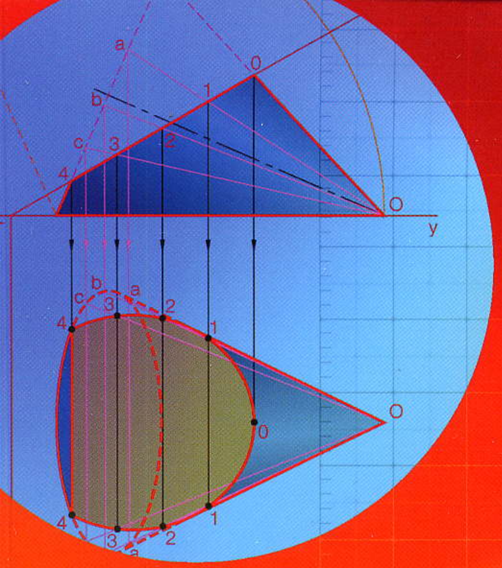
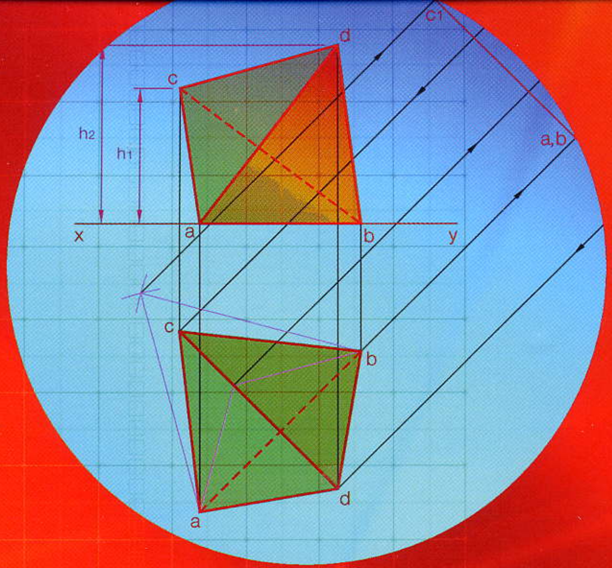
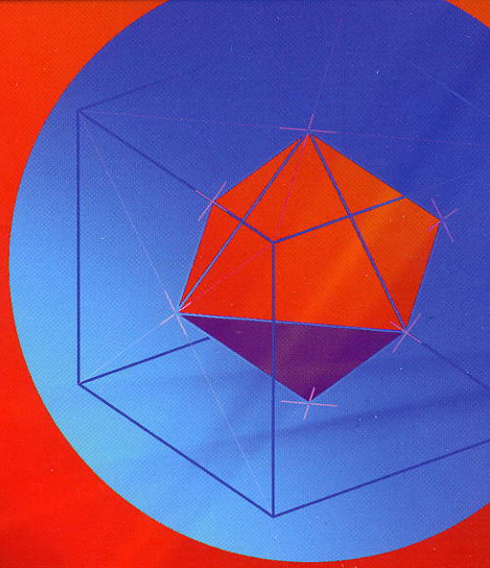
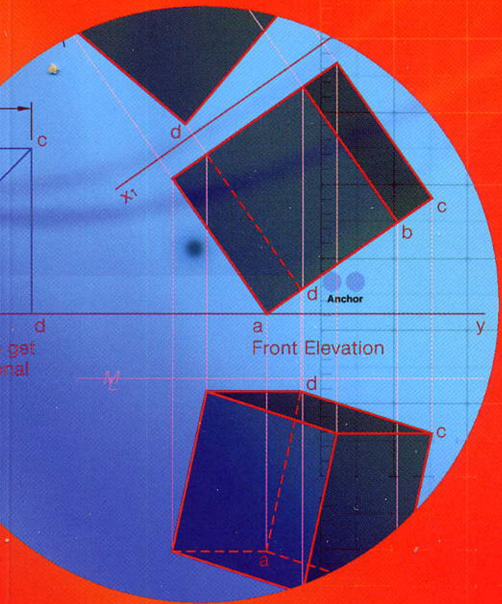


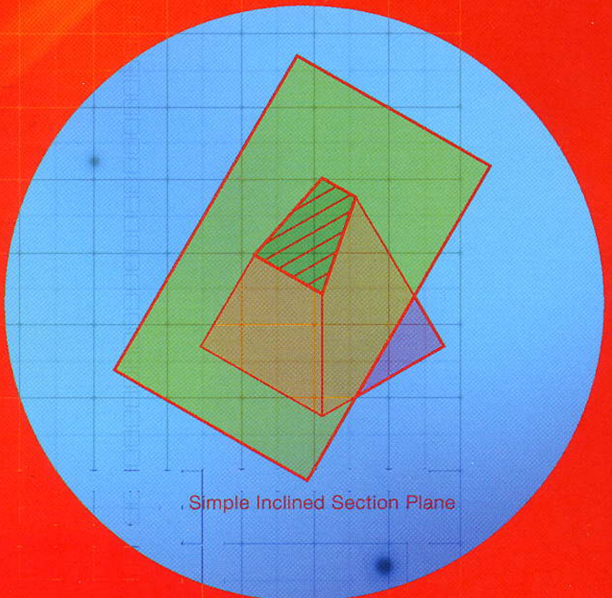


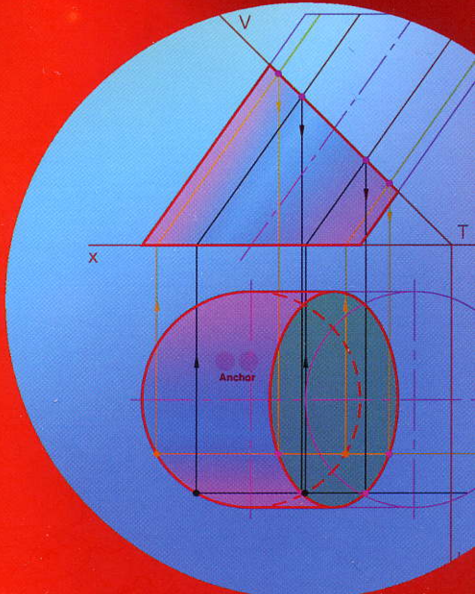


GRAPHICS IN DESIGN & COMMUNICATION 1

PLANE AND DESCRIPTIVE GEOMETRY







DAVID ANDERSON

9 The Oblique Plane

SYLLABUS OUTLINE

Areas to be studied:

• Definition of planes, simply inclined and oblique. • Determination of oblique and *tangent* planes. • True shape and inclinations of planes to principal planes of reference. • Intersection of oblique planes, lines and *dihedral angle.* • Sectioning of right solids by oblique planes. • *Treatment of planes as laminar surfaces given rectangular coordinates.* • *Properties and projections of skew lines.* • *Spatial relationships between lines and planes.*

Learning outcomes

Students should be able to:

Higher and Ordinary levels

- Distinguish between simply inclined and obliquely inclined plane surfaces.
- Determine the angle of inclination between given planes and the principal planes of reference.
- Determine the true length and inclination of given lines.
- Establish the true shape of an obliquely inclined plane.
- Determine the line of intersection between two planes.
- Determine the projections and true shape of sections of solids resulting from simply inclined and oblique cutting planes.

Higher level only

- *Construct obliquely inclined planes given the angles of inclination to the principal planes of reference and to include a given line or point.*
- *Establish the dihedral angle between two intersecting planes.*
- *Display knowledge of the relationships between planes and lines.*
- *Understand the concept of a laminar surface defined by spatial coordinates.*
- *Solve a variety of problems involving the intersection, inclination and positioning of laminar plane surfaces.*
- *Define the concept of skew lines and their use in solving practical problems.*
- *Establish various spatial relationships between skew lines and other lines and planes, including distance, inclination and direction.*

(6) The dihedral angle is found by finding a point view of line 2,3. The height of point 1 in the auxiliary elevation must be taken from the front elevation. A measuring line was introduced to shorten all the distances in the second auxiliary plan.

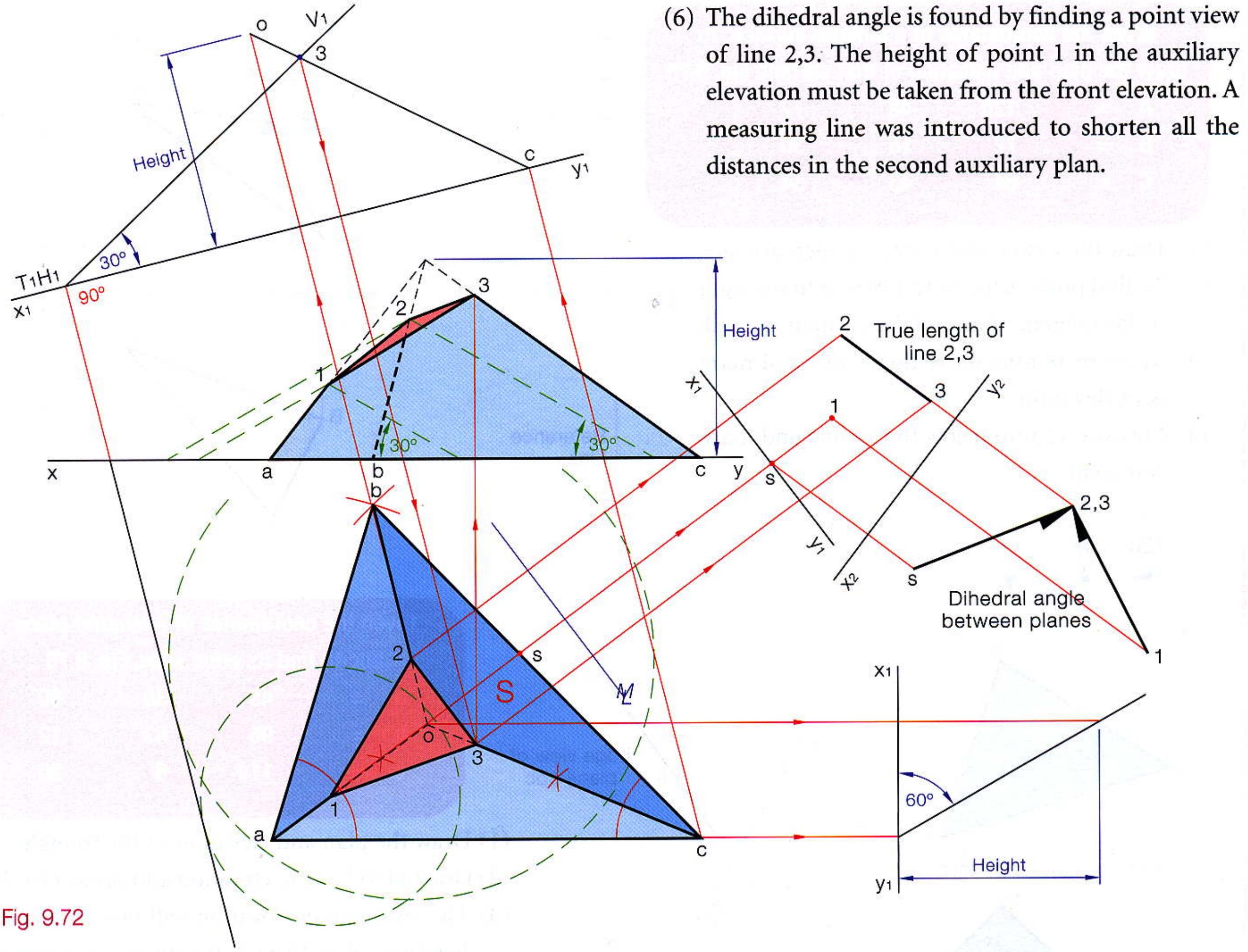


Fig. 9.72

Spatial Coordinates

All points in space can be defined by their coordinates. When dealing with three dimensions it is necessary to define a point using three coordinates, x, y and z. With reference to Fig. 9.73 we can see how point P can be established relative to a datum line, the horizontal plane and the vertical plane.

The first coordinate, coordinate x, refers to the distance to the right of the datum line. The second coordinate, coordinate y, refers to the distance the point P is above the horizontal plane. The third distance, coordinate z, refers to the distance point P is in front of the vertical plane.

Since we can define points in space using coordinates we can also define lines and planes using the same method.

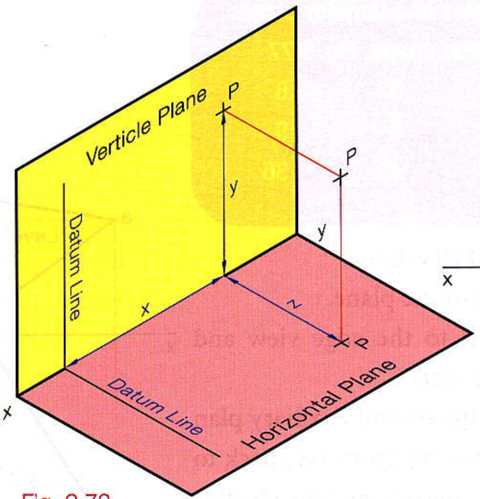


Fig. 9.73

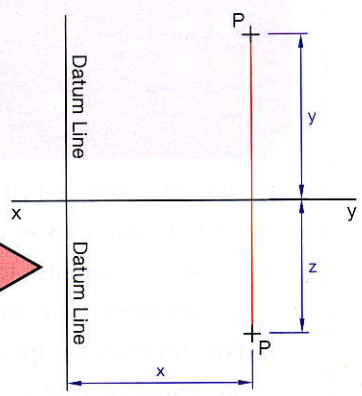


Fig. 9.74

HIGHER LEVEL

Given the coordinates of a laminar surface. To draw the plan and elevation of that surface Fig. 9.75

A	=	120	48	12
B	=	72	15	30
C	=	84	43	54

(1) Draw the xy line and a vertical reference line.

(2) To find point A measure 120 mm to the right of the reference line and draw a light vertical.

(3) Measure 48 mm above the xy line and mark A in elevation.

(4) Measure 12 mm below the xy line and mark A in plan.

x	y	z
120	48	12
→	↑	↓

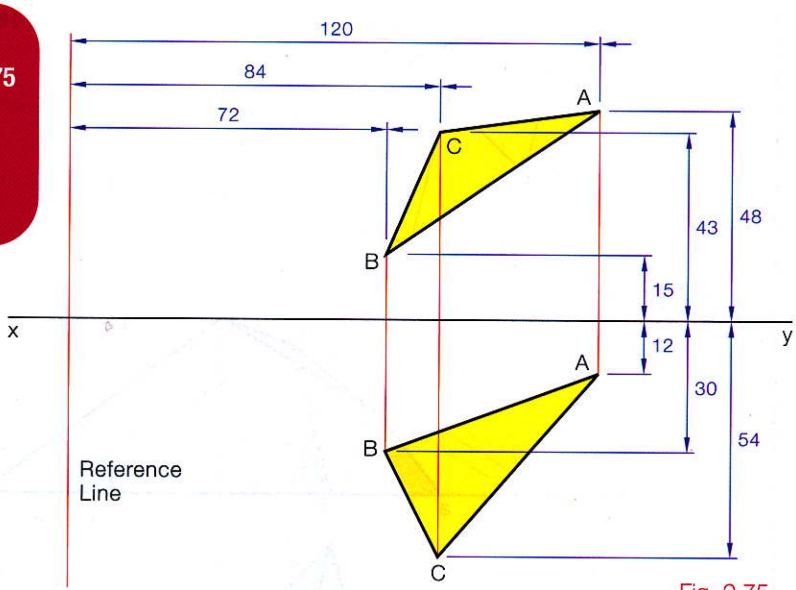


Fig. 9.75

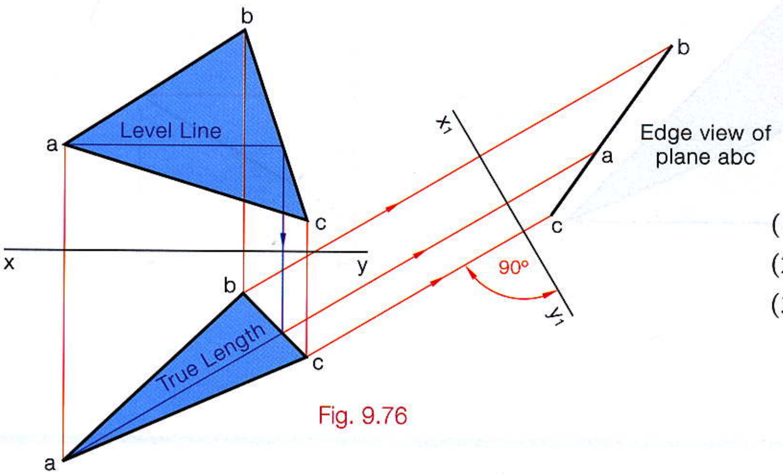


Fig. 9.76

Given the coordinates of a laminar plane abc to find its edge view. Fig. 9.76

a	=	45	30	60
b	=	96	63	12
c	=	114	9	30

(1) Draw the plan and elevation of the triangle.

(2) Draw a level line in elevation and project to plan.

(3) The level line in elevation will project as a true length in plan. View in the direction of the true length and project an auxiliary elevation. The plane will appear as an edge view.

Given the coordinates of a laminar plane abc, to find its true shape. Fig. 9.77

a	=	33	33	6
b	=	66	8	8
c	=	108	45	36

(1) Draw the plan and elevation.

(2) Find an edge view of the plane.

(3) Draw x_2y_2 parallel to the edge view and project the three points.

(4) The distances for the second auxiliary plan are found by measuring from x_1y_1 back to the plan or from a measuring line which is parallel to the x_1y_1 back to the plan.

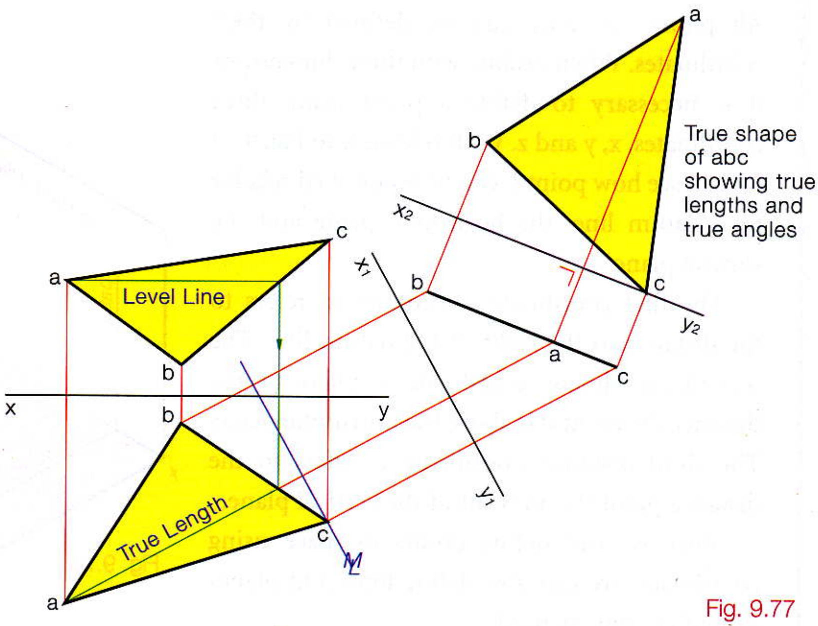


Fig. 9.77

Line of Intersection and Dihedral Angles for Triangular Lamina

When given the coordinates of meshing lamina it is often necessary to find the line of intersection between the planes and hence find the dihedral angle between the planes. There are three possible ways this problem can be presented:

(1) Given the line of intersection.
(2) Given one point on the line of intersection.
(3) Given no point on the line of intersection.

(1) GIVEN THE LINE OF INTERSECTION

Given the coordinates for two planes ABC and ABDE. Determine the dihedral angle between the planes, Fig. 9.78.

A = 35 5 115		B = 90 20 45	
C = 105 45 80		D = 70 75 10	
	E = 15 50 90		

Here we are given the line of intersection.

(1) Project an auxiliary view from plan showing the true length of the line of intersection between the planes. View perpendicular to AB in plan.

(2) Project from this auxiliary viewing down along the true length. The x_2y_2 will therefore be perpendicular to the true length found. Both planes will be seen as edge views thus showing the dihedral angle. Note that the distances for the second auxiliary are taken from the x_1y_1 back to the plan or from a suitable measuring line.

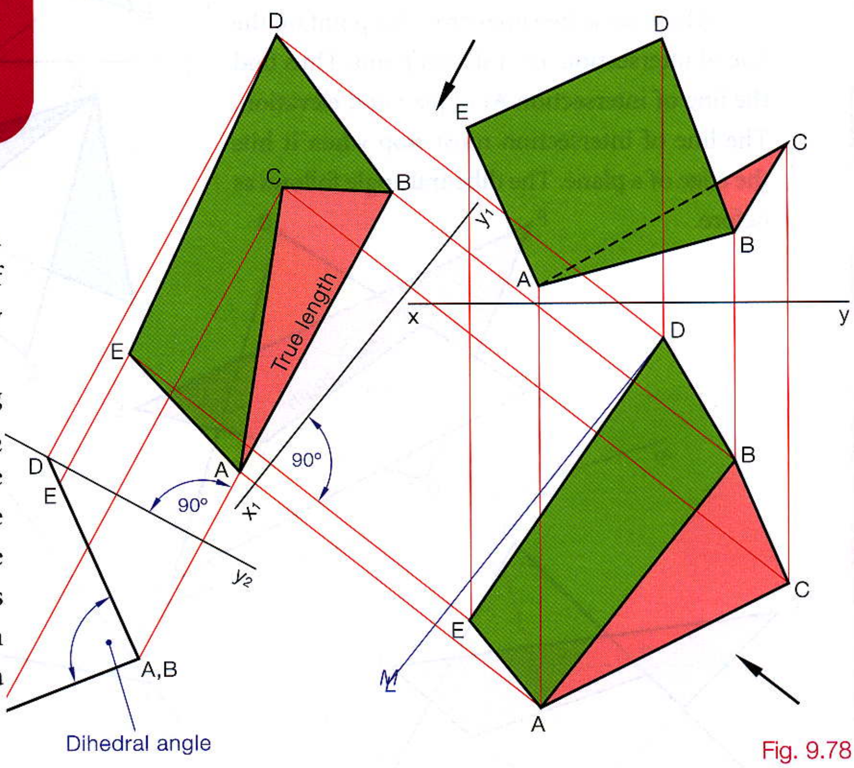


Fig. 9.78

(2) GIVEN ONE POINT ON THE LINE OF INTERSECTION

Given the coordinates of two planes ABC and ADE. Determine the line of intersection and the dihedral angle between the planes,. Fig. 9.79.

Point A is a shared point and therefore must be on the line of intersection

(1) Draw a horizontal cutting plane in elevation to cut both planes. In Fig. 9.79 this cutting plane gives points 1 and 2 on plane ABC and points D and 3 on plane ADE.

(2) Find these points in plan giving lines 1,2 and D,3. Where these two lines cross is a point on the line of intersection, i.e. a shared point. Thus find the line of intersection Ai in plan and elevation. The line of intersection must stop when it hits the edge of a plane. The dihedral angle follows as before.

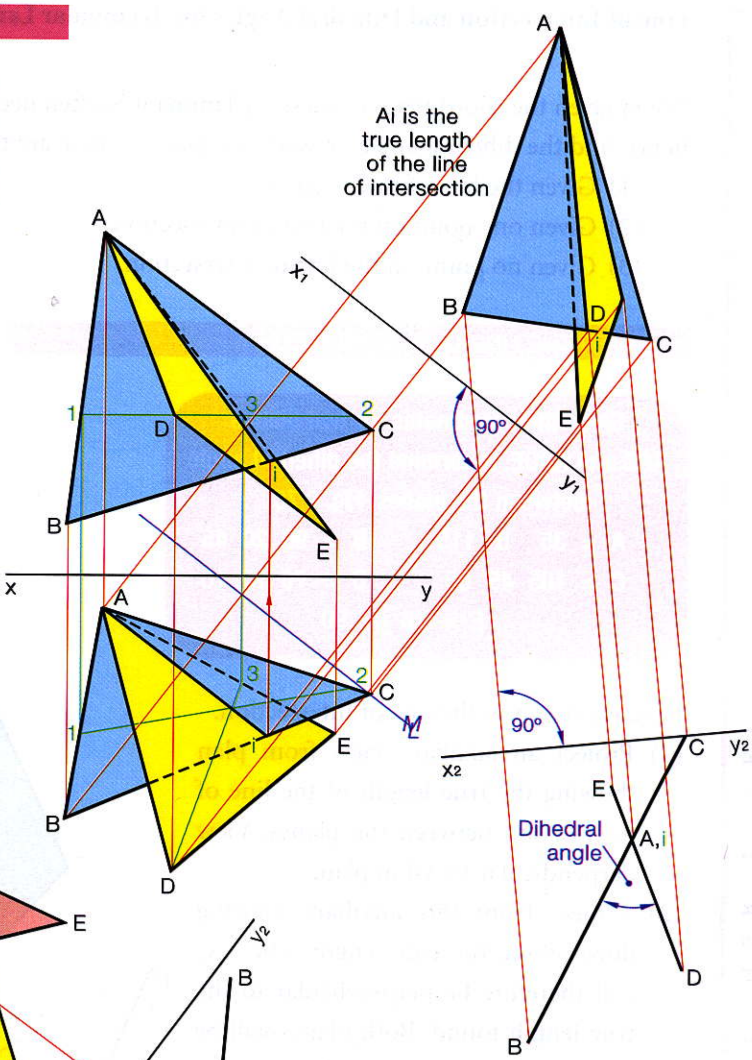
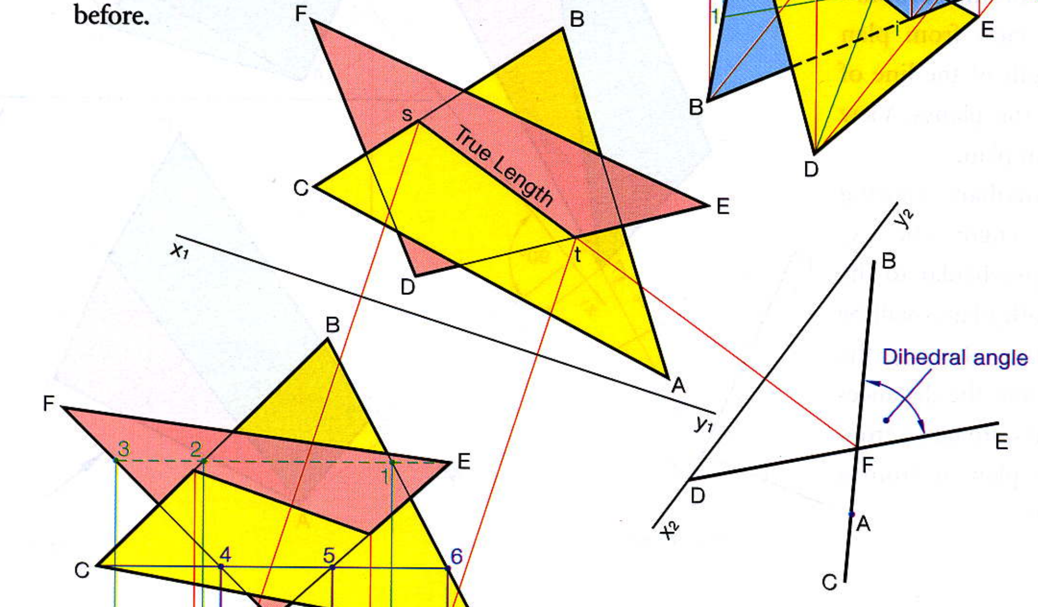


Fig. 9.79



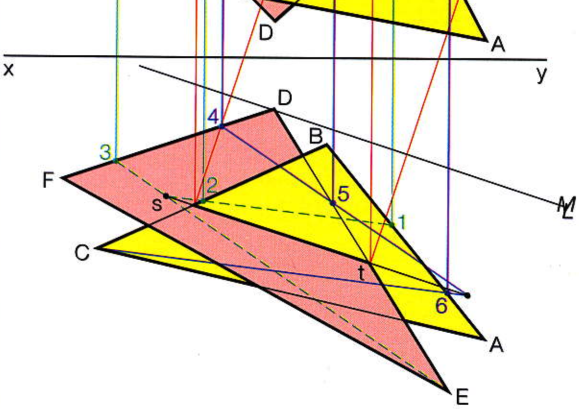


Fig. 9.80

(3) GIVEN NO POINTS ON THE LINE OF INTERSECTION

Given the coordinates of two planes ABC and DEF to find the line of intersection and the dihedral angle between the planes. Fig. 9.80

A = 140 5 80	**B = 95 90 25**	**C = 30 25 55**
D = 80 10 15	**E = 130 55 95**	**F = 20 70 35**

This method is the same as in the previous example except that two separate horizontal cutting planes are used.

These horizontal cutting planes can be drawn at any level as long as they cut both planes.

HIGHER LEVEL

Note 1: Line E,3 in plan will be parallel to 4,5. Also line 1,2 will be parallel to C,6 in plan.
Note 2: If the lines do not intersect they are extended until they do intersect.

To draw the shortest horizontal line to a plane from a given point P outside it, Fig. 9.81.

A = 75 75 10 B = 20 50 90
C = 40 5 115 D = 95 20 45
P = 110 45 80

(1) Draw an auxiliary showing the plane as an edge view. Project P onto this view.
(2) In the auxiliary draw the horizontal line from P to hit the plane at i. Project i to plan.
(3) Since the horizontal line from P to i in plan will be seen as a true length, then the line iP in plan must be the shortest distance from P to the line projected from the auxiliary elevation. Draw from P perpendicular to the projection lines to the auxiliary to find i.
(4) iP will be horizontal in elevation.

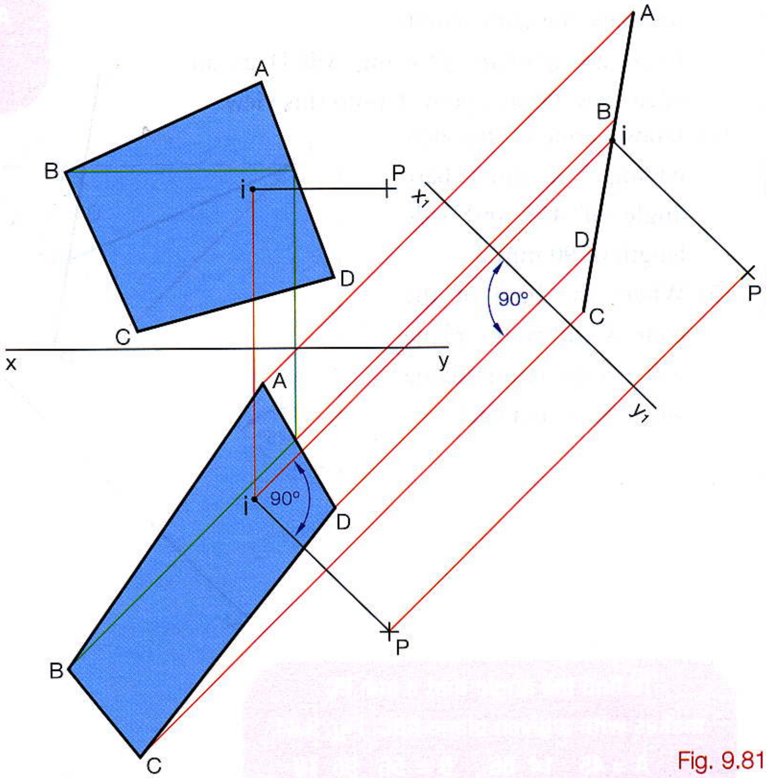


Fig. 9.81

To draw the perpendicular to a plane ABC from a point P outside it. Fig. 9.82

A = 120 88 2 B = 50 50 94
C = 160 20 84 P = 148 76 96

(1) Draw an auxiliary elevation showing the plane as an edge view and project point P onto this view.
(2) In the auxiliary, draw the perpendicular from P to the plane finding point q.
(3) It should be noted that **a perpendicular to a plane will appear perpendicular to the traces of that plane.** We can therefore draw the required line Pq in plan as it will appear perpendicular to the level line on the plane. The level line will be parallel to the HT line.
(4) Line Pq can be found in elevation as shown in Fig. 9.82.

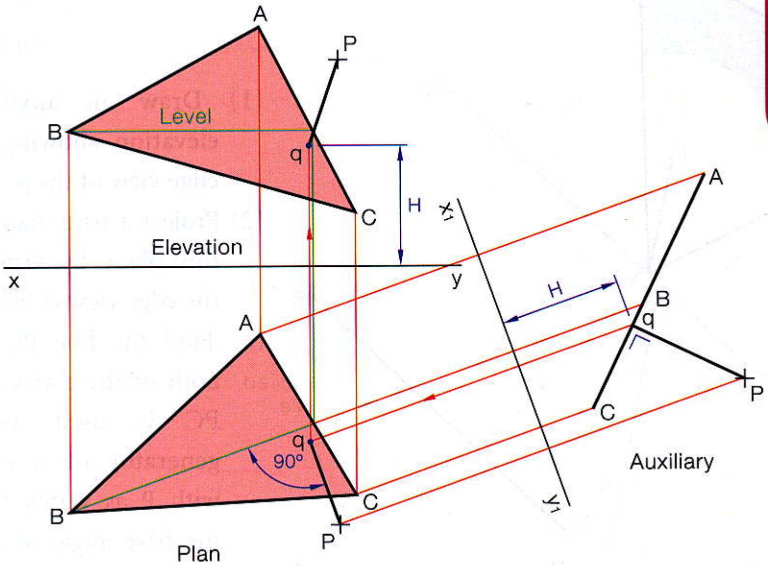


Fig. 9.82

To draw the projections of a line drawn from a point E outside a plane that will touch the plane at a distance of 90 mm from E and that shall be inclined at an angle of 45° to the horizontal plane. Fig. 9.83

A = 125 55 5 B = 165 15 85 C = 205 50 90
D = 185 80 40 E = 110 100 45

(1) This problem is solved under the principle that any line generator on the surface of a right cone will have the same inclination to the horizontal plane as the cone base angle. Furthermore all generators on a right cone will have the same length.
Draw an auxiliary showing ABCD as an edge view. Project point E onto this view.
(2) Draw a cone in this view having E as apex, base angle of 45° and side length of 90 mm.
(3) Where the base of the cone is cut by the plane will give the required line when projected back.

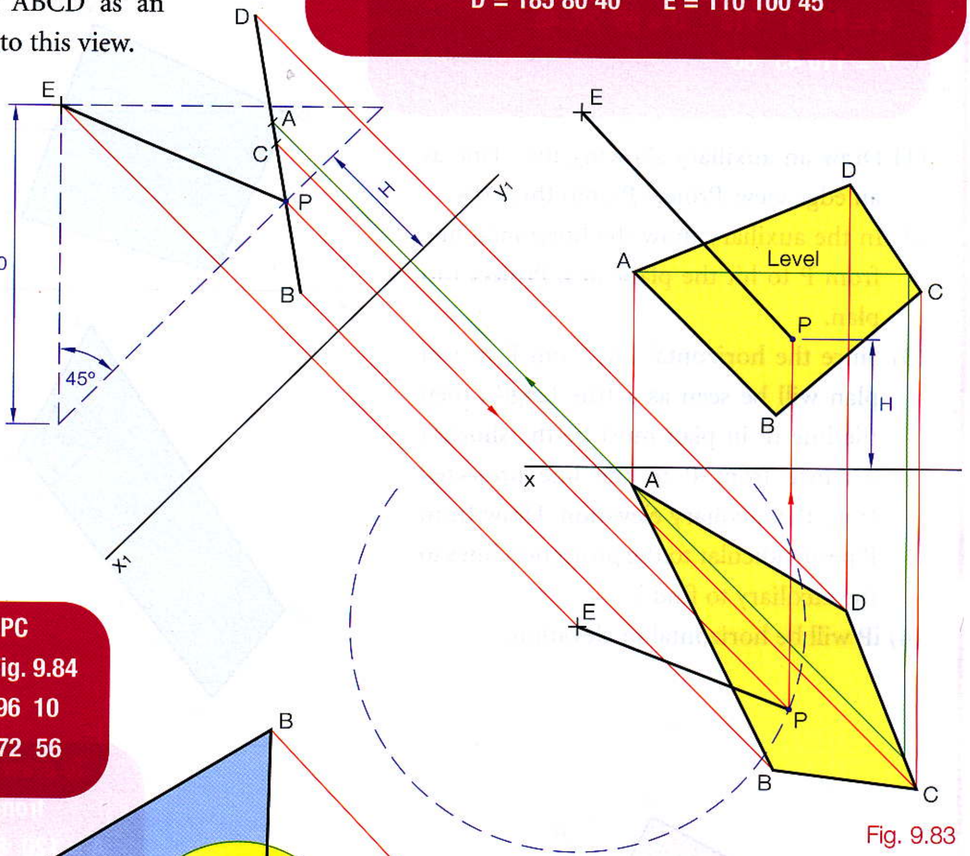


Fig. 9.83

To find the angle that a line PC makes with a given plane ABC. Fig. 9.84

A = 45 14 66 B = 56 96 10
C = 128 40 33 P = 105 72 56

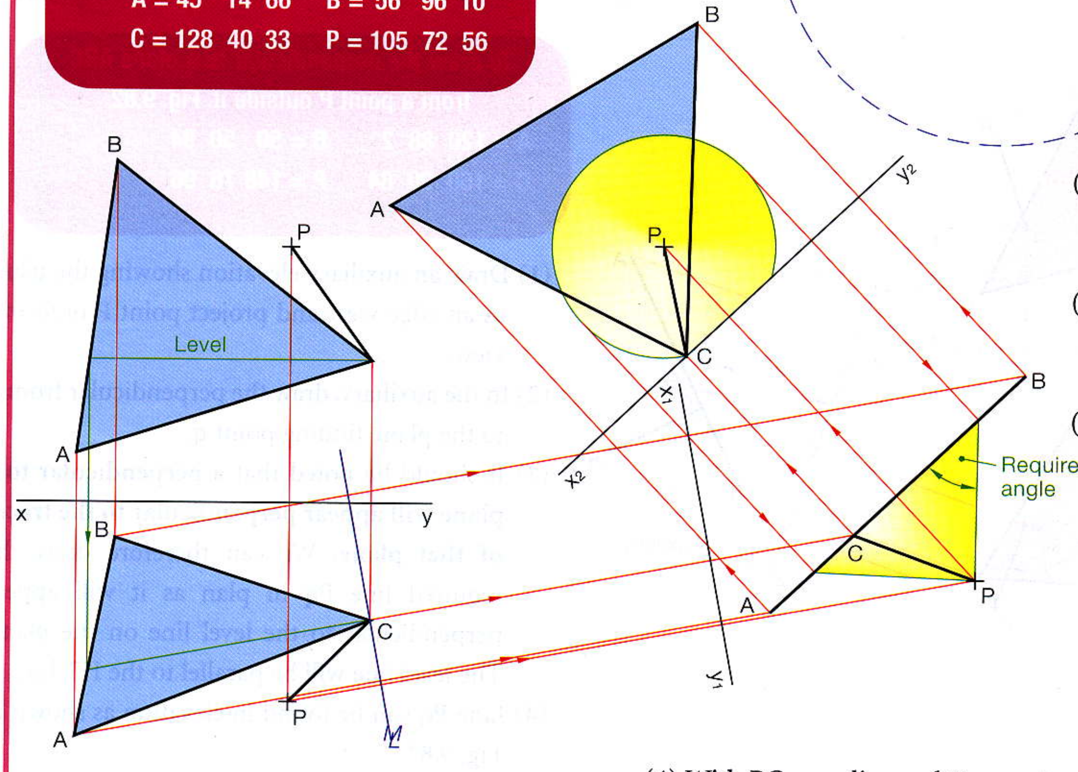


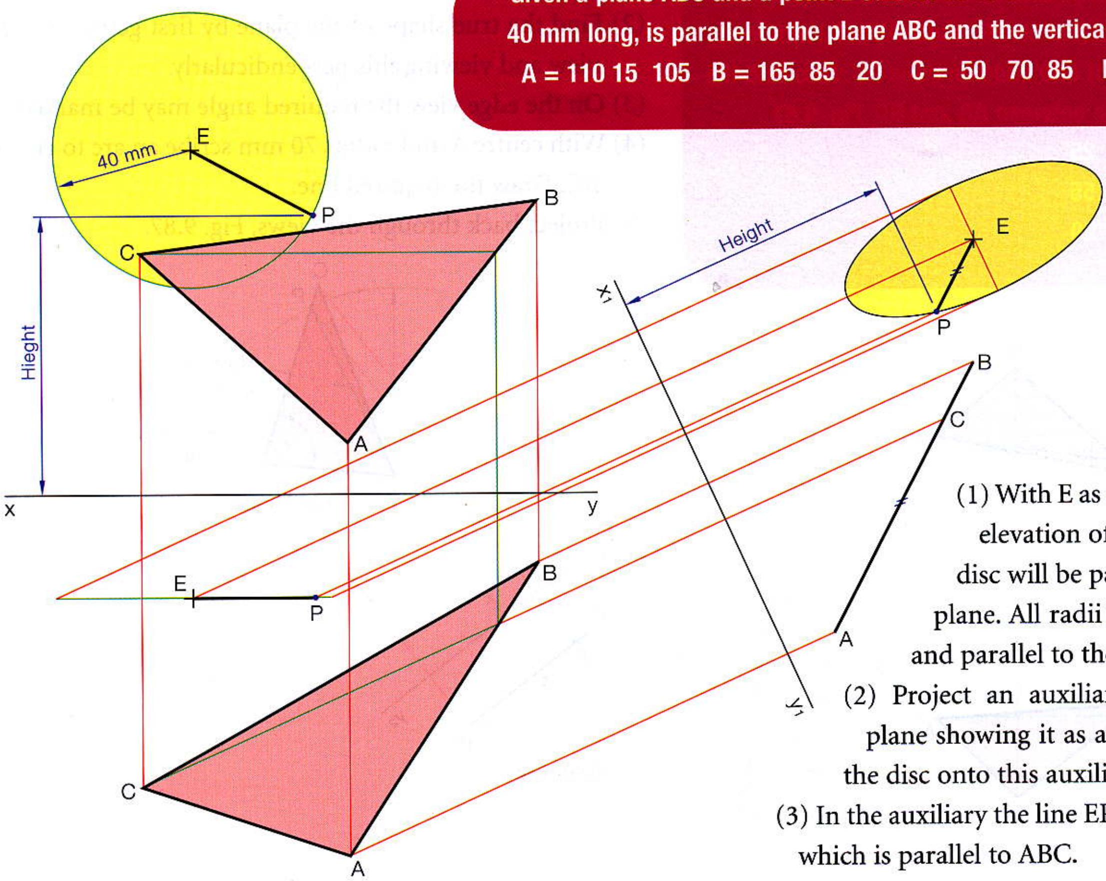
Fig. 9.84

(1) Draw an auxiliary elevation showing an edge view of the plane.
(2) Project a true shape of this plane by viewing the edge view at 90°.
(3) Find the line PC on both of these views. If PC is used as a generator of a cone with P as vertex then the base angle of that cone will equal the angle the line PC makes with the plane.
(4) With PC as radius and P as centre draw the plan of this cone in the second auxiliary plan. The edges of this circle produced back to the first auxiliary will give the required angle.

HIGHER LEVEL

Given a plane ABC and a point E outside it. To draw a line from E that is 40 mm long, is parallel to the plane ABC and the vertical plane. Fig. 9.85

A = 110 15 105 B = 165 85 20 C = 50 70 85 E = 65 100 30



(1) With E as centre draw a disc in elevation of 40 mm radius. The disc will be parallel to the vertical plane. All radii will be 40 mm long and parallel to the VP.

(2) Project an auxiliary elevation of the plane showing it as an edge view. Project the disc onto this auxiliary.

(3) In the auxiliary the line EP can be easily found which is parallel to ABC.

(4) Project EP back to plan and elevation. It satisfies all parameters.

Fig. 9.85

Given the coordinates of a plane ABC. Draw the projections of a line on the plane ABC, that passes through A and makes an angle of 60° with the edge BC. Fig. 9.86

A = 125 85 30 B = 170 10 100 C = 80 75 65

(1) Draw the plan and elevation of the plane.

(2) Get an edge view of the plane in the usual way.

(3) By viewing perpendicular to the edge view we can project a true shape of the lamina.

(4) On the true shape draw the required line. Project point p back through the views to find the line in plan and elevation as shown in Fig. 9.86.

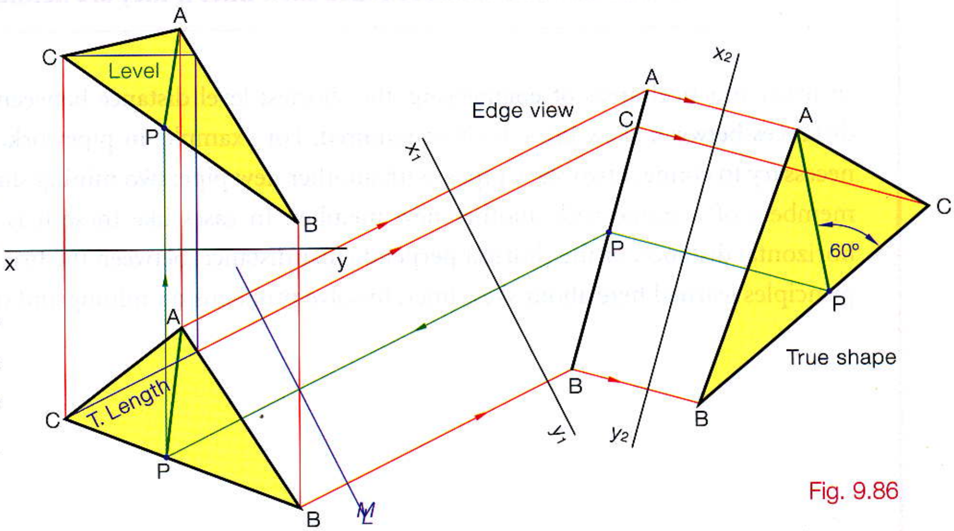


Fig. 9.86

Given the coordinates of a plane ABC. Draw the projections of a line on the plane ABC, 70 mm long, which starts at A and ends on edge BC. Also find the true angle between AB and BC. Fig. 9.87

A = 205 80 25
B = 175 105 55
C = 130 65 20

(1) Draw the plan and elevation of the lamina.
(2) Find the true shape of the plane by first getting the edge view and viewing this perpendicularly.
(3) On the edge view the required angle may be marked.
(4) With centre A and radius 70 mm scribe an arc to cut edge BC. Draw the required line.
(5) Project back through the views, Fig. 9.87.

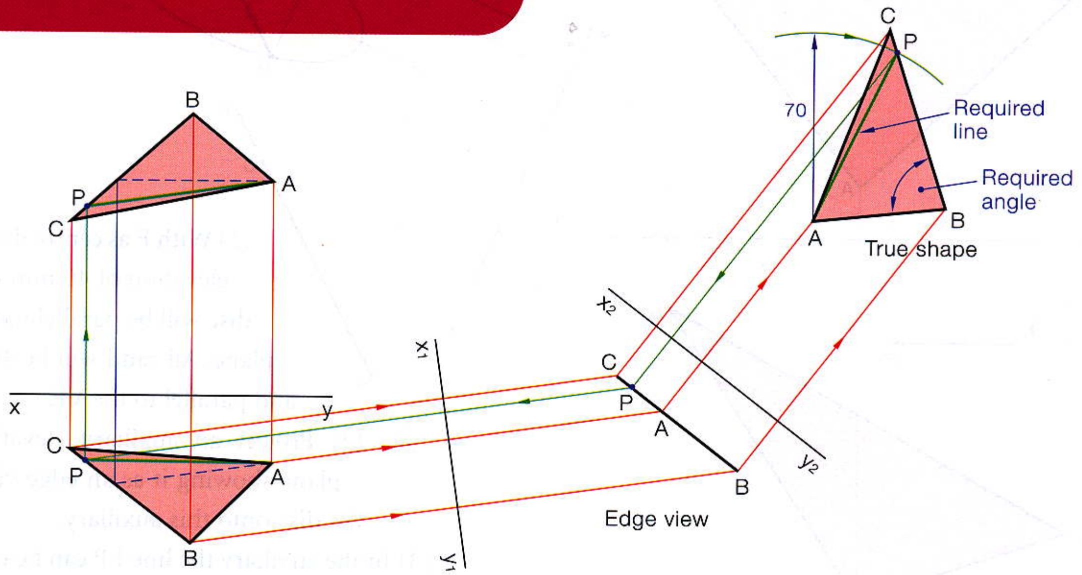


Fig. 9.87

HIGHER LEVEL

Skew Lines

Definition: Two lines are called skew lines if they are neither parallel nor intersecting.

In many practical areas of engineering, the shortest level distance between skew lines or the shortest perpendicular distances between skew lines, is often required. For example, in pipework, mining, structural frames etc., it is often necessary to connect two skew pipes, with another new pipe; two mining shafts, with another new tunnel; or two skew members of a frame with another new member. In cases like these it is of great advantage to know the shortest horizontal distance, or the shortest perpendicular distance, between the two elements. At a later stage we will apply the principles learned here about skew lines, to solve problems on mining and on the hyperbolic paraboloid.

If we produce a plane that contains one of the lines and has an edge that is parallel to the other line, then an edge view of that plane will show both lines as parallel.

To find the shortest horizontal distance between two skew lines AB and CD. Fig. 9.88

A = 45 55 5 B = 105 80 40

C = 30 100 45 D = 85 15 15

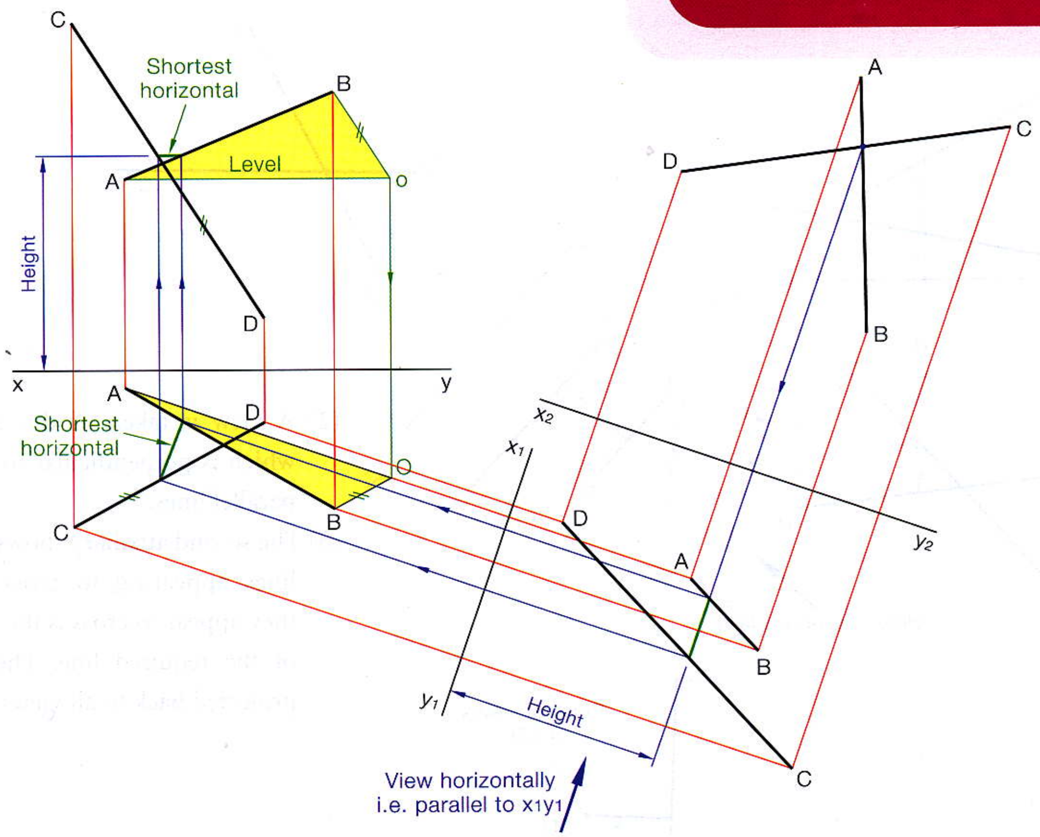


Fig. 9.88

(1) Draw the plane to contain AB and be parallel to CD. Draw a level line from A in elevation. From B draw a line parallel to the other skew line CD. These two lines intersect at O. This completes the plane in elevation.
(2) Drop O to plan.
(3) From B in plan draw a line parallel to CD in plan. This line intersects the line dropped from O in elevation to give point O in plan.
(4) Join O back to A thus completing the plan of the plane.
(5) An auxiliary elevation viewing along AO will show both lines as parallel.
(6) Project a second auxiliary plan by projecting horizontally, i.e. parallel to the x_1y_1. Both lines appear to cross. Where they appear to cross is the location of the shortest horizontal line.
(7) Project the line back through the views as shown.

HIGHER LEVEL

To find the shortest distance (shortest perpendicular distance) between two skew lines.
Fig. 9.89
A = 170 5 55 B = 250 20 65
C = 175 55 90 D = 230 25 30

(1) The initial part of this problem is solved in the same way as the previous example up to the stage of projecting an auxiliary showing the two lines appearing parallel.

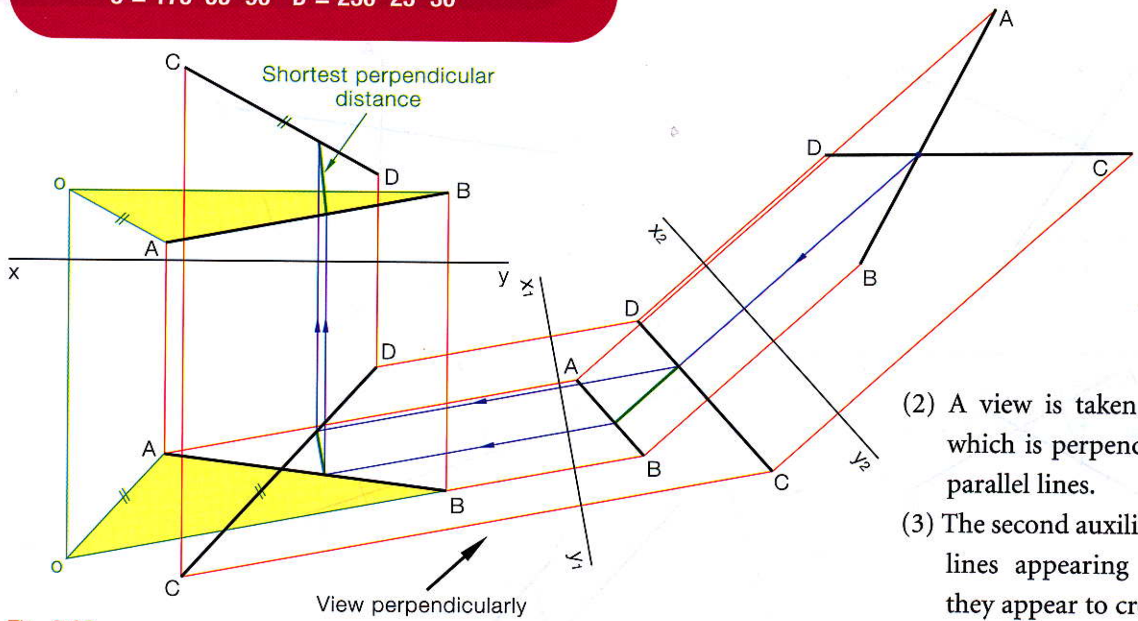


Fig. 9.89

(2) A view is taken of this auxiliary which is perpendicular to the two parallel lines.

(3) The second auxiliary shows the two lines appearing to cross. Where they appear to cross is the location of the required line. The line is projected back to all views.

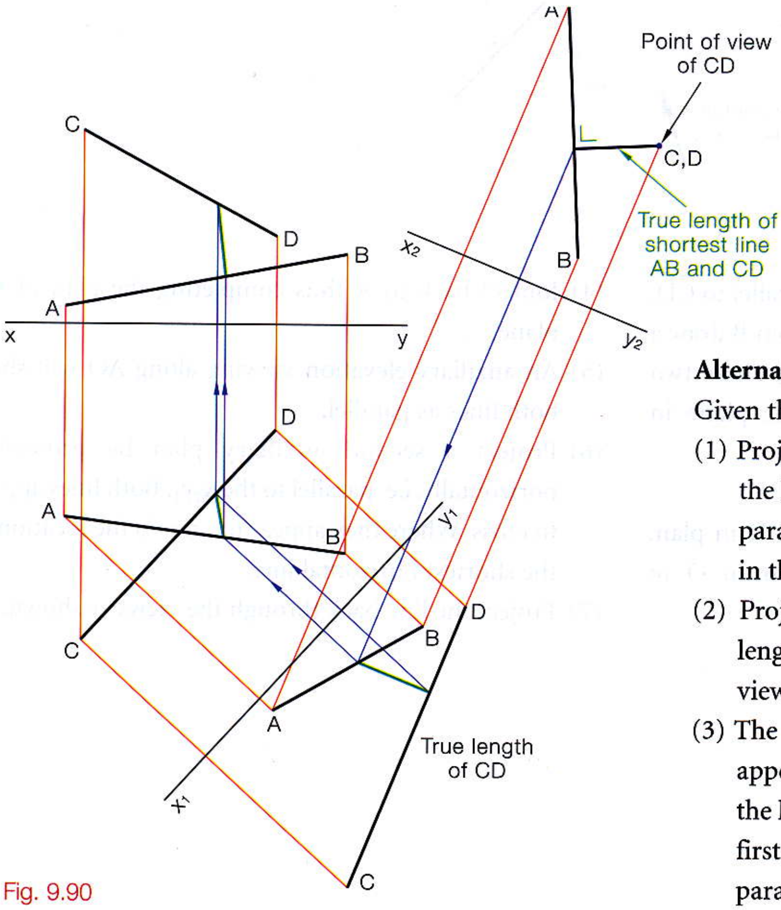


Fig. 9.90

Alternative Method: Line Method

Given the same problem.

(1) Project an auxiliary from plan that will show one of the lines as a true length. Fig. 9.90 shows x_1y_1 drawn parallel to CD thus showing line CD as a true length in the auxiliary elevation.

(2) Project a second auxiliary viewing along the true length line. This new auxiliary shows CD as a point view.

(3) The shortest line between two skew lines will always appear as a true length in a view that shows one of the lines as a true length. When projected back to the first auxiliary the shortest line must therefore be parallel to the x_2y_2 line.

(4) Project back to all views.

HIGHER LEVEL

Given the coordinates of the centre lines of two 15 mm diameter pipes. Determine the clearance between them using the line method. Fig. 9.91

A = 125 55 80 B = 175 10 10

C = 100 25 10 D = 165 75 50

The construction is as in the previous example. The pipes need only be drawn in the secondary auxiliary view.

(1) Draw the plan and elevation of the centre lines.

(2) Find the true length of one of these, e.g. CD, by auxiliary projection.

(3) Project a second auxiliary viewing along the true length CD.

(4) A point view of centre line CD is found. Draw in the pipe details which will show clearly the clearance between the pipes.

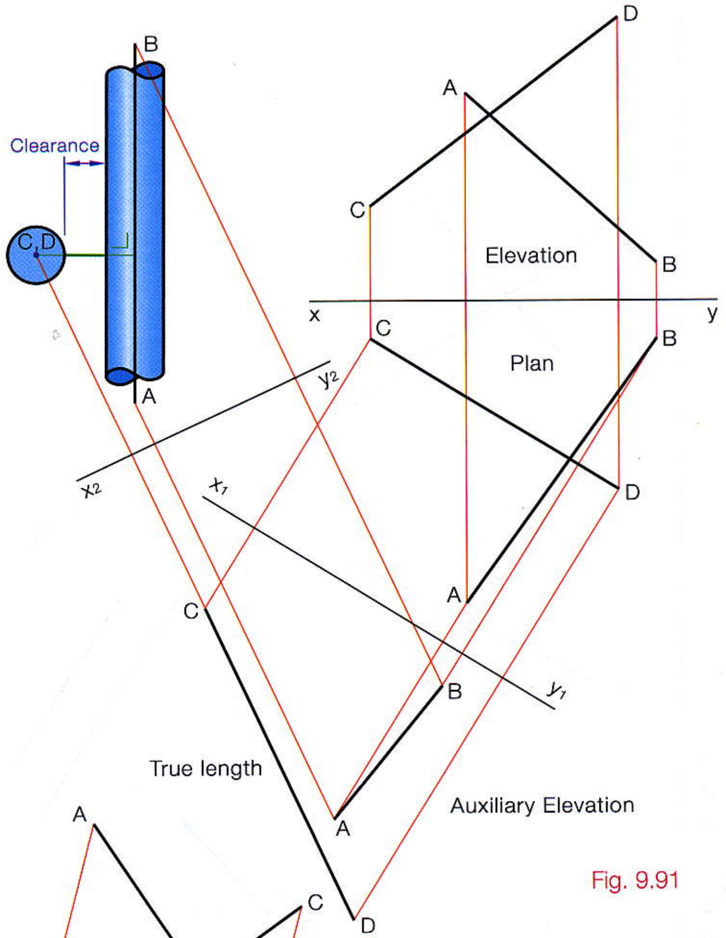


Fig. 9.91

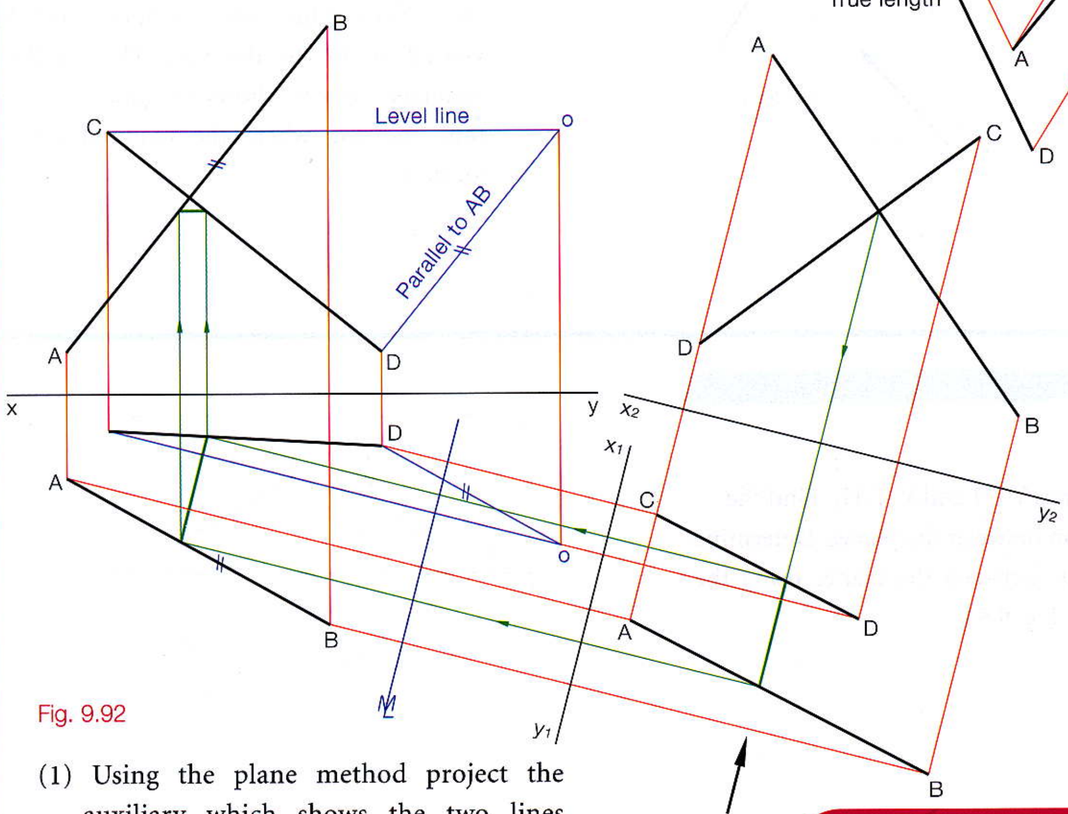


Fig. 9.92

(1) Using the plane method project the auxiliary which shows the two lines appearing parallel.

(2) View parallel to x_1y_1 (horizontally) for the second auxiliary. The lines appear to cross which is the location of the required brace.

(3) Project back through the views.

Given the coordinates of two struts. Show the projections of the shortest horizontal brace strut between them. Fig. 9.92

A = 30 8 16 B = 80 70 44

C = 38 50 7 D = 90 8 10

Given the coordinates of two skew lines. To find the shortest distance between them at 30° to the HP. Fig. 9.93

A = 120 9 33 B = 48 60 69

C = 60 25 16 D = 96 65 65

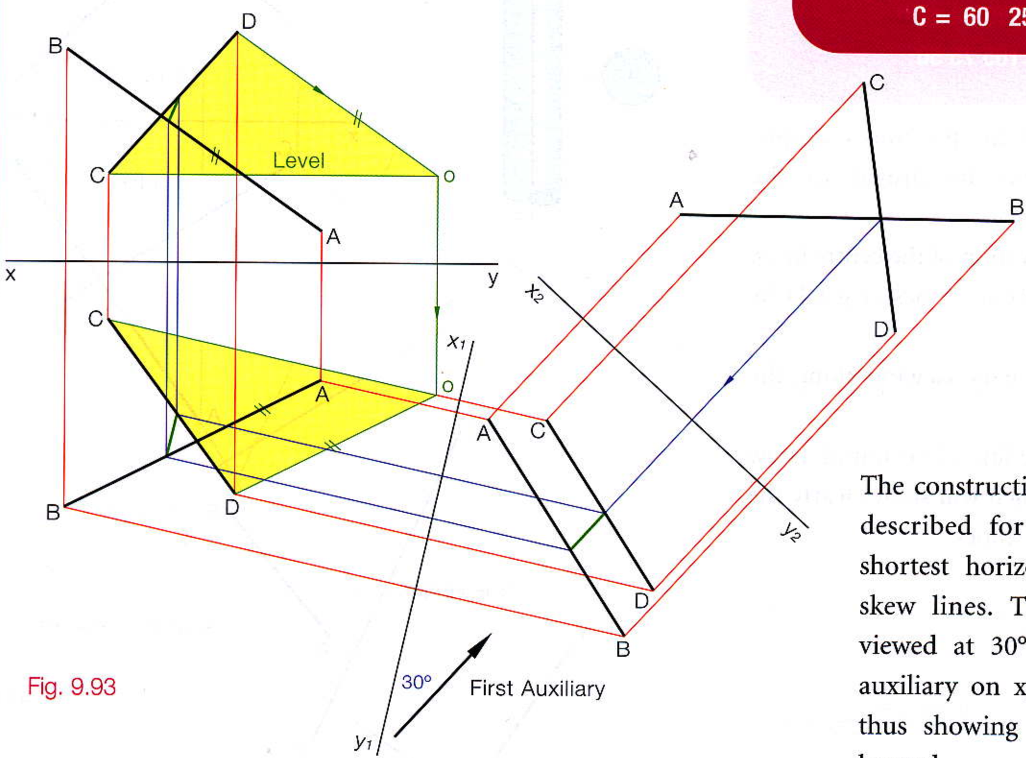


Fig. 9.93

The construction is nearly identical to that described for shortest perpendicular or shortest horizontal distance between two skew lines. The first auxiliary must be viewed at 30° to the x_1y_1. The resulting auxiliary on x_2y_2 shows the lines crossing thus showing where the required line is located.

Activities

DIHEDRAL ANGLE

Q1. Given two planes VTH and $V_1T_1H_1$. Find the line of intersection between the planes. Determine the dihedral angle between the planes using the triangle method, Fig. 9.94.

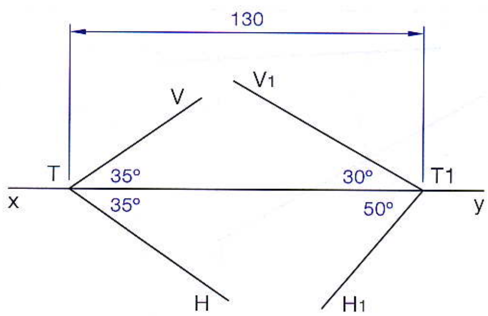


Fig. 9.94

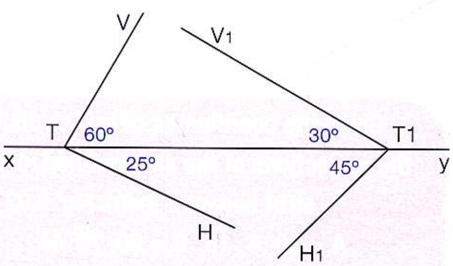


Fig. 9.95

Q2. Given two planes VTH and $V_1T_1H_1$. Find the line of intersection between the planes. Determine the dihedral angle between the planes using the point view method, Fig. 9.95.

HIGHER LEVEL

PLANE INCLINATION

Q3. Draw the traces of a plane VTH given the inclination to the vertical plane as 40° and the inclination to the horizontal plane as 70°.

Q4. Draw the traces of a plane VTH given the inclination to the vertical plane as 45° and the inclination to the horizontal plane as 55°.

Q5. Given the plan and elevation of a point P as shown in Fig. 9.96. Find the traces of a plane that contains point P and is inclined at 55° and 40° to the HP and VP respectively.

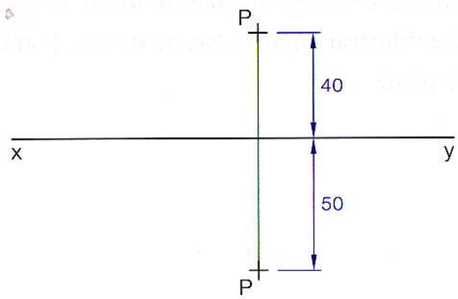


Fig. 9.96

Q6. Point P is to rest on an oblique plane which makes an angle of 60° to the HP and 50° with the VP, Fig. 9.97.

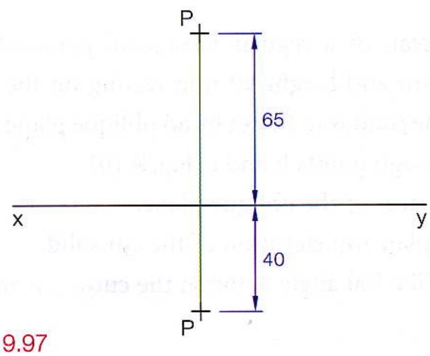


Fig. 9.97

Q7. Given the projection of a line AB. Find the traces of the plane that contains line AB and is inclined at 55° to the HP, Fig. 9.98.

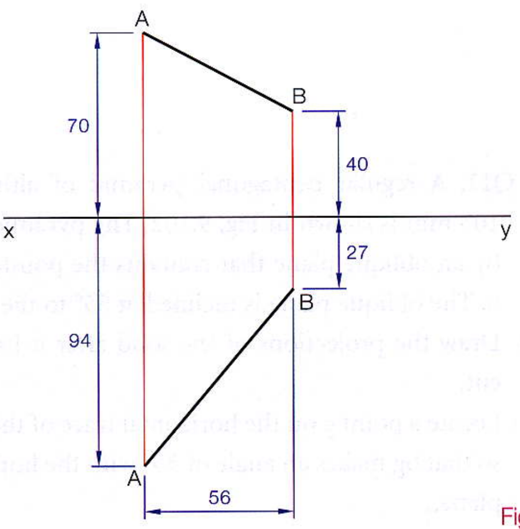


Fig. 9.98

Q8. Fig. 9.99 shows the plan of an oblique rectangular pyramid which has been cut by an oblique plane. Surfaces A, B, C and D are inclined at 50°, 60°, 70° and 35° respectively to the horizontal plane.

(i) Draw the plan and elevation of the cut solid.

(ii) Determine the inclination of the surface D to the vertical plan.

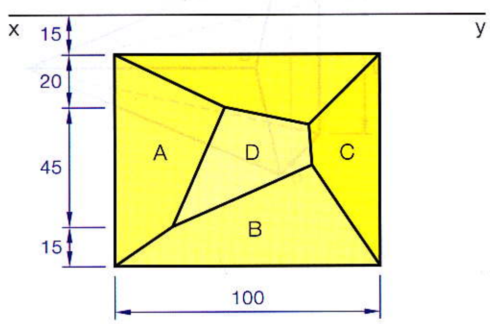


Fig. 9.99

Q9. Fig. 9.100 shows the plan and elevation of a prism with a square base of 60 mm side which has been cut by an oblique plane. The cut surface abcd is inclined at 45° to the horizontal plane and the edge ab is inclined at 20° to the horizontal plane.
(i) Draw the plan and elevation of the cut solid.
(ii) Find the traces of the oblique plane.
(iii) Determine the plane's inclination to the VP.
(iv) Find the dihedral angle between surface S and the oblique plane.

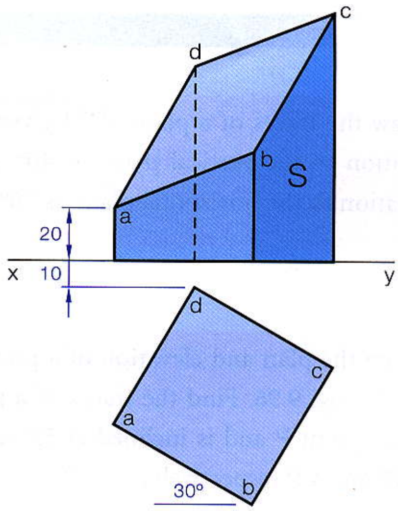


Fig. 9.100

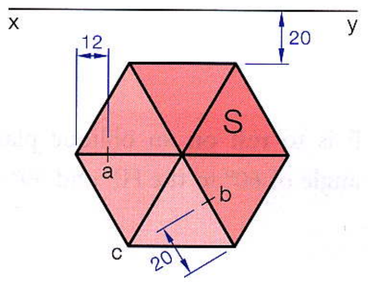


Fig. 9.101

Q10. The plan of a regular hexagonal pyramid of edge 40 mm and height 90 mm resting on the HP is shown. The solid is to be cut by an oblique plane which passes through points b and c, Fig. 9.101.
(i) Find the traces of the oblique plane.
(ii) Draw the plan and elevation of the cut solid.
(iii) Find the dihedral angle between the cutting plane and surface S.

Q11. A regular pentagonal pyramid of altitude 105 mm is shown in Fig. 9.102. The pyramid is cut by an oblique plane that contains the points a and b. The oblique plane is inclined at 55° to the HP.
(i) Draw the projections of the solid after it has been cut.
(ii) Locate a point g on the horizontal trace of the plane so that bg makes an angle of 35° with the horizontal plane.

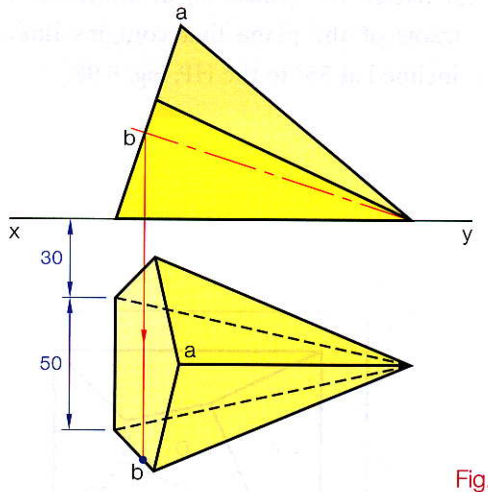


Fig. 9.102

H I G H E R   L E V E L

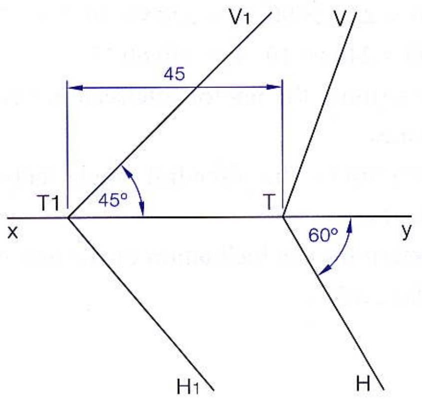


Fig. 9.103

Q12. The traces of two oblique planes are shown in Fig. 9.103. VTH makes an angle of 70° with the horizontal plane. $V_1T_1H_1$ makes an angle of 60° with the vertical plane. A square-based pyramid of base 50 mm and altitude of 105 mm is placed on VTH. One edge of the pyramid's base lies on the HT and one corner touches the vertical plane.

(i) Draw the given traces and the plan and elevation of the pyramid.

(ii) The pyramid is cut by $V_1T_1H_1$. Draw the projections of the pyramid when it is cut by this plane.

Q13. The traces of two oblique planes are shown in Fig. 9.104. VTH makes an angle of 50° with the horizontal plane. $V_1T_1H_1$ makes an angle of 75° with the vertical plane. A pentagonal-based pyramid of base 40 mm and altitude 95 mm is placed on VTH. One edge of the pyramid base lies on the VT and one corner touches the horizontal plane.

(i) Draw the given traces and the plan and elevation of the pyramid.

(ii) The pyramid is cut by $V_1T_1H_1$. Draw the projections of the pyramid when it is cut by this plane.

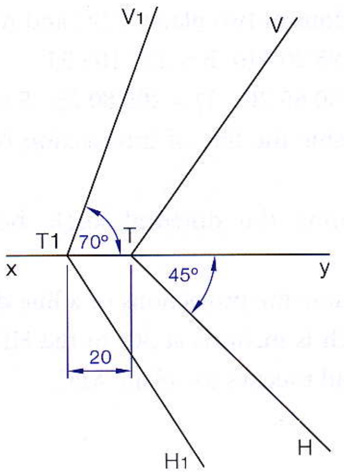


Fig. 9.104

LAMINAR SURFACES

Q14. Given the coordinates of two planes ABC and ABD.

A = 220 15 10 B = 270 35 90

C = 260 65 15 D = 230 60 75

(i) Determine the dihedral angle between the planes.

(ii) Determine the projections of a perpendicular from D to the plane ABC.

Q15. Given the horizontal and vertical projections of two planes ABC and ABD.

A = 130 90 10 B = 215 15 95

C = 165 100 100 D = 245 70 25

(i) Determine the dihedral angle between the planes.

(ii) Show the projections of a line drawn from C to the line AD and which shall be perpendicular to AD.

Q16. Given the horizontal and vertical projections of two planes ABC and ADE.

A = 125 85 30 B = 170 10 100 C = 80 75 65

D = 180 70 70 E = 75 20 105

(i) Determine the line of intersection between the planes.

(ii) Determine the dihedral angle between the planes.

(iii) Draw the projections of a line on the plane ABC, that passes through A and makes an angle of 70° with the edge BC.

Q17. Given the horizontal and vertical projections of two planes ABC and ADE.

A = 225 25 90 B = 220 95 50 C = 175 35 20

D = 240 60 10 E = 140 60 15

(i) Determine the line of intersection between the planes.

(ii) Determine the dihedral angle between the planes.

(iii) Determine the inclination of the line AD to the plane ABC.

Q18. Given the horizontal and vertical projections of two planes ABC and ADE.

A = 195 20 110 B = 175 105 55

C = 130 65 20 D = 205 80 25 E = 120 55 50

(i) Determine the line of intersection between the planes.

(ii) Determine the dihedral angle between the planes.

(iii) Determine the projections of a line drawn from E, which is inclined at 30° to the HP, is 45 mm long and touches the plane ABC.

Q19. Given the coordinates of two planes ABC and DEF.

A = 130 60 5 B = 210 10 90 C = 200 80 25

D = 240 20 30 E = 155 85 5 F = 130 45 60

(i) Determine the line of intersection between the planes.

(ii) Determine the dihedral angle between the planes.

(iii) Find the horizontal and vertical trace of DEF and find its true inclination to the vertical plane.

Q20. Given the coordinates of two planes ABC and DEF.

A = 185 25 15 B = 230 105 60 C = 120 45 75

D = 240 90 10 E = 195 35 95 F = 130 65 30

(i) Find the projections of the line of intersection between the planes.

(ii) Determine the dihedral angle between the planes.

(iii) Draw a line from E which is 35 mm long, is parallel to ABC and the vertical plane.

SKEW LINES

Q21. Given the coordinates of two skew lines AB and CD.

A = 125 85 30 B = 170 10 100

C = 75 20 105 D = 180 70 70

Show the projections of the shortest distance between them using the point view method.

Q22. Given the coordinates of two skew lines AB and CD.

A = 195 20 110 B = 130 65 20

C = 205 80 25 D = 120 55 50

Show the projections of the shortest distance between them using the plane method.

Q23. Given the coordinates of two skew lines AB and CD.

A = 160 90 20 B = 225 5 10

C = 130 25 70 D = 235 30 25

Show the projections of the shortest horizontal line joining them.

Q24. Given the coordinates of two skew lines AB and CD.

A = 225 25 90 B = 240 60 10

C = 220 95 50 D = 175 35 20

Show the projections of the shortest horizontal line between them.

Q25. Given the horizontal and vertical projections of two skew lines AB and CD.

A = 130 90 10 B = 245 70 25

C = 215 15 95 D = 165 100 100

Show the projections of the shortest line between them inclined at 15° to the HP.

Q26. Given the horizontal and vertical projections of two skew lines AB and CD.

A = 140 100 10 B = 130 15 70

C = 160 45 85 D = 205 10 45

Show the projections of the shortest line at 20° to the HP between them.