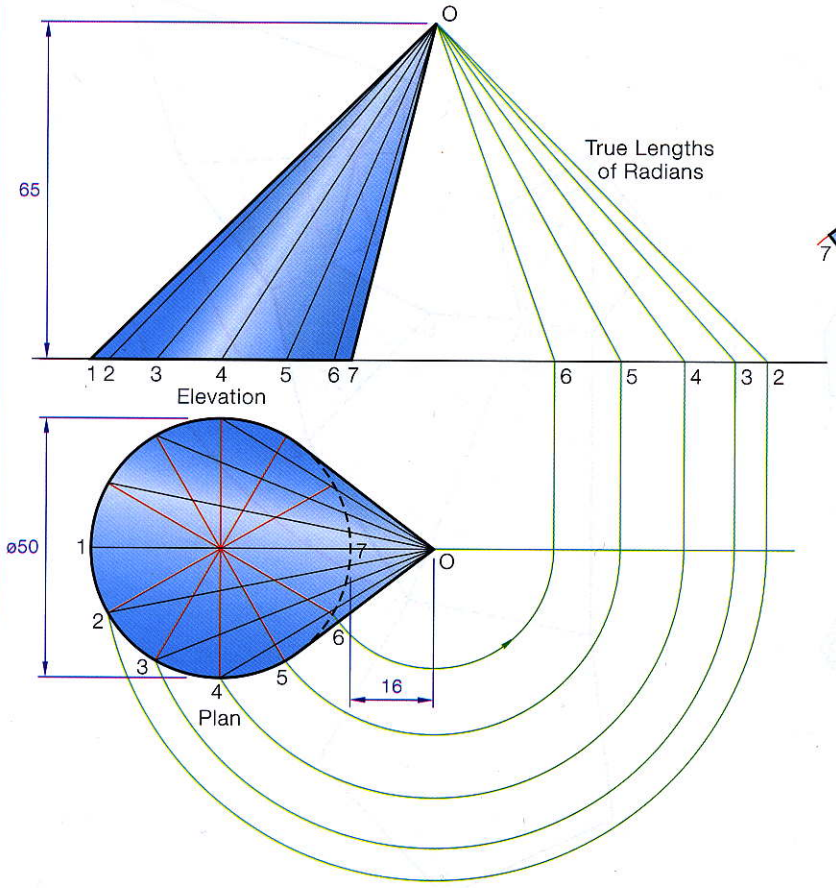


Fig. 11.63

To develop the surface of an oblique cone.
Fig. 11.64

(1) Divide the base circle and draw in the radians.
(2) Find the true length of each radian. Radian O,1 and O,7 are already shown as true lengths.
(3) Start the development with radian O,1.
(4) With the true length of O,2 as radius and O as centre, scribe an arc. With chord length 1,2 as radius and point 1 as centre, scribe an arc to cut the previous arc giving point 2 on the development.
(5) Continue in this way to complete the development.



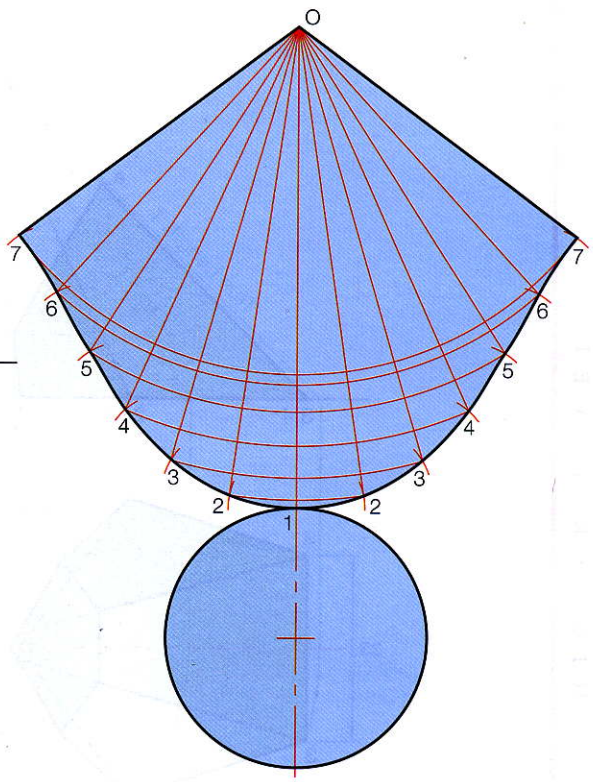
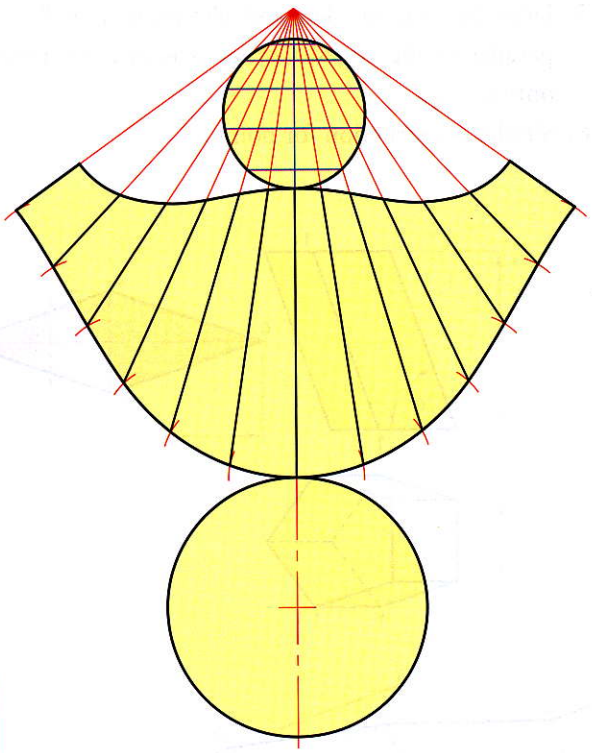


Fig. 11.64

To develop a truncated oblique cone. Fig. 11.65

(1) Develop the full cone as above.
(2) Transfer the cut length of each radian in elevation across to its true length.
(3) Transfer these true lengths to the development.
(4) Add the true shape of the cut surface.



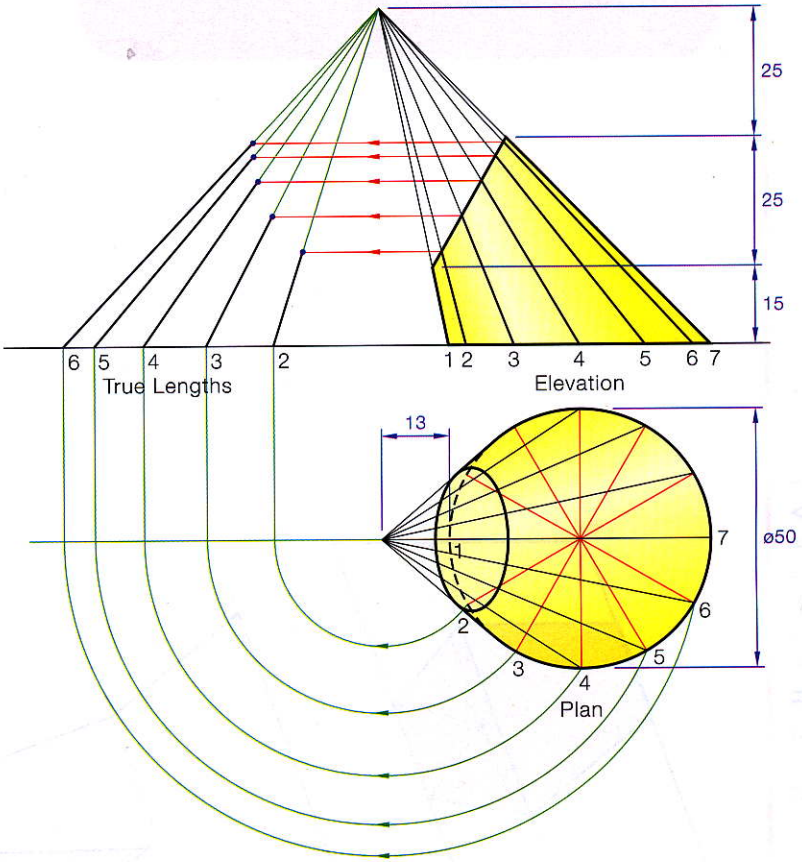


Fig. 11.65

Given the plan and elevation of an oblique pentagonal prism and the development of a label that is to be wrapped around it. Draw the plan and elevation of the prism when the label is in position. Axis ab on the label is to be placed centrally on edge 1,1 of the prism. Fig. 11.66

(1) Develop the surface of the prism as described earlier.
(2) Place the label in position on the development.
(3) All corners and fold points are easily projected back, except corners c and d.
(4) On the development, point d is projected parallel to the edges onto 2,3 which is a true length. Point q is found on 2,3 in plan which is also a true length.
(5) Draw line qq in plan and elevation. It will run parallel to the edges. Corner d is now projected onto line qq.
(6) Similar construction for point c.

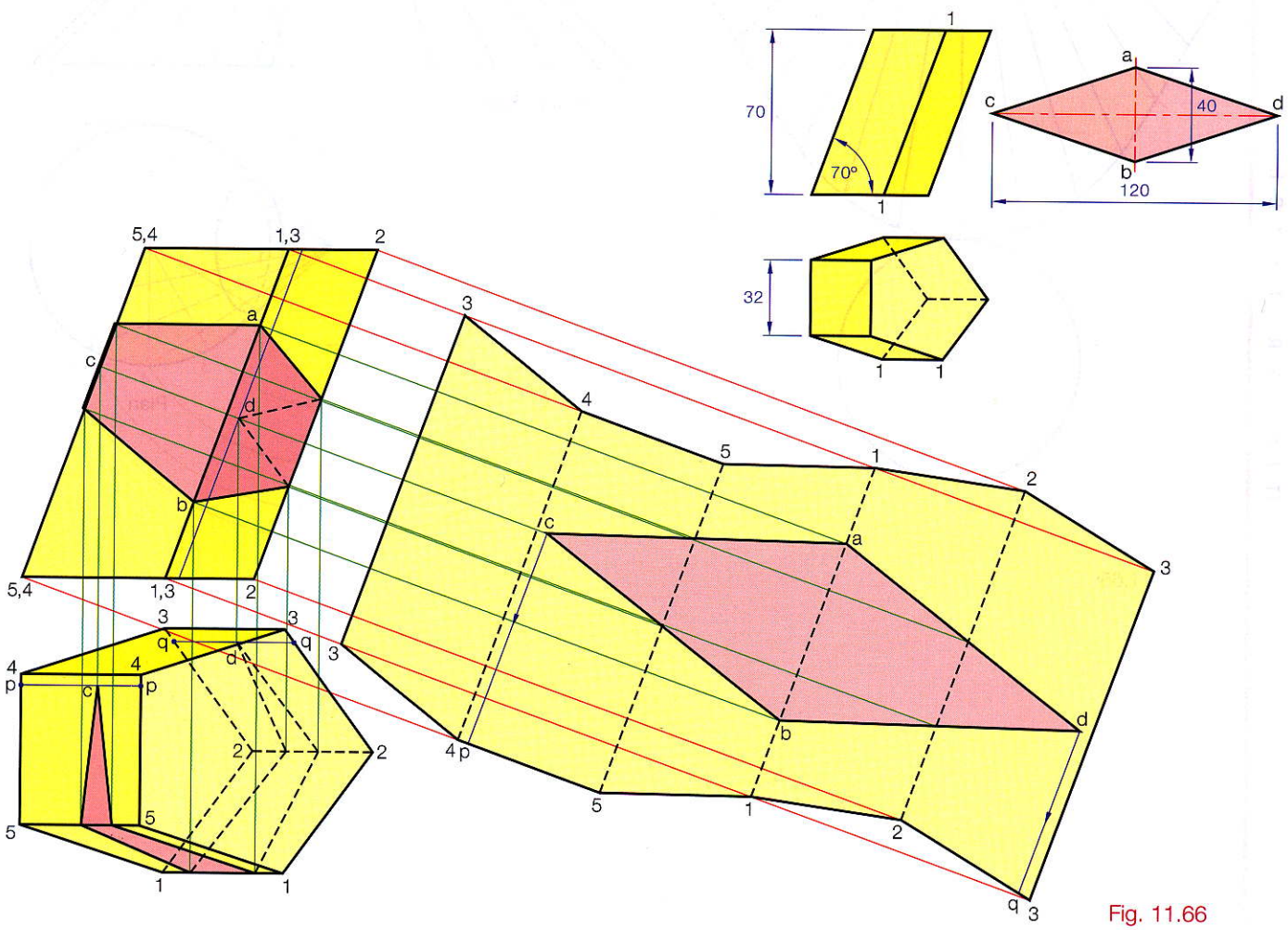


Fig. 11.66

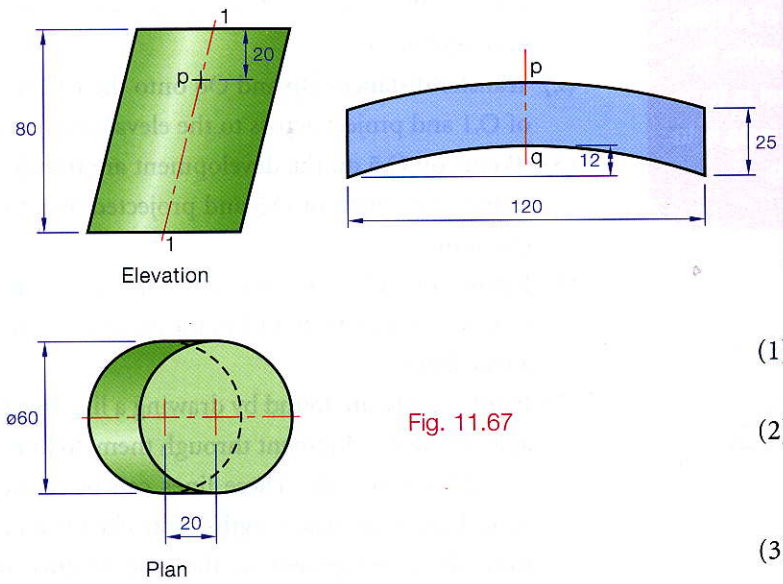


Fig. 11.67

Given the plan and elevation of an oblique cylinder and the development of a label that is to be wrapped around it, Fig. 11.67. Draw the plan and elevation of the cylinder when the label is in position. Point p on the label is to be placed on point p on the cylinder. Axis pq is to line up with line 1,1 on the cylinder surface.

(1) Develop the cylinder and place the label in position.

(2) Where the label crosses the generator lines in the development the points are brought back to elevation and then down to plan.

(3) A generator is drawn through the end of the label. Distance d is a true length and is stepped-off on the plan. The generator is drawn in plan and elevation and the end of the label is projected onto it, Fig. 11.68.

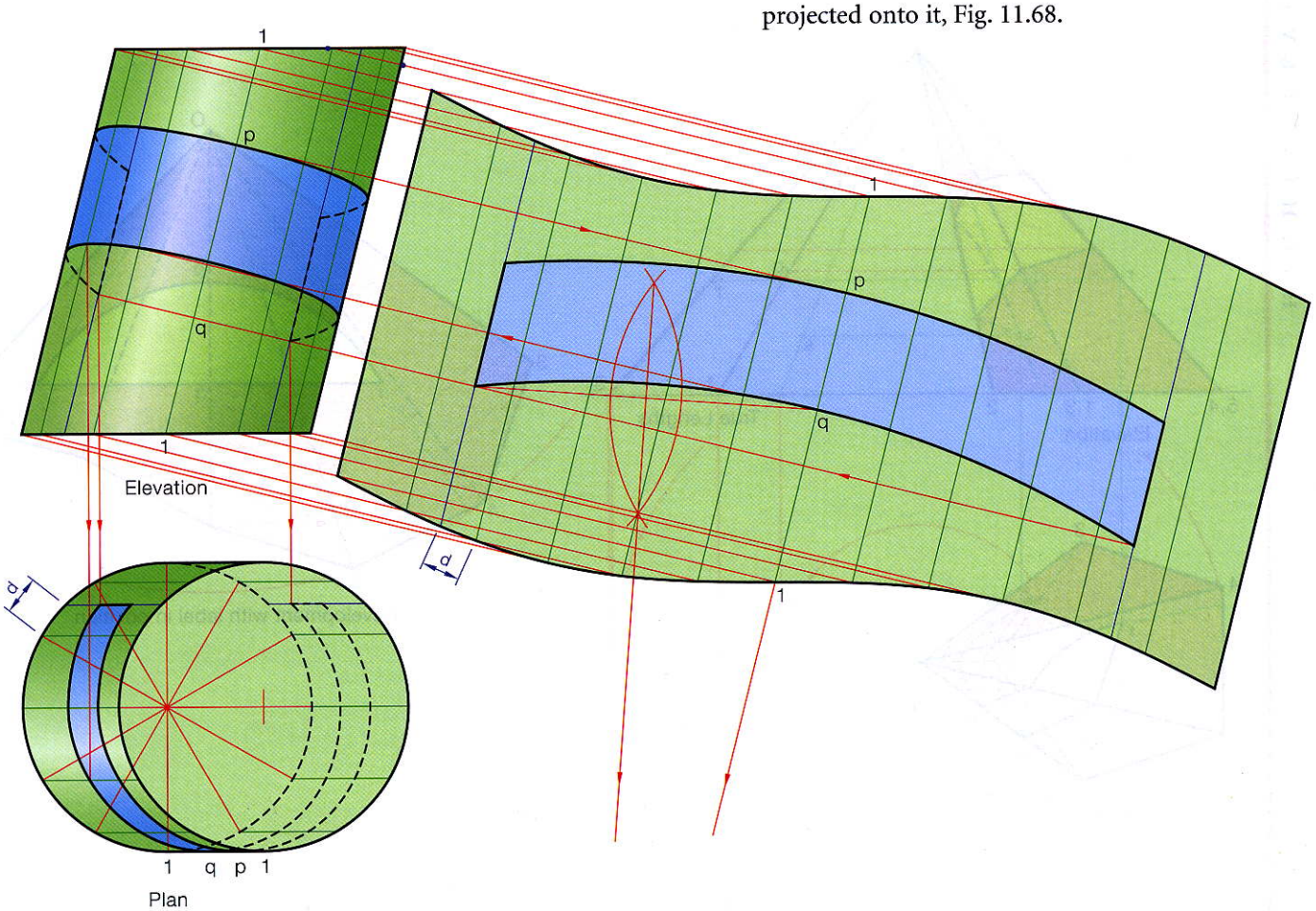


Fig. 11.68

Given the plan and elevation of a pentagonal-based oblique pyramid and the development of a label. Draw the plan and elevation of the pyramid when the label is wrapped around it. Point p on the label is to be placed on point p on the pyramid, and the axis pq is to be placed on the edge O,1. Fig. 11.69

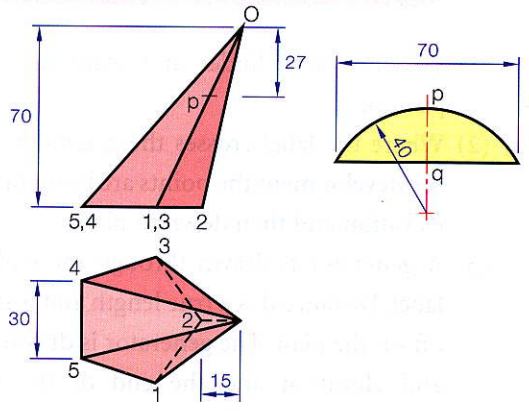


Fig. 11.69

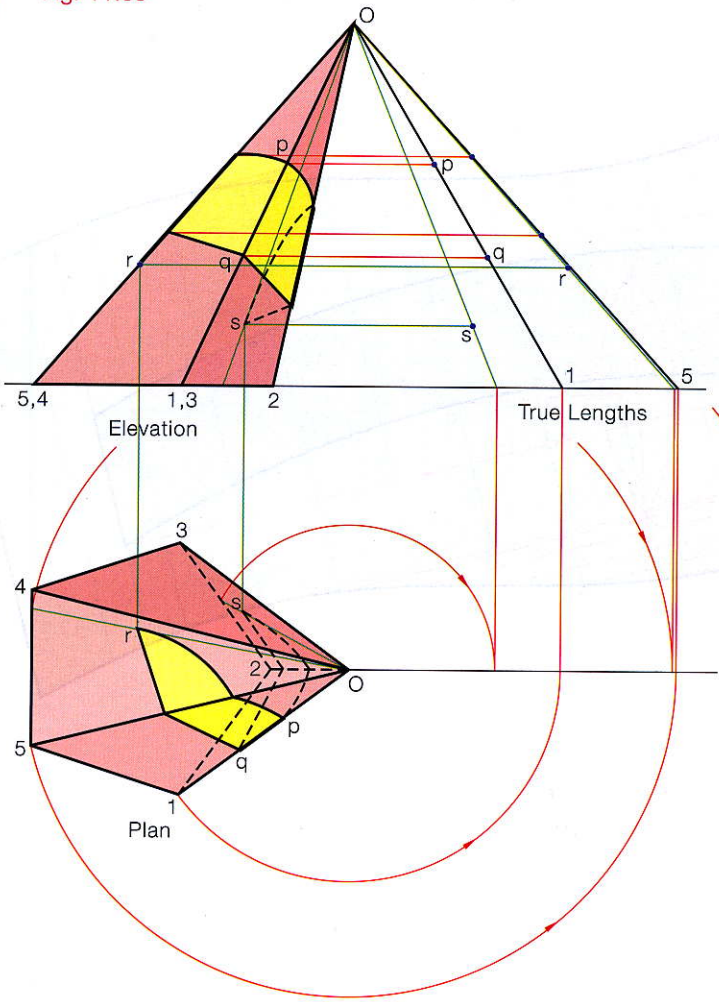
(1) Develop the surface of the pyramid as explained earlier. Place the label in position on the development.

(2) Transfer distances Op and Oq onto the true length of O,1 and project across to the elevation.

(3) Points on O,5 on the development are transferred to the true length of O,5 and projected over to the elevation.

(4) Points on O,2 in the development can be transferred directly to O,2 in the elevation as this is a true length.

(5) Points r and s are found by drawing a line from the apex in the development through them, to lines 4,5 and 2,3 respectively. These lines can be found in plan. Find their true lengths. Transfer Or and Os from the development to the true lengths, then project to elevation and plan, Fig. 11.70.



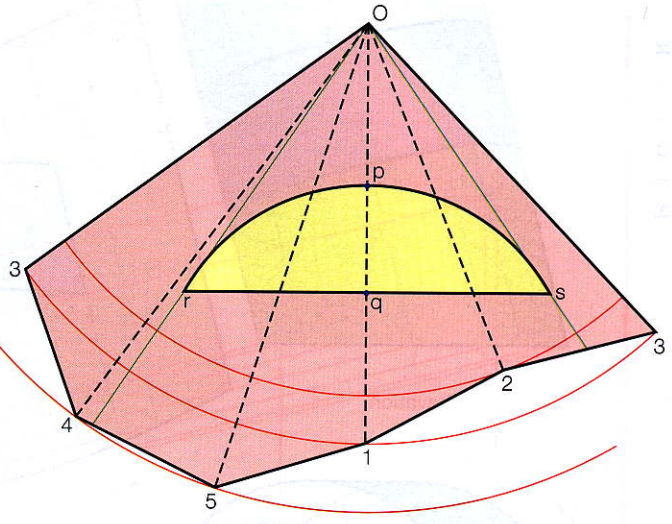
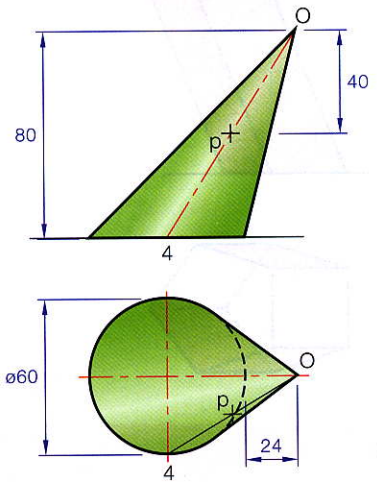


Fig. 11.70

Given the plan and elevation of an oblique cone and the development of a label. Draw the plan and elevation of the cone when the label is wrapped around it. Point p on the label is to be placed on point p on the cone and axis pq is to be placed on the generator 0,4. Fig. 11.71



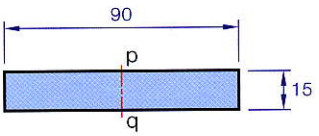


Fig. 11.71

(1) The true lengths of the radians are needed for the development.

(2) Place the label in position.

(3) Distances are transferred from the development to these true lengths and projected across to the elevation.

(4) Special treatment is given to the label corners a, b, c and d.

Fig. 11.72b

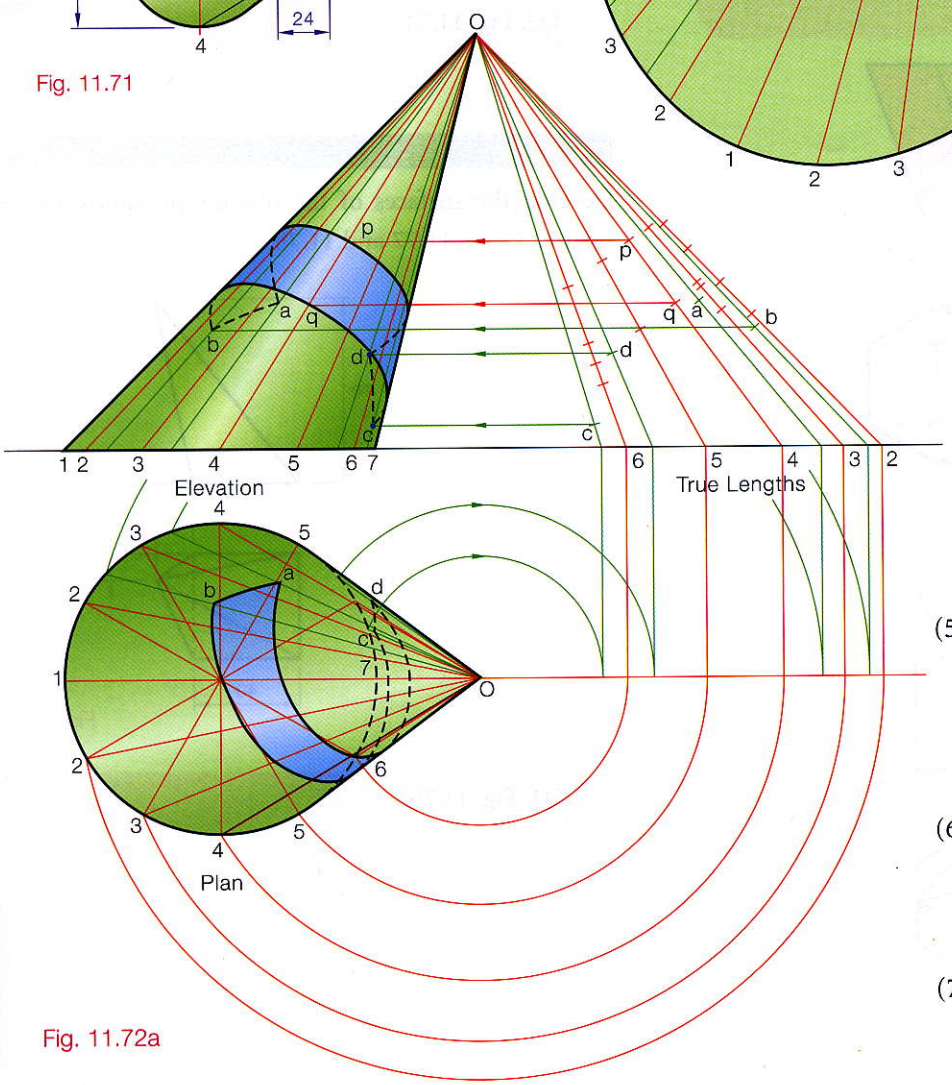


Fig. 11.72a

(5) A radian is drawn through corner b, for instance, on the development. This radian is found in plan and then in elevation, Fig. 11.72a.

(6) Find the true length of the radian in elevation and transfer distance Ob from the development onto this true length.

(7) Point b is projected across to elevation and plan, Fig. 11.72b.

Activities

HIGHER LEVEL

Q1. TO Q3.

Develop the surfaces of the oblique prisms shown in Figures 11.73, 11.74 and 11.75.

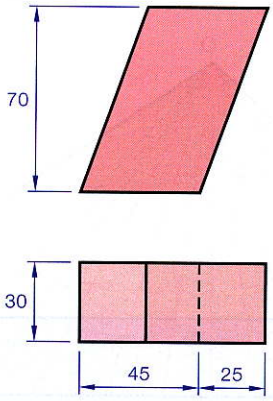


Fig. 11.73

Q1. Fig. 11.73

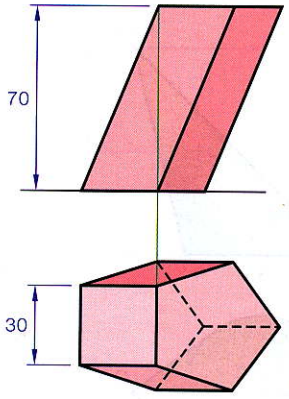


Fig. 11.74

Q2. Fig. 11.74

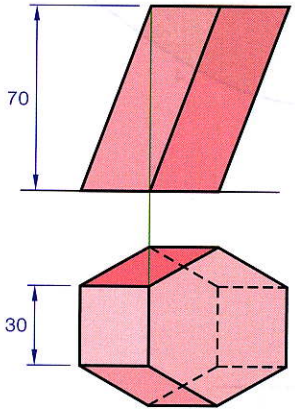


Fig. 11.75

Q3. Fig. 11.75

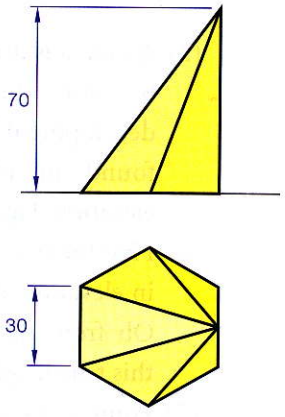


Fig. 11.77

Q5. Fig. 11.77

Q4. TO Q6.

Develop the surfaces of the oblique pyramids shown, Figures 11.76, 11.77 and 11.78.

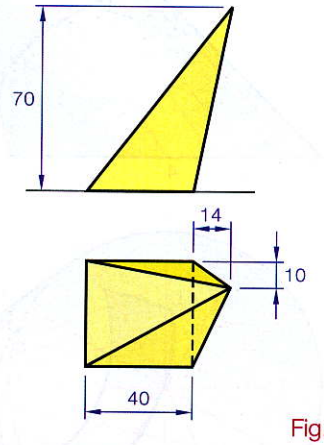


Fig. 11.76

Q4. Fig. 11.76

HIGHER LEVEL

Q6. Fig. 11.78

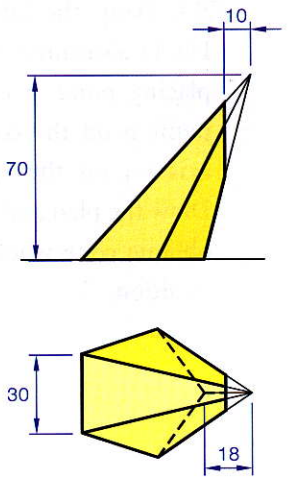


Fig. 11.78

Q7. TO Q9.

Develop the surface of the oblique cylinders and oblique cones shown in Figures 11.79, 11.80 and 11.81.

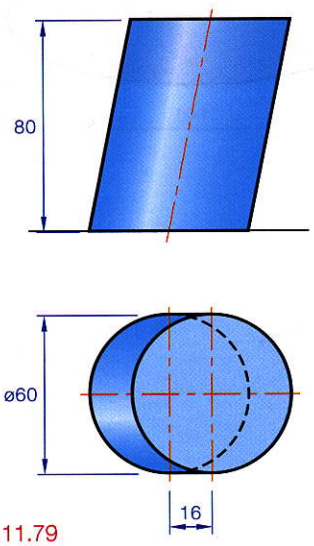


Fig. 11.79

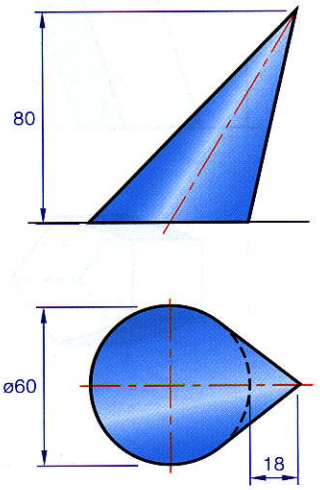


Fig. 11.80

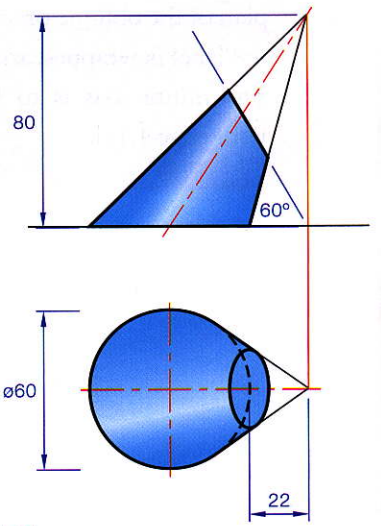
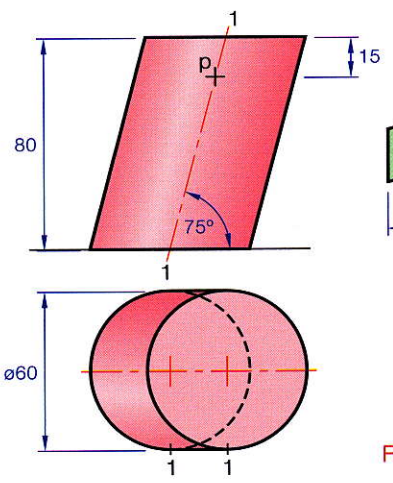


Fig. 11.81

Q10. Given the plan and elevation of an oblique cylinder and the development of a label. Draw the plan and elevation of the oblique cylinder when the label is wrapped around it. Point p on the label is to be placed on point p on the cylinder. Axis pq on the label is to be lined up with radian 1,1 on the cylinder. Fig. 11.82



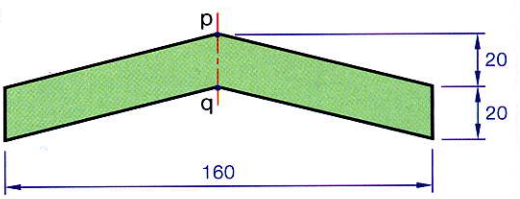


Fig. 11.82

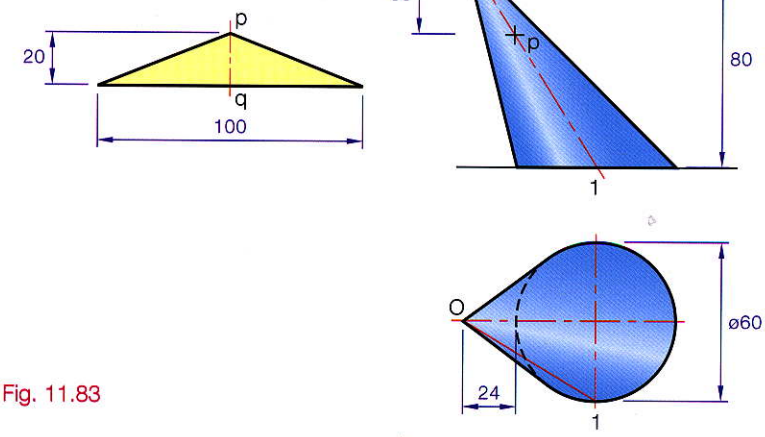


Fig. 11.83

Q11. Wrap the label shown in Fig. 11.83 around the oblique cone, placing point p on the label on point p on the cone and placing axis pq on the radian line O,1. Draw the plan and elevation of the oblique cone when the label is in position.

Q12. Draw the elevation and plan of the oblique prism when the label is wrapped around it. The minor axis is to line up with edge 1,1.

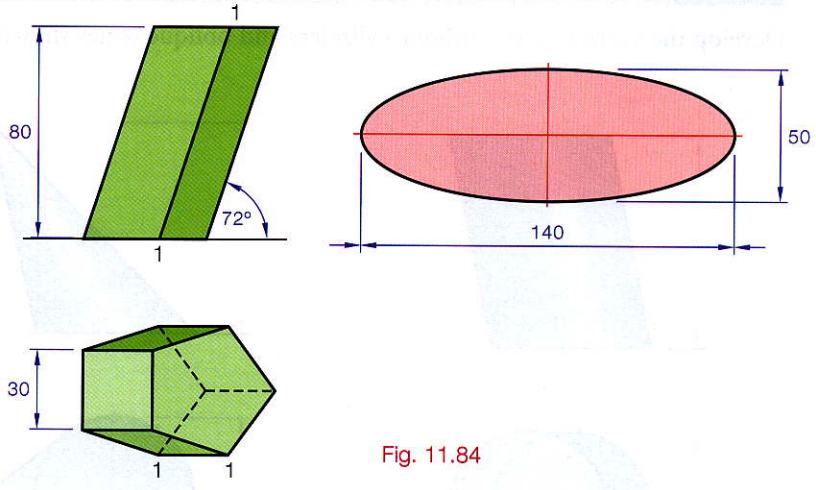


Fig. 11.84